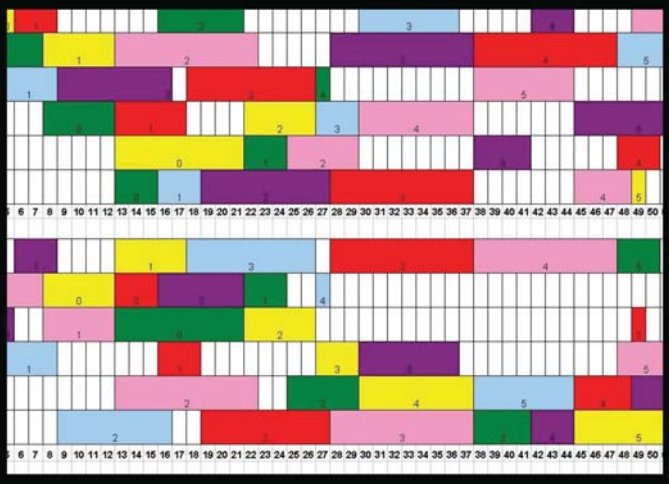

A Gentle Guide to Constraint Logic Programming *via* ECL^{i}PS^{e}

Antoni Niederliński

3-rd edition, revised and expanded

PRACOWNIA KOMPUTEROWA JACKA SKALMIERSKIEGO

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3rd edition, revised and expanded

Antoni Niederliński

pkjs.com.pl

Gliwice 2014

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"Sweet are the uses of adversity!"
William Shakespeare (1564-1616), *"As You Like It"*

"Alle Beschrȃnkung beglűckt."
Arthur Schopenhauer (1788-1860), *"Parerga und Paralipomena"*

"What good are books without pictures and stories?"
Lewis Carroll (1832-1898), *"Alice in Wonderland"*

" Three friends, a Politician, a Doctor and a Mathematician, started on a summer walk-out in the enchanting Silesian Beskidy Mountains, when the Politician noticed a single black sheep in the middle of a grassland. 'All Silesian sheep are black', he remarked. 'No, my friend', replied the Doctor, 'Some Silesian sheep are black'. At which point the Mathematician, after a few second's thought, said blandly: 'In the Silesian Beskidy Mountains, there exists at least one grassland, in which there exists at least one sheep, at least one side of which is black.'"
Anonymouse

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Foreword

0.1 Main assumptions

This is to be a painless introduction into an exciting software technology named *Constraint Logic Programming*, in the sequel abbreviated by *CLP*. The book aims to teach modeling decision problems and solving them using CLP. It addresses the needs of all interested in quickly finding feasible and optimum solutions to combinatorial and continuous decision problems using a well-established tool. It serves to create a basic foothold on CLP for all those wishing to get some operational experience of using it before eventually dwelling into more advanced realms of theory. Therefore:

- it starts with an introduction to CLP's predecessor - the Prolog language. It is the first language containing in a nutshell the basic ideas of declarative programming later developed and extended in CLP languages;
- the book is based on a series of extensively commented examples of increasing difficulty. The Author strongly believes that *an ounce of application is worth a ton of abstraction*[1]. He believes that the best way to learn and master advanced abstractions (Prolog and CLP are full of them) is by seeing them applied to concrete examples. Examples - especially in a logic-saturated discipline - are easier to understand by beginners than theories;
- the book presents basic ideas and methods of *CLP*, the emphasis being not on theory but on intuitive understanding. Obviously, not each student with interest in CLP intends to make a M.Sc. or Ph.D. in CLP. Most of them just want to know what can be done with CLP, and how. So this

[1] This is sometimes referred to as Booker's Law.

book is not addressed to Ph.D candidates, although it seems that most of them could profit from reading it before plunging into more advanced, mathematically-saturated texts;

- all examples discussed are running under one of the most popular and intensively supported CLP platforms, the

$$ECL^iPS^e\ Constraint\ Programming\ System\ \ (ECL^iPS^e\ CPS)$$

platform (see [ECLiPSe-10]), freely available under *Cisco-style Mozilla Public License* from `http://www.eclipseclp.org/`.

A survey of some of the earlier tools for solving *CSP* and *OCSP* may be found in [From-94].

The impetus of this book goes back to a series of lectures and projects on Prolog and CLP, run in the years 1984-2007 at the Faculty of Automatic Control, Electronics and Computer Science of the Silesian University of Technology in Gliwice, Poland, and in the years 2008-2013 at the Faculty of Informatics and Communication of the University of Economics in Katowice, Poland. The first teaching assignments made use of the platforms *Visual Prolog* and *CHIP*, the last one - of ECL^iPS^e.

The authors educational experience in teaching Prolog and CLP convinces him that a major stumbling block for those learning it is *modelling*, i.e. translating verbal problem statements into Prolog or CLP programs. This can be dealt with by a series of stepping stones leading the learner through a broad range of verbal problems of increasing complexity, translated into Prolog or CLP programs. To practice the art of translation, sets of unsolved problems are provided as well. Thus the core of the book are examples: most of the book is devoted to presenting them, discussing them and solving them. The programs that solve them are build using a broad range of various powerful "black boxes" referred to as *built-in predicates* and embedded into the ECL^iPS^e platform: they have a precisely defined functionality, the user always knows what to feed them and what to obtain in return, but their algorithmic mechanism - being part of the excluded theory - is hidden. The interested reader may find it in a number of theoretically-oriented books and publications, e.g. [Apt-03], [Apt-07], [Bartak-10], [Bratko-01], [Dechter-03], [Jaffar-94], [Marriott-98], [Rossi-06], to mentions just a few. An extensive in-depth animated and multi-version digital CLP lecture series for the ECL^iPS^e platform has been presented by Simonis ([Simonis-10]). It provides as well a number of interesting examples.

The art of translating real-world problems into Prolog or CLP programs is best learned using puzzles. However, solving puzzles using Prolog or CLP is not only an excellent exercise in learning modelling. The Author fully agrees with the ideas advocated by Michalewicz (see [Michalewicz-07] and [Michalewicz-08]):

1. Puzzles are educational, as they illustrate many useful (and powerful) problem-solving rules in a very entertaining way.

2. Puzzles are engaging and thought-provoking.

3. It is possible to talk about different techniques (e.g. simulation, optimization), or application areas (e.g. business, management, industrial engineering, finance) and illustrate their significance by discussing some simple puzzles.

What is perhaps more important is that some of the main business, management and industrial combinatorial applications of CLP languages, like resource allocation, timetabling, crew rostering, scheduling, planning, vehicle routing and a multitude of others, are just *mega-puzzles* or *giga-puzzles* with a very large number of variables; to get a sure foothold for starting to solve them, it seems necessary to learn the *CLP-way-of-thinking* and master some basic techniques by solving a series of *micro-puzzles* first.

Last but not least, nowadays a student textbook has to compete for the students time and attention with the Internet, computer games, social activities, and a number of other distractions. So it should not bore the student stiff. A good lecturer is expected to say from time to time something unusual, something paradoxical, some joke, just for the sake of keeping students from falling asleep and alerting their minds. I think the same applies to textbooks. They should not be dull. Here puzzles come in handy as excellent vehicles for introducing moments of relaxation into prolonged intellectual exertions.

0.2 What is in the book?

The contents of the book are organized as follows:

The initial Chapter 1 presents an introduction to general ideas underlying CLP. There the basic notions of *Constraint Satisfaction Problems* (CSP) and *Constraint Optimization Problems* (COP) are defined and illustrated. The concept of *constraint* as used in the book is explained. Attention is drawn to

the relations between CLP and Operation Research, Artificial Intelligence and Knowledge Engineering.

Chapter 2 (*In the beginning was Prolog*) presents Prolog - the predecessor of all CLP languages. It was the first language that allowed the programmer to specify only what is known about the problem and what goal is to be pursued, while abstracting from mechanisms used to exploit the knowledge. The emphasis is on ideas that were later on developed and extended in CLP languages, but which are perhaps easier to grasp in a more simple setting. The examples presented there belong to four categories, like all other CLP programs:

1. Examples of determining a *feasible state* (FS) in the problem state-space, or simply said, at determining a full descriptions of some situations on the basis of partial but sufficient data. They include a.o. a set of examples about configuring a system consisting of three different components with different costs and different compatibility requirements. Their purpose is to introduce the reader to the basic mechanisms of tree search and constraint propagation as unification. The examples illustrate exhaustive search and backtracking search for finding feasible configurations.

2. Examples of determining an *optimum state* (OS) in the problem state space. This is illustrated by determining the least expensive feasible configuration using *branch-and-bound* search.

3. Examples of determining a *feasible state trajectory* (FST) in the state-space from some well-defined initial states to some well-defined final states. This is illustrated by the well-known *Towers of Hanoi* problem.

4. Examples of determining an *optimum state trajectory* (OST) in the state-space. This is illustrated by a number of well-known maze-walking, river-crossing and jug filling puzzles.

The classes seem to exhaust all possible Prolog and CLP applications. The Author never came across Prolog or CLP problems that could not be accommodated in one of those categories.

All examples in Chapter 2 are using Prolog as provided by the ECL^iPS^e platform, referred to as ECL^iPS^e *Prolog*. Prolog programs have the extension *.pl*. Compiling any Prolog program makes ECL^iPS^e use only those mechanisms and standard constraints that belong to standard Prolog.

The question may well be asked, "Why start with ECL^iPS^e *Prolog*, why not with the much more powerful ECL^iPS^e *CLP*?" All the more so because the mechanism of Prolog (standard backtracking with constraint propagation *via* unification) differs from the mechanism of CLP (enhanced backtracking with constraint propagation *via* consistency techniques). The answers to those objections concentrate on purely tutorial reasons and are as follows:

1. Ideas that were later on developed and extended in CLP languages (like declarativity, constraint propagation, logical inference by search with backtracking, *branch-and-bound*) have their roots in Prolog, and are easier to grasp in the more simple Prolog settings[2].

2. The basic elements of Prolog programs are the same as those for CLP programs.

3. Prolog programs structure closely resembles CLP programs structure.

4. Prolog (in contrast to CLP) may be used to easily program *exhaustive search*, which is a rather inefficient search method and may be used only for simple problems, but is the ancestor of all other search methods and the knowledge of its mechanism promotes understanding of more efficient and advanced search methods.

5. Last but not least - Prolog is readily available on the ECL^iPS^e platform.

Many examples solved in Chapter 2 are again invoked in later chapters to show their solution with the help of more advanced CLP mechanisms.

All examples in Chapters 3,...,6 are using CLP as provided by the ECL^iPS^e platform, referred to as ECL^iPS^e *CLP*. CLP programs have the extension *.ecl*. Compiling any CLP program makes ECL^iPS^e use only those mechanisms and standard constraints, which are supported by the CLP libraries declared at the head of the program.

The topics presented in Chapters 3,...,6 have been dichotomized into following categories:

1. *Goal*-dependent categories:

   - the goal is to determine *feasible solutions*, which may be given by either *feasible states* (FS) or *feasible state trajectories* (FST) ;

[2]Thew idea is: "Start with the simple, gain mastery, move gradually to the complex".

- the goal is to determine optimum solutions, which may be given by either *optimum states* (OS) or *optimum state trajectories* (OST) .

2. *Built-in*-dependent categories:
   - only *elementary built-ins* are used;
   - *global built-ins* are used as well.

Because both dichotomizations are independent, they give rise to four chapters:

1. Chapter 3 (*CLP with elementary constraints for feasible solutions*) starts with discussing basic differences between Prolog and CLP languages, like differences in domain declarations, differences of backtracking strategies and differences of constraint propagation. Problems that can be solved using constraint propagation only, and problems that need to supplement constraint propagation with search, have been presented, explained and solved. Some of the problems solved in Chapter 2 using Prolog have now been solved using CLP; some new problems, for which a Prolog solution would be quite expensive, are solved as well.

2. Chapter 4 (*CLP with global constraints for feasible solutions*) introduces the notion of *global constraints* and presents properties and applications of three important global constraint used for finding feasible solutions. They are the `alldifferent/1`, `element/3` and `occurrences/3` built-ins. The chapter presents also a discussion of data handling in ECL^iPS^e CLP, with special attention to iterations, practically not used in any Prolog but being of great importance in ECL^iPS^e CLP. The application of data handling predicates for a range of problems has been presented.

3. Chapter 5 (*CLP with elementary constraints for optimum solutions*) shows that the solution of rather varied optimization problems can be obtained using elementary constraints only. Upgrades of the standard *branch-and-bound* approach (as used for Prolog programs) are presented. Basic built-ins (`bb_min/3` - `bb_min/5` and `search/6`), used for implementing *branch-and-bound* in ECL^iPS^e CLP, are presented and applied to range of optimization problems.

4. Chapter 6 (*CLP with global constraints for optimum solutions*) presents properties and applications of two important global constraints used for finding optimum solutions of complex scheduling problems: `cumulative/4`

(`cumulative/5`) and `disjunctive/2` built-ins. They are applied to a range of scheduling problems, starting with job-shop problems (including the famous `MT10` benchmark), and ending with traveling salesman problems.

Chapter 7 (*CLP for continuous variables*) is a departure from combinatorial problems considered in previous chapters. Now an extension of CLP to continuous variables is presented, and all problems discussed are defined for continuous domains. They are either *Continuous Constraint Satisfaction Problems* (CCSP), or *Continuous Constraint Optimization Problems* (CCOP). The chapter starts with highlighting the basic differences between them and the CSP/COP discussed so far. This is followed by a set of CCSP examples concerned with compound interest problems. Next, a set of CCOP examples concerned with linear programming problems is presented. The examples are chosen so as to highlight the fact that - contrary to OR approaches - CCOP does not need problems to be cast into some canonical form.

Each chapter, save the first one, terminates with a set of exercises. Most exercises concerned with solving CSPs are *Internet-born*. They seem to belong to the folklore of puzzle-lovers and most of them have not (to the best of the Authors knowledge) been solved using CLP approaches. Some of them may be found on so many puzzle websites, that to state their whereabouts would be pointless. Most exercises concerned with solving COPs are good old *Operation Research* problems, well known from a number of excellent textbooks and websites. Although most of them have not been solved using CLP approaches, their origins are always cited.

Some remarks concerning *semantic discipline* and *parsimony of vocabulary* have to be made at this place. The Author tried hard to avoid any synonyms, well aware that they are a curse for any diligently studying beginner. This approach may even be defended by such fundamental principle as *Ockham's razor*[3].

Unfortunately, this inclination brought the Author sometimes into conflict with established ECL^iPS^e terminology. The most important case is perhaps the one involving terms "predicate", "function", and "compound term". Any function is a relation (although not all relations are functions), relations are de-

[3]The following Latin saying "*Entia non sunt multiplicanda praeter neccessitatem*" meaning "*Entities should not be multiplied beyond necessity*", attributed to the 14th-century English logician, theologian and Franciscan friar Father William of Ockham (1285–1349), is known as *Ockham's razor*. The saying is often used as a heuristics to choose between two hypotheses explaining the same observations equally well, but having different "degrees of complication".

scribed by predicates, so the term "predicate" seems to obliviate the term "function": there are no discernable operational differences between their meaning. So the term "function" will not be used further. Similarly, the name "compound term" denotes either a "predicate" or a "structure". No "compound terms" may be found that are not defined as "predicates" or "structures".

0.3 How to use the book?

The reader is encouraged to solve all examples discussed in the book, in their original version and in any conceivable modification, as well as examples provided in the *Exercise* sections. While doing this it should be remembered that learning CLP is essentially a mix of trial and error with explorations aimed at finding why something doesn't work. While learning CLP the old "ski principle" holds: *if you don't fall, you won't learn*! Learning CLP gives ample opportunities for making mistakes, from simple formal mistakes detected by $ECL^iPS^e/, CLP$, to sophisticated, difficult to diagnose, logical mistakes.
The basic software needed, available on the ECL^iPS^e website

`http://www.eclipseclp.org/`,

has to be downloaded. At the time of writing this book (2013), the software available was in the `Release 6.0_201` file dated 19-Feb-2013. The user is encouraged to read the installation notes, `README_UNIX` for Unix/Linux systems, or `README_WIN.TXT` for Windows systems. The download results (for Windows systems) in the directory

`Program Files\ECLiPSe6.0`,

in the `C` catalogue, and from the `ECLiPSe6.0` directory the `TKEclipse6.0` icon may be put onto the desktop, (Figure 1).

Figure 1: The $TKECL^iPS^e$ icon

A click on the icon makes the Main Window of ECL^iPS^e to appear, see Figure 2.

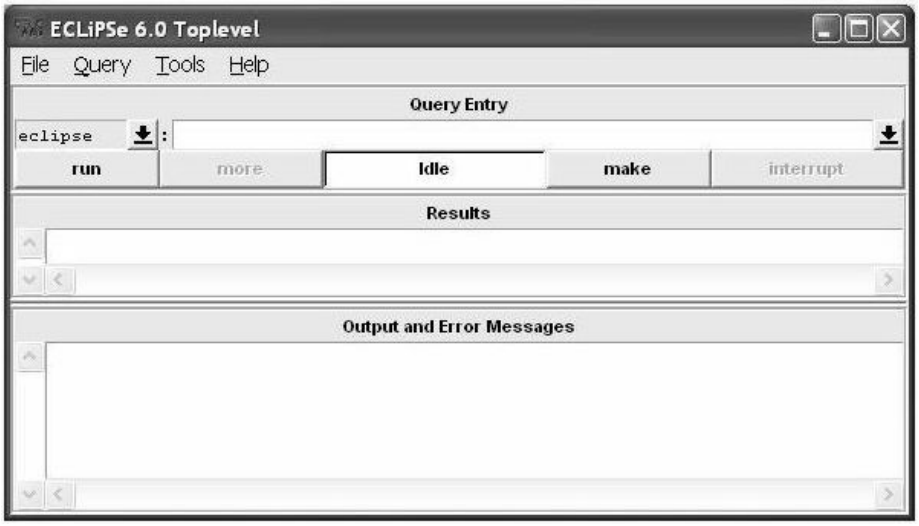


Figure 2: Main Window of ECL^iPS^e

The option *File* makes available the menu from Figure 3

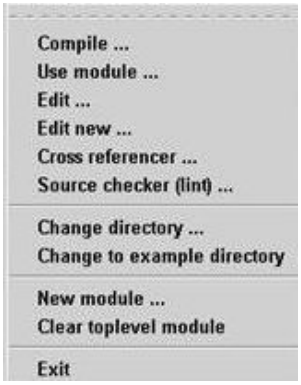


Figure 3: *File* menu

The *Compile* option from this menu enables the loading and compilation of any program with extension *.pl* (a Prolog program) or with extension *.ecl* (a CLP program). All programs presented in this book are activated by inputting the universal query `top.` Because `top` is used for all programs, it is worthwhile to clean the memory before using it for another program. This can be done by activating the option *Clear toplevel module*.
The option *Help* makes available the menu from Figure 4.

Figure 4: *Help* menu

Its most often used sub-option is *Full documentation...*, which makes available a broad range of documents as shown in Figure 5.

Here the user may find a full list of all standard predicates or *built-ins* (option *Alphabetical Predicate Index*), libraries (option *Constraint Library Manual*), a tutorial (*ECLiPSe Tutorial Introduction*) and *User Manual*. Easy immediate access to all definitions is the reason standard predicates won't be defined (save some important and difficult ones) in this book. The compilation of any *.ecl* program makes use of ECL^iPS^eCPS libraries declared in the program head. For all standard predicates the *Alphabetical Predicate Index* documentation provides data about libraries needed for supporting those predicates.

Short CLP programs may also be run using the *command mode* with the path leading to `eclipse.exe`. Then entering the command `eclipse` in the command window invokes the command mode (see Figure 6), prompting the

user to paste a small CLP program and activate it with `ENTER`. The command mode may be also used to run any CLP program by clicking its `.ecl` name.

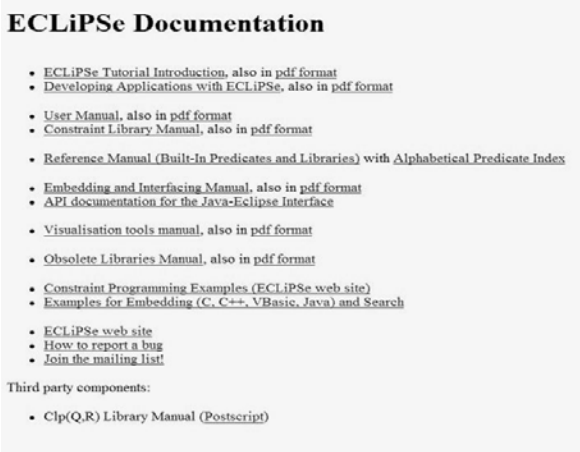


Figure 5: Documents available through *Full documentation...*

0.4 Acknowledgments

The Author did not have the good luck to meet - early in his career - people knowledgeable in Prolog or CLP, and enthusiastic about them. Having a control-engineering academic background and position, his education on Prolog and CLP was entirely self-inflicted, with the help of books and papers he read, and software he used; they were authored mostly by people he has never even met. Nevertheless, it seems they deserve to be given credit for the inspiration they provided by their writing and their software.

The first and most influential book to be mentioned was authored by K.L. Clark and F.G. McCabe (see [Clark-84]). It was a splendid tutorial, which caused the Author to get Prolog-infected. He went through all their examples in the early 80-ties, using a SINCLAIR ZX Spectrum microcomputer for a `microProlog` interpreter running under `CP/M`, and distributed on audio cassette tapes. It was with the help of this software that the Author started running courses on Prolog for students at the "Automation and Robotics" stream at the

Faculty of Automatic Control, Electronics and Computer Science of the Silesian University of Technology in Gliwice, Poland.

Later, the Author started to use Turbo-Prolog and its *PDC*[4] - developed descendants, *PDC Prolog* and *Visual Prolog*. *Visual Prolog v. 5.2* has been used to design four expert system shells *rmes*, which are the subject of another book. The Author had innumerable opportunities to marvel at the quality of software produced by *PDC* people while working on *rmes* and while teaching Prolog.

Next, while staying with Professor Mietek Brdyś at the University of Birmingham, UK, the Author first came across CLP by reading the excellent and inspiring book by van Hentenryck ([van Hentenryck-89]). This was followed by using the CHIP v.5.2 software for a course on CLP for students majoring in "Computer Controlled Systems". The Author continued to use CHIP for a number of years and was always impressed by its elegance and power. It's main drawback is the price of the software and lack of public-domain or educational versions.

In 1997 the Author came across two excellent websites by Roman Barták from Charles University, Praha, Czech Republic, see [Bartak-10] and [Bartak-10a].

[4]PDC stands for Prolog Development Center, a Copenhagen based software company.

Figure 6: Running ECL^iPS^e in command mode

That was the beginning of fruitful and inspiring friendly contacts. Professor Bartáks insightful talks at a series of "Workshops on Constraint Programming for Decision and Control", run at the Institute of Automation of the Silesian University of Technology in Gliwice, Poland, during years 1999-2005, was an important boost to the Authors work and the work of some of his Ph.D students as well.

Thanks to the initiative of Professor Jerzy Gołuchowski from the Economic University (EU) in Katowice, the Author had the good chance to pursue his CLP interest at the Chair of Knowledge Engineering (EU), starting 2009 with a series of CLP lectures based on $ECL^iPS^e\,CPS$. The Author is indebted to Professor Gołuchowski for relieving him from chores like attending meetings about planning, proposals and policy, and from activities like fund raising, consulting, interviewing. The writing of this book has been also inspired by the interest shown by colleagues from the Chair.

The Author is grateful to his former Ph.D. students, especially Dr. Łukasz Domagała and Dr. Wojciech Legierski, for many fruitful discussions on CLP and interesting examples of CLP.

His former colleagues from the Computer Control Group at the Faculty of Automatic Control, Electronics and Computer Science of the Silesian University of Technology in Gliwice, Poland: Drs. Jerzy Mościński, Dariusz Bismor and Krzysztof Czyż, helped him a lot by explaining peculiarities of *Miktex* used for writing this book. He owe thanks to Dr. Jacek Loska for constantly keeping his hardware and software alive and up-to-date.

The Author is grateful to Hakan Kjellerstrand from Sweden, who - at a rather short notice - read the typescript of the first edition of the book and provided valuable feedback on many topics of importance.

Last but not least, the work done on this book would be unthinkable but for the understanding and support of the Author's Wife Teresa, who patiently tolerated for years his prolonged spiritual absence at home.

Obviously, the Author is solely responsible for all mistakes and misrepresentations that may eventually be found in this book.

Finally, the Author offers his deepest apologies to whomever he has neglected to mention.

Gliwice, January 2014

Chapter 1

Introduction

1.1 What is Constraint Logic Programming?

Constraint Logic Programming (CLP) is a tool for solving *constraint satisfaction problems*(CSP). For the important combinatorial case CSP is characterized by following features[1]:

- a finite set S of integer variables $X_1, ..., X_n$, with values from finite *domains* $D_1, ..., D_n$;
- a set of constraints between variables. The *i-th constraint* $C_i(X_{i_1}, ..., X_{i_k})$ between k variables from S is given by a *relation* defined as subset of the Cartesian product $D_{i_1} \times, ..., \times D_{i_k}$ that determines variable values *corresponding* to each other in a sense defined by the problem considered . Quite often the constraints may not be stated as relations, but by equations, inequalities, subroutines etc. The number of variables present in a constraint is named *arity* of this constraint. A constraint for a single variable is *unary*, for two - *binary*, for `k > 2` - *k-ary*;
- a CSP *solution* is given by any assignment of domain values to variables that satisfies all constraints. It may be *non-unique* or *unique*.
- a CSP *solution* may additionally minimize or maximize an *objective function*. Then it is usually referred to as *constraint optimization problem*

[1]This not so gentle (but general and precise) definition will hopefully be more obvious and lucid after working through some examples from chapters 3,...,6.

(COP), and its solution as *optimum* solution.

Let us spend a moment unpacking these features. This is best done by simple examples, see Figure 1.1 for a non-unique solution, Figure 1.2 for a unique solution and Figure 1.3 for an optimum solution.

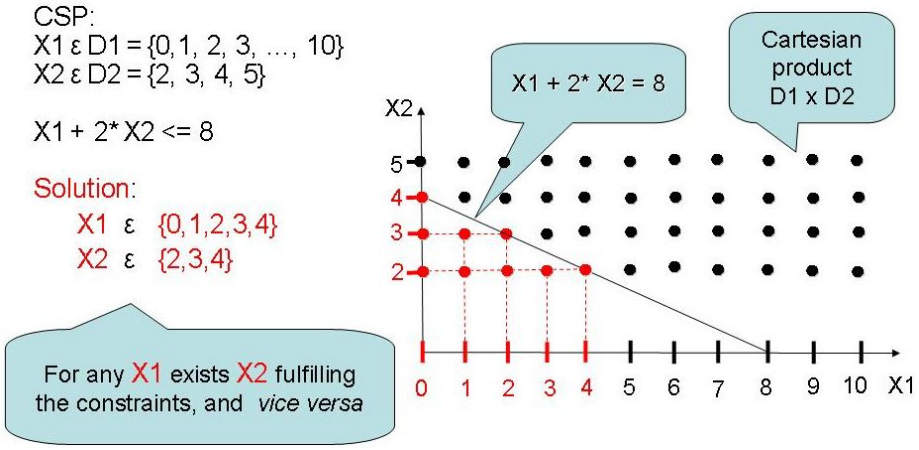


Figure 1.1: Simple CSP example with non-unique solution.

Readers familiar with *Integer Programming* will recognize the problem from Figure 1.3 as such, see chapters 5 and 6.

1.2 Why use Constraint Logic Programming?

A salient feature of combinatorial *CSP* and *COP* is that all variables take values from *finite* domains. It follows that in theory any *CSP* and *COP* can either be shown to have no solution or be solved using an algorithmically simple *exhaustive search* or *direct enumeration*[2] approach. Therefore the wisdom of developing special tools for such problems may be questioned. Why are present-day tools for solving combinatorial *CSP* and *COP*, outlined in this book, better than exhaustive search? The answer to this question is as follows:

[2]I.e. generating one by one all *n-tuples* of the Cartesian product of variable domains and testing whether they satisfy all constraints of the problem.

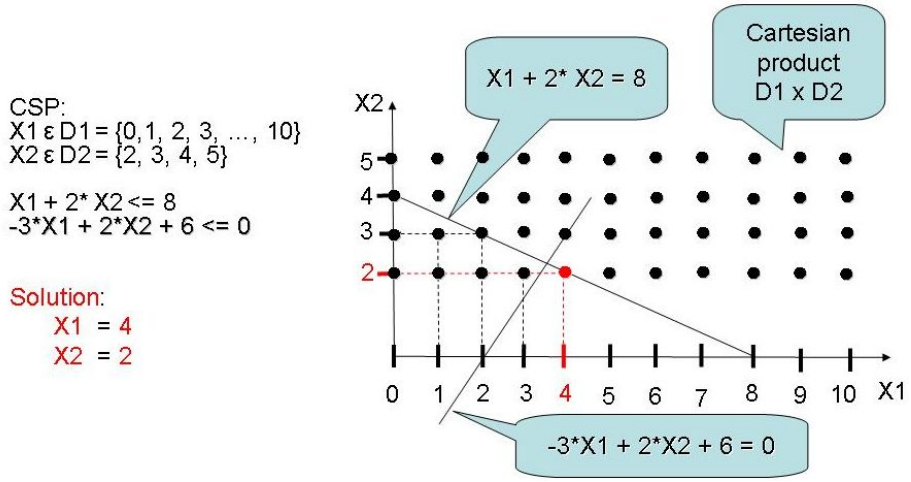


Figure 1.2: Simple CSP example with unique solution.

1. Because of the numerical *effectiveness* of determining *CSP* and *COP* solutions, which for exhaustive search and large numbers of variables is very bad indeed. It means that the number of enumerations needed to get those solutions may be exorbitant. E.g. consider a particular case of `30` variables, each one of them assuming `100` different values[3]. The total number of 30-variable sets (constituting what is usually called the *state space*) amounts to $100^{30} = 10^{60}$. Because humans are notoriously bad at understanding how *large* is a *large number*[4], it is worthwhile to convert such numbers into time. Suppose that the evaluation of a particular set of constraints for any of the 30 variable sets will take a microsecond. Evaluating all sets will take 10^{54} seconds or $10^{54}/3600$ hours or $10^{54}/(3600 * 24 * 365)$ years. Obviously:

   $$10^{54}/(3600 * 24 * 365) > 10^{54}/(10000 * 100 * 1000) = 10^{45},$$

   so exhaustive search would need more than 10^{45} years, i.e. a time considerably in excess of the estimated age of the universe ($1.4 * 10^{10}$ years). This is what we mean by *combinatorial explosion*, or what Richard Bellman (see [Bellman-61]) referred to as the *curse of dimensionality*. CLP lan-

[3] This is really a small problem compared with e.g. average university timetabling problems.
[4] This is best seen while watching budgetary discussions in any Parliament.

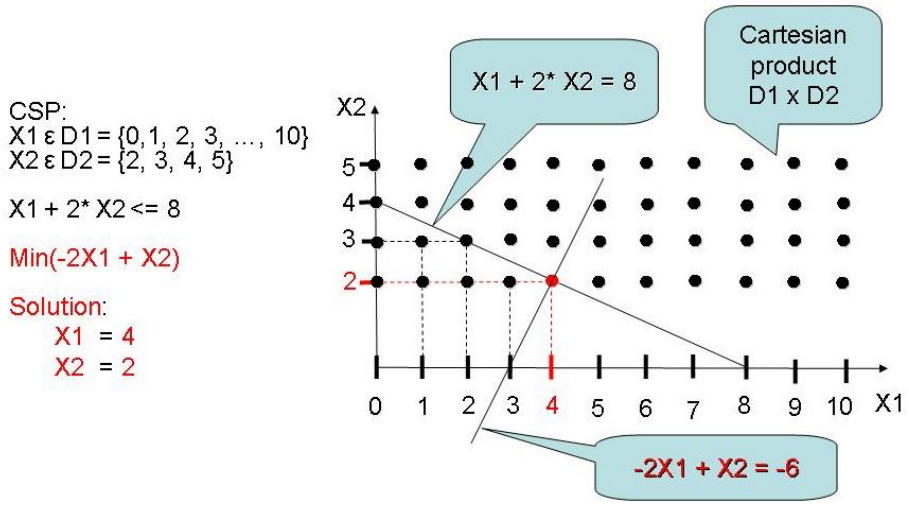


Figure 1.3: Simple COP example.

guages cope (to some extent, not entirely) with such problem by early and judicious use of problem constraints[5] and use of implicit feasible problem-specific heuristics in order to substantially decrease the number of sets to be tested.

2. Because of the *declarativity* of Prolog and CLP programs. *Declarativity* means that a *properly formalized* description of the solved problem is tantamount to the program solving the problem. It is contrasted with *imperativity* (*procedurality*) based on designing algorithms needed to solve problems. Declarativity means further that while using Prolog or CLP languages no algorithms for problem solving need to be designed. The algorithms, which are of course necessary for any computer-based problem solving, have been embedded into Prolog or CLP compilers. To simplify a bit, it may be stated that the art of Prolog and CLP consists in designing such problem descriptions that are understood by Prolog or CLP language compilers, and that ensure an efficient determination of the solution[6]. However, it should be kept in mind that non-trivial complete Prolog

[5] Those are the Shakespearean sweet uses of adversity.
[6] Although thinking declaratively is considered to be much easier than thinking procedurally

and CLP programs cannot entirely get rid of imperativity since they need to some extent the fixing of order for clauses to be executed, and need commands for data to be imported and messages to be generated.

1.3 What do we mean by 'constraints'?

The term 'constraints" deserves some attention. It is understood to mean anything that limits the freedom of action. Constraints are ubiquitous: any program we write in any language is full of them. However, their meaning in imperative languages (like Pascal, C, C++) differs considerably from their meaning in CLP languages. In imperative languages constraints are *passive*; that means they may be used only if all their variables are grounded, and they are used as tests for choosing the next step taken, see Figure 1.4.

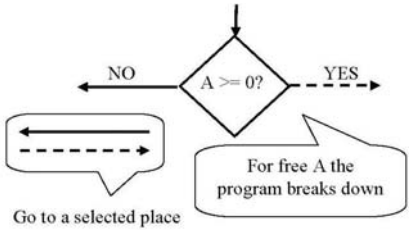


Figure 1.4: A passive constraint example

Constraints in CLP languages are *active*; that means they may be used also if some or all of their variables are free. Active constraints (denoted by various symbols like `#` for finite domains or `$` for real or symbolic domains) are used for initiating a search for such variable groundings that satisfies them, see Figure 1.5.

(see e.g. [Apt-07]), and declarative programs are easier to understand, develop and modify, it does not mean that using Prolog or CLP techniques is always plain sailing. It simply means that difficulties experienced while producing efficient algorithms are no longer present, but instead a new set of difficulties (luckily less formidable) appear while attempting to design *efficient* declarative programs making judicious use of available *built-ins*. We never get something for nothing.

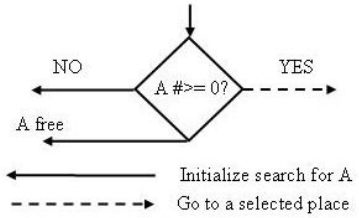


Figure 1.5: An active constraint example

1.4 Constraint logic programming and artificial intelligence

Artificial Intelligence (AI) is usually understood to be this branch of computer science that deals with creating tools for jobs usually considered to need considerable human intelligence, see [Poole-98], [Luger-98] and [Russel-03]. E.g. a time-tabling program for a large university department (see e.g. [Legierski-06]) surely deserves to be considered as such, as it needs to satisfy a variety of curricula, balance a large number of conflicting demands by staff and students, and make best use of facilities available. The label AI is also relevant for programs that support the design of complicated *vehicle routing* tasks for a set of vehicles located in one or more depots, operated by a crew of drivers, having to deliver an assortment of goods from some spatially dispersed warehouses to some spatially dispersed clients in a way that minimizes the total cost of delivery, see e.g. Toth-Vigo [Toth-02]. Solving timetable or vehicle routing problems *manually* can put a high demand on the intelligence of humans doing it, because they need to take into account a huge number of relations, conflicting factors, and trade-offs.

Some authors (e.g. Puget [Puget-08]) are of the opinion that constraint programming is one of the most successful application of Artificial Intelligence. Puget quotes the following achievements of constraint programming for one of the most often met application field, which is scheduling.:

- scheduling operations of a paint shop in a car assembly plant. The paint shop is one of the most critical zones in the process, because whenever a paint color is changed, the shop's machinery must be completely purged;

this is an operation costing both time and money. The developed application has minimized the number of times paints need to be changed in filling customer orders, resulting in considerable savings;

- scheduling production at a large manufacture and marketer of home appliances in order to better match customer demand and reduce response time, while keeping low inventories of finished goods;
- designing multi-constrained time-tables for engineers monitoring on a 24-hour basis all computer and telecommunication systems in a large financial institution.

This set of examples has been considerably extended by Simonis (see [Simonis-10]), who quotes the following interesting applications:

- assembly line scheduling for Mirage 2000 fighter aircraft production;
- various crew rostering systems like personnel planning for the guards in jails or nurses in hospitals;
- production of Belgian chocolates;
- design of advanced signal processing chips;
- design of print engine controller in Xerox copiers;
- assigning ships to berths in container harbor;
- scheduling Bandwidth on Demand.

Researchers, designers and users of AI products have always been confronted with the need to solve difficult complex problems, see e.g. [Luger-98]. Exactly the same problems are solved using constraint programming technology.

It is also worthwhile to note that the closeness of the connection between *Prolog/CLP* and *AI* has deeper, more fundamental roots. This is so because *AI* as known today may be dated from the failure of the *General Problem Solver* (GPS) project[7].The critical step in solving a problem with `GPS` was the definition of the problem space in terms of the initial state, the goal state to be achieved, and the transformation rules defining feasible moves from state to state. Using an inference method called *means-end-analysis*, `GPS` would determine the

[7]GPS was a computer program created in 1959 by Herbert A. Simon, J.C. Shaw, and Allen Newell at the Carnegie-Mellon University in Pittsburgh, PA, USA.

so called syntactic difference between any initial state and the final state, as well as determine a logical operator that decreases this difference. This strategy proved successful for solving formalized symbolic problem, like e.g. theorems proofs, geometric problems and chess playing. Encouraged by initial success, Newell and Simon made attempts to increase the prowess of `GPS` by incorporating smarter reasoning techniques using more clever search algorithms, and hoping it will eventually allow them to solve real-world problems outside the "find a trajectory in problem space" scheme. By and large, it proved a failure: developing programs that could prove theorems of logic did not seem to provide techniques that could be readily adapted to other tasks. At the end of the day these programs were very smart at logic, but still stupid when it came to anything else. It was then widely recognized that a main characteristic of intelligent behavior was not so much general principles of reasoning applicable to any field of human activity, but rather detailed concrete knowledge of the very narrow areas relevant to the problem solved. It turned out that *for solving real-world problems plenty of relevant problem knowledge is needed, but the necessary logical instrumentation is rather modest.* Because it was impossible to model intelligent behaviour which did not rely both upon specific domain knowledge and sound reasoning, an *AI* paradigm emerged based on the requirement to put both components in any *AI* programs. An obvious next step was to separate (logically, structurally) in *AI* programs those two crucial components: domain knowledge (usually presented in some declarative form and residing in one part of the program) and reasoning available as a service provided by some other program or another part of the entire program[8].

1.5 Constraint logic programming and operations research

Operations Research (OR) is a discipline that aims to calculate optimum or sub-optimum solutions to complex decision-making problems, characterized by some clearly defined *objective function* and limited *resources*. It is basically concerned with optimizing the objective function, i.e. determining its maximum (in case it represents profit or yield) or minimum (in case it represents loss or cost). The objective function depends upon some *decision variables* that

[8]This type of programming, known as *knowledge based programming, is typical for* Prolog *and* CLP*: the program contains domain knowledge relevant to the problem solved, the compiler contains the reasoning system.*

can be manipulated to achieve the aim, see [Winston-94], [Williams-99], and [Taha-08]. Originating in military efforts before World War II, its techniques have developed and turned useful for problems in a variety of industries. The most widely used numerical tools of operations research are known as various kinds (*linear*, *integer*, *mixed*) of *programming*; the term has no connection with computer programming, but has its roots in the history of the discipline. The techniques usually stipulate and need the existence of a *canonical form* of the decision problem: determine

$$\min_{\mathbf{x}} \mathbf{c}^T\mathbf{x}$$

under constraints:

$$\mathbf{Ax} = \mathbf{b}$$

where $\mathbf{x}$ is an n-dimensional column vector of *real* or *0-1* decision variables, $\mathbf{A}$ is an $m \times n$ matrix of reals or 0-1 elements, and $\mathbf{c}$ is an m-dimensional column vector of reals or 0-1 elements. Modern CLP platforms (including ECL^iPS^e) provide efficient *solvers* for this type of problems. What's more, CLP modelling and solving of operation research problems usually do not need the prior transformation of those problems into some canonical form, and provide a large number of *global constraints* that simplify both problem formulation and solution. The CLP- and OR- approaches to solving optimization problems have been compared in a number of insightful publications, see e.g. [Hansen-03] and [Milano-04]. A trend to integrate traditional OR techniques with CLP is also clearly visible, see [Hooker-00] and [Hooker-07].

1.6 Constraint logic programming and knowledge engineering

What do we mean by *knowledge* while speaking about *knowledge engineering*? To explain this lets start with some more simple concepts like *data* and *information*. Quite often they are defined as follows:

- *data* is given by sets of *0-1* vectors staying for numbers, letters, signs, words, pictures, sounds. They originate usually as results of some measurements, human actions or processing of other data. They are represented as *bits*, *bytes*, *words*, *lists*, *arrays*, *records*;

- *information = data + meaning of data + purpose of data*. Information is thus a purpose-oriented meaningful set of data. Information appears as the result of some target-oriented human action. It is stored in *data bases* and *data warehouses*;

- *knowledge = information + goal + ability to use information to reach the goal*. Knowledge consists thus of information relevant to some goal and the ability to process the information in a way that procures the goal. The goal is usually given as some state estimation or decision. Knowledge is represented by *facts*, *rules* and *mathematical models*.

Not so long ago knowledge was considered to be an exclusively human attribute. However, in the last 30 years more and more inroads into the realm of knowledge have been struck by computer technology. They cover knowledge discovery (data mining), knowledge storing (knowledge bases), knowledge representation and knowledge application (reasoning) for some small and well-defined domains. It also became obvious that computer-assisted knowledge discovery and computer-assisted knowledge application may be a source of large economic and social benefits. Those circumstance cumulated in the raise and development of *Knowledge Engineering* as a discipline that forms an umbrella covering all computer-assisted knowledge activities and presents a set of basic concepts to speak about them, see e.g. [Brachman-04] and [Gołuchowski-07].

It so happens that Prolog and CLP excel in almost all those features and activities that are crucial for knowledge engineering. Perhaps the most important is the ability to present knowledge in a declarative form using logic and mathematics, and apply this form for computer assisted reasoning, aiming at proving or disproving some statements. This is behind one of the widespread knowledge engineering applications, namely expert systems (see e.g. [Niederliński-06]) and business rule management systems (see e.g. [Morgan-08], [Ross-03], and [von Halle-02]).

1.7 Classifying problems

From a tutorial point of view similarities between verbally different problems and the resulting similarities of programs that solve them are important. In order to exploit them effectively a classification of problems solved in this book is introduced. *Prolog* problems (and *CLP* problems as well) may be classified as belonging to one of the following four categories:

1. FS-type problems concerned with finding *feasible states* i.e. states satisfying all constraints of the problem. To this category belong most puzzles and mind-teasers, for which partial data describing some situation is given and the solver is expected to provide the missing facts so as to get a consistent situation. The importance of such puzzles for learning *Prolog* is well illustrated by a number of specialized *Prolog*-Puzzle websites (see e.g. [Edmund-10] or http://brownbuffalo.sourceforge.net/). They convey in simple form problems that are present in such complicated real-world applications as university time-tabling and industrial time-tabling. Unfortunately, those real-world applications are so complex and need so many variables that they are hardly suitable for learning *Prolog* and *CLP*. FS-type problems may be farther divided into:

   - *Feasible configuration problems* , which aim at selecting - from some set - subsets meeting constraints of belongness and compatibility constraints .
   - *Feasible assignment problems*, aiming at finding - for any element of some set - elements of another sets so as to fulfill some constraints. A type of assignment problems is often referred to as *transportation* problems.
   - *Feasible timetabling problems*, aiming at pairing elements of some set with elements of a set of time intervals.

2. FST-type problems concerned with finding *feasible state trajectories* i.e. sequences of feasible states from some well defined feasible *initial state* to some well-defined feasible *final state*. This class of problems is generally more difficult than the previous one. To this category belong puzzles and mind-teasers, for which some moves need to be accomplished, e.g. finding the way out of a maze, bringing people across a river or finding the shortest path a traveling salesman has to take. They convey in simple form problems that are present in many important industrial and business applications. like scheduling of operations or routing of vehicles. FST-type problems may be farther divided into:

   - *Feasible sequencing problems*, aiming at ordering elements of some set so as to fulfill some precedence constraints.
   - *Feasible scheduling problems*, aiming at ordering elements of some set so as to fulfill some precedence constraints and constraints on available resources.

3. OS-type problems concerned with finding *optimum states* i.e. feasible states optimizing some objective function. Those problems have a number of feasible states, and therefore it is possible to find such feasible state that is *best* from some point of view. To this category belong *optimum configuration problems*, *optimum assignment problems* and *optimum timetabling problems*, which differ from FS-type problems by aiming additionally at minimizing some objective function , most often a cost function.

4. OST-type problems concerned with finding *optimum state trajectories* i.e. feasible state trajectories optimizing some objective function. Those problems have a number of feasible state trajectories, and therefore are open to select such feasible state trajectory that is *best* from some point of view. To this category belong *optimum sequencing problems* and *optimum scheduling problems*, which differ from FST-type problems by aiming additionally at minimizing some objective function , most often a cost function.

This classification seems to be exhaustive and all-encompassing. The Author never came across *Prolog* or *CLP* applications with goals that could not be put into one of those four categories.

Chapter 2

In the beginning was Prolog

The first programming language offering basic *CLP* methods (like backtracking search and propagation of constraints) was *Prolog*[1]. Because of the simplicity and transparency of *CLP* methods used, it is worthwhile to start the discussion with *Prolog*. The more so that it is implemented as option in $ECL^iPS^e\,CPS$.

2.1 Prolog basics

Prolog[2](an acronym meaning *Pro*gramming in *log*ic) is based on a fruitful and inspiring ideas of writing programs consisting neither of *instructions* (like *procedural*, *imperative* languages) nor of *functions* (like *functional* languages), but of *relations* (expressed by*predicates*) between *logical variables*. This makes the language *declarative*: relations (predicates) and variables cannot be used to formulate *commands*, i.e. to formulate *algorithms*, but can be used to describe the problem under consideration. This makes *Prolog* (and *CLP*, which is inheriting those properties) an excellent tool for presenting problem-relevant *knowledge*. However, for *Prolog* to be a useful tool for solving problems, a system capable of drawing inferences from this knowledge is needed. Such a system, referred farther as *inference system*, is embedded in the *Prolog*/*CLP* compiler and is

[1] *Prolog* was conceived as joint effort by a group around Alain Colmerauer in Marseille, France, and Robert Kowalski in Edinburgh, UK, in the period 1971-1974.

[2] The word 'prologue' of Greek origin denotes originally an introduction to some large entity like a book or play. This coincidence is rather uncanny because the computer language *Prolog* happened to be an introduction to a large computing paradigm, described in this book.

(for a limited set of predicates) of universal character. This means that problem descriptions are in *Prolog* separated (in a conceptual and in a software sense) from techniques needed to solve the problems. This means also that *Prolog/CLP* are *declarative*: the *problem description* is the *problem model* is the *problem modeling program* is the *problem solving program*. By *problem description* is meant a description understandable for the *Prolog/CLP* compiler and assuring an efficient determination of the solution. Once more, the *art* of *Prolog/CLP* programming consists of formulating such descriptions.

2.1.1 Domain of inference

Prolog's domain of inference is the domain of *terms*. A term is defined by its type: it may be an *atom*, a *variable*, a *number*, a *predicate*, a *structure* or a *list*:

- an *atom* is given as any sequence of characters starting with a lower case letter, or starting with lower or upper case letter but put between double or single quotes[3]. Atoms are non-numerical (i.e. *logical* or *symbolic*) *constants*. E.g. `blu_sky` is a logical constant, because in a given situation it may be true or false, and `"Antoni Niederlinski"` is a symbolic constant because no logical value can be assigned to it. The use of quotes distinguishes atoms that start with upper case letters from variables;

- a *variable* given as any sequence of characters starting with an upper case letter or underscore, e.g. `X, A, John, Who, _who, _how_much`. Variables in *Prolog* and *CLP* are used as *unknowns*, similar as in logic and algebra. This is contrasted with variables in procedural programs, where they are place-holders for varying but known entities. A discussion of modes of variables is presented in Section 2.1.3. A single underscore `(_)` denotes an *anonymous variable* and means "any term"; it is used to preserve the defined predicate arity in case the value of the variable occupying the place of the anonymous variable is of no interest;

- a *number* is given as any integer constant (like `-10`, `-6`, `0`, `2`, `8`) or floating-point constant (like 2.71, 3.14) with decimal points only[4];

[3]Single (straight) quotes will be avoided in programs discussed in this book. This is so because they are converted in text files (e.g. PDF files) into lexicographic (curly) quotes that are not recognized by ECL^iPS^e. So a scan of a PDF-file program with such quotes cannot be activated.

[4]No decimal commas are allowed.

- a *predicate* is a *relation* between variables, e.g.

  `likes(Somebody, Something)`,

  where `likes` is the predicate *name* (always starting with a lower case letter), while `(Somebody, Something)` is a *tuple* i.e. an *ordered sequence* of the predicate *arguments*. The number of arguments is referred to as *arity* of predicate, to be used in references like `name/arity` (`likes/2` in our example). For the relation to be meaningful, sets (or in *CLP*-parlance: domains) for both variables have to be defined: e.g. the variable `Somebody` may take values from the set of names of 20 students attending my lecture, and the variable `Something` may take values from the set of names of `5` popular programming languages.

  The above predicate is a formalized prefix version of the colloquial infix sentence

  `"Somebody likes Something"`;

  the predicate *prefix* form, although awkward for colloquial use, has the undeniable advantage of providing easy access to the subject (`Somebody`), object (`Something`) and predicate (`likes`) of the sentence. What's more, the prefix form can easily accommodate a large number of arguments.

  Having defined the predicate arguments as variables `Somebody` and `Something`, a predicate as such has no logical value: it is neither true nor false, but ambiguous. However, it forms a blueprint for a *grounded* predicate, with variables bounded to some constants, e.g.

  `likes("John Smith","Prolog and CLP")`

  that may be either a false (unsatisfied) statement or a true (satisfied) statement for some particular `"John Smith"` from my group of 20 students[5].

  It should be stressed that predicate arguments form a *tuple*, i.e. *their order matters*; it must be the same for any usage of the predicate. This is so because *Prolog* and *CLP* identify variables across clauses not by their names, but by their position in the tuple. Thus

  `likes("John Smith","Prolog and CLP")`

  and

  `likes("Prolog and CLP","John Smith")`,

  should not appear in the same program, although they may mean (from the programmers point of view) exactly the same thing.

  Predicates may be *nested*: any predicate may serve as argument of another

[5] Because any bounding of the predicate arguments to some constants produces a *proposition* that is either true or false, a predicate is sometimes referred to as *propositional function*.

predicate, e.g.:

```
likes(graduate_student(Somebody),
      computer_science_subject(Something)).
```

A special case of predicates are *functions*[6]: all functions are predicates, but not all predicates are functions. Therefore no distinction will further be made between them. Because for some functions *infix* notation with standard operators is normally used (e.g. `X1 + X2`, where `"+"` is the standard operator), such infix notation is also accepted by *Prolog* and *CLP*.

Predicates may be divided into:

1. *Standard predicates* (*built-in predicates*), defined and designed by *Prolog* or *CLP* language designers, and made available to users. They are farther divided into *elementary predicates*, defining basic relations as given in libraries `ic` and `branch_and_bound`, with arguments contained at most in one *list*, and *global predicates*, defining advanced relations as given in libraries `ic_global`, `ic_cumulative`, `ic_edge_finder`, `ic_edge_finder3`, with *arguments* usually contained in many *lists*.
2. *Private predicates*, defined and designed by *Prolog* or *CLP* program designers, with names different from those of standard predicates, and with arbitrary number of lists.

- a *structure* is presenting a tuple of a fixed number of atoms, called its *arguments*. Any structure has a name (which looks like an atom). The number of arguments of a structure is called its *arity*. The name and arity of a structure are together called its *functor* and is often written as *name/arity*. Functors could be seen as general data types, arguments as defining instances of those data types. Structures correspond to records in other languages. Although structures look deceivably like predicates, they differ from them because they do not contain variables; therefore are always true.

- a *list* of terms, including an empty list. A (nonempty) list may look like: `[a, b, "CDE", 5, F]`, an empty list is denoted by `[]`. For details see Section 2.1.8.

[6] For a set of n variables with declared domains, a function is declaring - for some subset of values of `n-1` variables (called arguments) a unique value of the n-th variable, called (called outputs)

2.1.2 Prolog and CLP programs

Prolog and *CLP* programs are declarations of constraints. Constraints in *Prolog* (and *CLP*) programs have the form of *clauses*, which are either *facts* or *rules*, ended with a full stop:

1. *Facts* are structures or predicates with all arguments grounded, considered by the program designer to be true. The following is a clause representing a fact:

```
likes("John Smith","Prolog and CLP").
```

   It means that `"John Smith"` from my group of students does indeed likes `"Prolog and CLP"`. Facts are, by their very nature, *singular* and *specific*.

2. *Rules* are conditional statements of the form:

```
conclusion(_) :-
    condition_1(_),
    condition_2(_),
    ...,
    condition_n(_).,
```

   where `conclusion(_)` is a predicate with some free arguments referred to as *head* of the rule, the sequence `condition_1(_), condition_2(_),... condition_n(_)` being a *conjunction* of predicates with some free arguments referred to as the *body* of the rule, the comma (,) is the *conjunction operator* read *and*, the symbol (`:-`), being a way to write the rule implication arrow ←, denotes *Prolog implication* and is read *if*. Thus the rule is read like this:

```
if condition_1(_) and condition_2(_) and...condition_n(_)
are satisfied, then conclusion(_) is satisfied.
```

   The presence of variables in the head and body of rules makes rules *general*: they are valid for a set of variables, as contrasted with facts. The indentation in the rule expression has no logical meaning: it is used to

enhance the readability of rules.

It should be remembered that *Prolog implication* differs from the better known *implication of logic*: if any condition of the *Prolog* implication is false (unsatisfied), the conclusion is considered as false (unsatisfied), see Table 2.1. This assumption is known as *Closed World Assumption*[7]. Its aim is twofold:

`Condition`	`Conclusion`	`Conclusion :- Condition`
True	True	True
False	False	True
False	True	False
True	False	False

Table 2.1: Definition of implication in *Prolog*

`Condition`	`Conclusion`	`Condition` $\Rightarrow$ `Conclusion`
True	True	True
False	False	True
False	True	True
True	False	False

Table 2.2: Definition of implication in logic

- to avoid the nondeterminism existing for the *implication of logic* for which, if any condition is false (unsatisfied), the conclusion may be true (satisfied) or false (unsatisfied), see Table 2.2;
- to force the program designer to put all available relevant knowledge into the *Prolog* (or *CLP*) program.

Prolog naming conventions deserve some comments:

[7]The "world" that is subject of the programs reasoning is "closed" in the sense that everything that matters for the problem has been taken care of in the program, or can be inferred from the program.

1. The naming of *private predicates*[8] is entirely arbitrary save they are different from names of *built-ins*[9].

2. Private predicate names may not convey any meanings, e.g. instead of writing:
   ```
   likes(Somebody, Something)
   ```
   we could write as well:
   ```
   blah_blah(Somebody, Something)
   ```
   swapping `likes` in the entire program by `blah_blah` without affecting the functioning of the program. However, humans inspecting such program may have problems in guessing what it's all about. For the sake of program readability, modifiability and maintenance, it pays to use predicate names that correspond to the predicate meaning.

3. The naming of variables needs to be consistent only in rules, but not outside rules. The same variable may bear different names in different rules without affecting the program functioning. *Prolog* (and *CLP*) recognizes variables not by their names, but by their position in predicates. However, for the sake of program readability, modifiability, and maintenance, it pays to use the same variable names in different rules.

The described features have an advantage and a disadvantage:

- the advantage is the ease of incorporating third party programs into our own programs: it suffices to paste them and provide calls from within our program. No name adjustment is necessary;

- the disadvantage is the possible muddle caused by using inappropriate names for variables and predicates. In extreme cases it may make the understanding of a *Prolog* (or *CLP*) program a really tough job.

2.1.3 Modes of variables

The word "mode" denotes the role played by a variable as argument of a built-in predicate: the variable may be:

- an *input*, i.e. it is determined outside the predicate considered: it must be declared as *bounded* to some other predicate, or list, or atom or number;

[8] Private predicates are predicates defined and designed by the user.
[9] Built-ins are predicates defined and designed by *Prolog*/*CLP* language designers and made available for users of those languages.

- an *output*, i.e. it is determined by the predicate considered.

In order to avoid mode errors while using standard predicates, their variables are distinguished in the documentation by attribute names and corresponding symbols:

- Input variables that are (by program statements) bounded to some other predicate, a list, an atom or number, are referred to as *instantiated* and denoted by a *plus* prefix, like `+X`, in the standard predicate definitions.

- Input variables that are (by program statements) bounded to some grounded predicate, grounded lists, atoms or numbers are referred to as *grounded* and denoted by a *double plus* prefix, like `++X`, in the standard predicate definitions.

- Output variables are denoted by a *minus* prefix, like `-X`. They are of course not bound to anything.

- A distinctive (rather valuable) feature of *Prolog* and *CLP* is the existence of predicates with variables serving as either inputs or outputs. They are then in the predicates definition distinguished by a question mark, like `?X`.

The definitions are summarized in Table 2.3.

Variable instantiated (+X)	Variable grounded (++X)	Variable free (-X)	Variable any mode (?X)
An input bounded to any predicate or list, to an atom or number	An input bounded to a grounded predicate or list, to an atom or number	An output bounded to nothing	An input or output, bounded or free

Table 2.3: Modes of variables

The differences between various variables are additionally illustrated by Figure 2.1: any *grounded variable* is *instantiated*, but some *instantiated variables* may not be *grounded*.

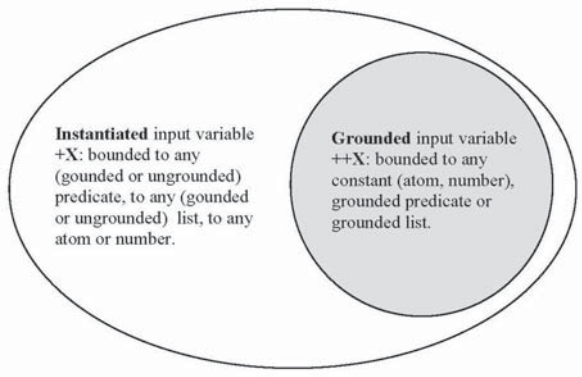


Figure 2.1: Venn diagram for input variables

2.1.4 Operations

The basic arithmetical operations available in *Prolog* are shown in Table 2.4. They may be used in either *infix* form or *prefix* form, see *Full documentation...* in Figure 5.

Symbol	Operation
+	addition
-	subtraction
	multiplication
/	real division
//	integer division
mod	modulus
^	power

Table 2.4: Standard arithmetic operations

The standard order of operations (strength of binding, precedence) are expressed in Table 2.5[10].

This means that if a number or other symbol, or an expression grouped by one or more symbols of grouping, is preceded by one operator and followed by

[10]What follows in this section may be omitted while first reading.

Operation	Binding strength	Precedence value
terms inside brackets	strong	low
exponents and roots	$\bigwedge$	$\mid$
multiplication and division	$\mid$	$\bigvee$
addition and subtraction	weak	high

Table 2.5: Standard order of operations

another, the operator higher in the table should be applied first. As in *Edinburgh Prolog*, a lower precedence value means that the operator binds stronger (1 strongest, 1200 weakest)[11]. Arrows in Table 2.5 indicate directions of increase.

In *Prolog*, the user is able to modify the syntax dynamically by explicitly declaring new operators. The built-in `op/3` performs this task. Its structure is:

```
op(+Precedence, +Associativity, ++Name)
```

where `Precedence` is an integer from the range `1` to `1200`, `Name` is the operator with the chosen `Precedence`, and `Associativity` is an argument that distinguishes between different classes of operators. Denoting by:

f - an operator with declared precedence,
x - an argument whose precedence must be strictly lower than that of the operator
y - an argument whose precedence is lower or equal to that of the operator,

and farther assuming that:
- arguments enclosed in parentheses or unstructured arguments have precedence equal zero,
- structured arguments have precedence equal to the precedence of the operator,

then possible operator classes and their associativity are shown in Table 2.6
These concepts may be illustrated by a following simple example: consider the expression

`u - v - w`,

with operator '`-`' having the precedence 500. It is understood as

[11] This terminology is indeed unfortunate, since a higher precedence value in *Prolog* indicates lower precedence (in normal English). The lowest precedence value in *Prolog* binds the strongest.

Operator class	Associativities
prefix	fx, fy (unary) or fxx, fxy (binary)
infix	xfx, xfy, yfx
postfix	xf, yf

Table 2.6: Operator classes and their associativity

`(u - v) - w`,
and not as:
`u - (v - w)`.
To get the correct interpretation, the operator '`-`' has to have the associativity `yfx`. For a more advanced example see Section 5.8.3.

2.1.5 Constraint propagation

Constraint propagation is a process initiated by grounding a free variable from some constraint. The propagation aimes at letting know about this event all to *which it may concern* and at performing all operations relevant to the mentioned grounding. In Prolog it is performed by two actions:

1. Value spreading.
2. Unification.

Value spreading denotes the process by which the grounding done for a variable from some constraint is repeated for all instances of this variable in the body of this rule and for all other instances of the constraint in bodies of other rules.

Unification denotes the process of matching values of other instances of the grounded variable in order to obtain equality of terms. The principles of unification are:

- in the Herbrand domain unification my be done only for *syntactically equivalent terms*. Two terms are *syntactically equivalent* if:
  - `they are of the same type and format, e.g.`
    `"likes(A,B)" and "likes(X,Y)",`
    `or "[X,Y,Z]" and "[P,_,R]",`
  - `one of the unified terms is a` *free variable*;
- free variables can be unified with anything, including other free variables. This is consistent with the property that variable names have meaning only inside rules;

- different atoms are not unifiable;
- different numbers are not unifiable.

Unification is invoked by the built-in infix predicate `=/2`. E.g. the unification:

```
likes(Somebody, Something) = likes("John Smith",prolog)
```

is feasible and results in

```
Somebody = "John Smith"
Something = prolog
```

So in *Prolog* `1 + 1 = 2` does not hold because the left hand term and the right hand term are not syntactically equivalent. Instead the built-in `is` has to be used and the equation is written as `1 + 1 is 2`. Of course not all syntactically equivalent terms may be unified (are unifiable). E.g.:

```
likes(Somebody,pascal) = likes("John Smith",prolog)
```

is false, because the different constants `pascal` and `prolog` are not unifiable. This is also the case for:

```
likes("Jim Taylor",prolog) = likes("John Smith",prolog)
```

because the different constants `"Jim Taylor"` and `"John Smith"` are not unifiable. The equality sign (`=`) is thus meaningful only between unifiable terms.

The outcome of propagation may be twofold:

1. For a successful propagation the next free variable is grounded.
2. For an unsuccessful propagation the last grounded variable is degrounded and backtracking starts.

Constraint propagation in Prolog is not an autonomouse activity: it can only be used in conjunction with search.

2.1.6 Tree search with no trees

To proceed, it would be handy to introduce the concepts of *state*, *state space*, *feasible state* and *contracted state*. *State* means any grounding of domain values

to *all* decision variables, the *state space* is given by all groundings of domain values to all decision variables, a *feasible state* is a state for which all constraints are satisfied, a *contracted state* means any grounding of domain values to *some* decision variables[12].

The goal of any *Prolog* (and *CLP*) program is to satisfy a *query* that is the head of some rule. The *Prolog* (*CLP*) compiler contains an *inference system* that *searches* the *state space* for a *feasible state* that will satisfy the query. This is done by generating sequences of *contracted states* leading to the feasible state, provided a feasible state exists. If so, *Yes* is followed by some detailed messages. If no feasible state exists, *No* will be printed. All *Prolog* (and *CLP*) programs discussed farther will always have the query *top*; this makes for convenient testing.

Search denotes the following sequence of steps:

1. *Selecting* a *decision variable* from the body of the rule defining the query;
2. *Grounding* the selected *decision variable*, i.e. assigning to it a value from its domain. Thereby a *contracted state* is generated and the selected decision variable is termed *grounded*;
3. *Spreading* the value of the grounded decision variable to all its instances in the body of the rule;
4. *Testing* the satisfaction of all predicates in the body of the rule using *unification*:
   - if this is not possible because some predicates are not grounded, steps 1, 2 and 3 are repeated for the next nearest variable or for the rule defining this predicate, until eventually all predicates are grounded and satisfied;
   - if all predicates in the body of the query are grounded and satisfied, the query is satisfied and the variable values used for grounding are displayed as the program solution;
   - if some predicate in the body of the query fails, the latest selected variable is *degrounded*, a return is performed leftwards to the nearest tested predicate with variables not yet grounded to some values from

[12]The concept of *state* is - to the best knowledge of the Author - not particularly *en vogue* in the *CLP* community. The Author, because of his control-engineering and dynamic system background, is missing it from ever since, and uses this opportunity to show its broad usefulness while discussing *CLP*.

their domains, and one of the variables is *regrounded*. While returning, all variables that have been successfully grounded between the said nearest tested predicate and the failed predicate, are degrounded as well. The return, the degrounding, and the regrounding is named *backtracking*, and the predicate with variables of yet untested values to which the return was performed, is named *choice point*.

It should be emphasized that any variable grounded to some value may be grounded to another value only as the result of backtracking.

This is illustrated by the following simple *Prolog* program `2_1_search.pl`:

```
/*1*/  a(X,Y) :-
/*2*/       b(X),
/*3*/       c(X,Y).
/*4*/  b(1).
/*5*/  b(&).
/*6*/  c(&,"A").
```

The programs query is `a(X,Y)`. This means that the program aims at finding such values for decision variables `X` and `Y` that satisfy `a(X,Y)`. The program contains one rule (lines `/*1*/`, `/*2*/`, and `/*3*/`) and three facts (lines `/*4*/`, `/*5*/`, and `/*6*/`). The indentation for lines `/*2*/` and `/*3*/` is used to make the rule better readable. The domains for variables `X` and `Y` are defined implicitly by the facts: the domain of `X` is (`1`,`&`), the domain of `Y` is `"A"`.

The rule states that in order to satisfy `a(X,Y)` such value for `X` has to be found that satisfies `b(X)`, and such value for `Y` has to be found that together with the value for `X` satisfies `c(X,Y)`. The conditions for the rule are queried in a top-down fashion, so the first value found for `X` is `X=1`. Because the domain of `X` contains another value `&`, a choice point is created for `b(X)`. Next, the value `X=1` is spread to line `/*3*/` resulting in `c(1,Y)`, which does not unify with `c(&,"A")` from line `/*6*/`. So `c(1,Y)` is unsatisfied, `X` is degrounded from its value `1` and a return to the choice point for `b(X)` follows. Now `X` is regrounded with `&`, the regrounding is spread to line `/*3*/` resulting in `c(&,Y)` that successfully unifies with `c(&,"A")` from line `/*6*/` giving the solution `X = &`, `Y = "A"`.

The process described may be interpreted as running according to the *search tree* from Figure 2.2 that reflects the program structure.

For obvious reasons the search from Figure 2.2 is known as *top-down* search or *depth-first* search. The way returns are generated (as the result of violating

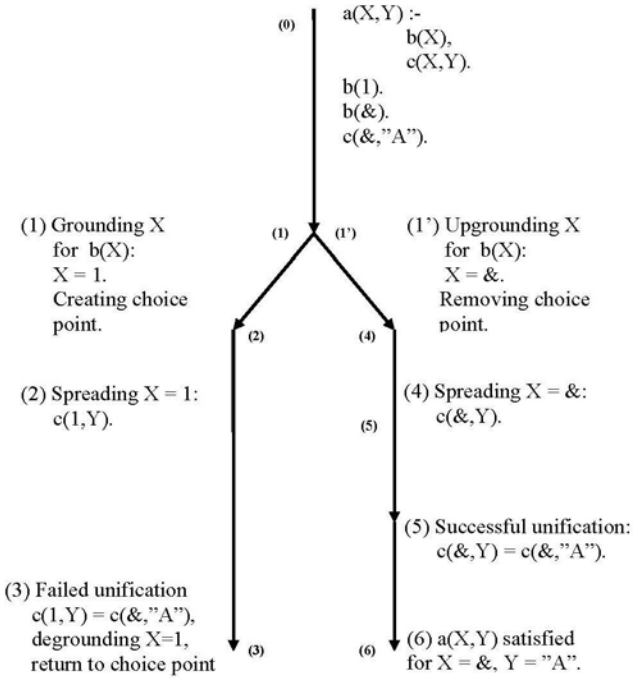


Figure 2.2: Search tree for simple *Prolog* program

some constraint) is known as *standard backtracking*. Therefore the full name of this search is *Depth-First Backtracking Search* or *Top-Down Backtracking Search*.

The search tree is defined by all states of the decision variables that constitute the leaves of the search tree. For this example they are `(X,Y) = (1,"A")` and `(X,Y) = (&,"A")`. The intermediate node (there is only one node for this example) of the search tree corresponds to the choice point, where the value of `X` is chosen.

The amazing thing is that search trees are never explicitly present in its entirety, but simply generated piecewise, *on-the-fly*. In the discussed example the left-hand branch from Figure 2.2 is generated first, but after the failed unification in line `/*3*/` it is dropped save the choice point `(1)- (1')`, to be used for generating the right-hand branch. The mechanism of making search

trees with no trees present in its entirety is of great practical significance because it allows *Prolog* and *CLP* languages to deal with problems corresponding to search trees of exorbitant sizes.

An important regularity from the example deserves to be emphasized:

- *grounding* of free variables occurs in top-down search any time a predicate with free variables is encountered;
- *degrounding* of grounded variables occurs when the last grounding results in some constraint to be unsatisfied and a return to the nearest choice point is done, where this (or some other free variable) may be *regrounded*. There is no other way for grounded variables to change their values.

It should be emphasized, that *Prolog*s search and unifications constitute a *complete inference method*. It means that if a solution to a *CSP* modelled in Prolog exists, it will be determined[13].

2.1.7 Failing usefully

As it had been already stressed, backtracking is initiated when some grounded predicate fails. However, there are situation when backtracking is forced by deliberately using an "always false" atom called `fail/0`. This is illustrated by program `2_2_fail.pl`:

```
/*1*/  top:-
/*2a*/      who_are_your_friends_1.
/*2b*/      %   who_are_your_friends_2.
/*2c*/      %   who_are_your_friends_3.

/*3*/  friend("Mark").
/*4*/  friend("Jack").
/*5*/  friend("Andrew").

% For 'who_are_your_friends_1' there is no backtracking,
% just one solution (the first one from top) is given:

/*6*/  who_are_your_friends_1:-
/*7*/  friend(Who),
/*8*/  write("Friend:  "),write(Who),  nl.

% For 'who_are_your_friends_2', 'fail' generates backtracking,
% all friends are displayed but eventually the program fails with a 'No':
```

[13] Well, it may sometimes take quite a time!

```
/*9*/  who_are_your_friends_2:-
/*10*/       friend(Who),
/*11*/        write("Friend:  "),write(Who),nl,
/*12*/       fail.

% For 'who_are_your_friends_3', 'fail' generates backtracking,
% but when no backtracking can be performed any more, the second
% definition of 'who_are_your_friends_3'i invoked
% and the program end with a Y'es'

/*13*/ who_are_your_friends_3:-
/*14*/       friend(Who),
/*15*/       write("Friend:  "),write(Who), nl,
/*16*/       fail.

/*17*/ who_are_your_friends_3:-
/*18*/       write("Those are all my friends."),nl.
```

The messages are:
Message for `who_are_your_friends_1`: `Friend: Mark.`

```
Yes.
```

After clicking twice "more" in the Main Window from Figure 2 (to enforce other solutions), the message is:

```
Friend:  Jack
Yes.
Friend:  Andrew
Yes.
```

Message for `who_are_your_friends_2`:

```
Friend:  Mark
Friend:  Jack
Friend:  Andrew
No.
```

Message for `who_are_your_friends_3`:

```
Friend:  Mark
Friend:  Jack
Friend:  Andrew
Those are all my friends.
Yes.
```

Well, obviously `fail/0` is an explicitly procedural operator that clearly

shows the impossibility of writing declarative programs that work without some procedural crutches. It is worth remembering that `fail/0` is functioning like any predicate which is always false, e.g. *2 is 3*.

2.1.8 Recursive definitions

A *recursive* predicate definition is given by:
1) a *rule* with the *head* being the defined predicate and the *body* containing this very predicate with different argument structure;
2) a *fact*, most often the grounded predicate.
The conciseness, declarativity and power of *Prolog* (and *CLP* as well) is largely due to the widespread usage of recursive definitions of predicates. As example may serve the *list* definition. *Lists* are basic *Prolog* data structures. They are *n-tuples* of elements, beginning with a left-hand square bracket and closing with a right-hand square bracket:

```
List = [Element_1, Element_2,...,Element_n] .
```

It may be decomposed as follows:

```
List = [Head|Tail]
```

where `Head` is the first element of the list `List`, and `Tail` is the list that remains after removing the first element. Because `Tail` is a list, it must obviously contain a `Head_of_Tail` and a `Tail_of_Tail`. The last one is a list, therefore we can speak about the `Head_of_Tail_of_Tail` and the `Tail_of_Tail_of_Tail`, and so on, until the empty list `[]` is reached, which has no head. So the `list` concept is in fact defined recursively. And most predicates with lists as arguments use recursion as well. The most simple illustration is provided by defining a predicate that determines list membership. It has the structure:

```
membership(Member,List),
```

which is intended to mean that `Member` is an element of `List`. It is defined by stating the fact that the head of the list is a list member, no matter what the tail is:

```
/*1*/      membership(Member,[Member|_]).
```

and stating the recursive rule that a list member is the member of the list tail, no matter what the head is:

```
/*2*/      membership(Member,[_|O]) :-
/*3*/                membership(Member,O).
```

The underscore _ denotes an *anonymous variable*; it means we do not care about the value it may be grounded with and are not interested in knowing this value. Let's have a look at how this definition is working. The program `2_3_list.pl`:

```
/*1*/      top:-
/*2*/              membership(E,[1,2,3,4]),
/*3*/              writeln(E),
/*4*/              fail.
/*5*/      top:-
/*6*/              writeln("Those are all elements of the list.").

/*7*/      membership(Member,[Member|_]).

/*8*/      membership(Member,[_|O]) :-
/*9*/              membership(Member,O).
```

generates the message:

```
1
2
3
4
Those are all elements of the list.
```

The programs *query* is `top`. The logical constant `top` will be used in the sequel for all *Prolog/CLP* programs in the book. *Prolog* compiler attempts to satisfy the query, i.e. make `top` true. In order to do it, it has to satisfy the predicate in line `/*2*/`. This invokes the definition from line `/*7*/`, `E` is grounded to `/*1*/`, and because of the other part of the definition (lines `/*8*/` and `/*9*/`, a choice point is created for the predicate `membership()`. The predicate `fail/0` is a built-in that cannot be satisfied, so a backtrack to the choice point is made generating the next element of the list and another choice point, and so on, until (thanks to removing head after head by the action of line `/*7*/`, `/*8*/` and `/*9*/` predicates) the list becomes empty.

The example presented is more general than it seems: *Prolog* recursion is always defined between a list (in the head of the recursive rule) and the tail of this list (in the body of the recursive rule).
For ECL^iPS^e *Prolog* the built-in predicate `member(?Member,?List)` functions exactly as the above `membership()` predicate: while using it instead of membership in line `/*2*/`, lines `/*7*/`, `/*8*/`, and `/*9*/` are not needed, see top1 in `2_3_list.pl`.

2.1.9 Basic list operations

There are only two such operations:

1. Removing successive heads from a non-empty list and constraining them. This is continued until the list is empty, as shown in the following example:

```
recursive_predicate([H|T],....):-
        % The head is removed and processed:
        constraining_the_head(H),
        ........................
        recursive_predicate(O,....).


        % Removing of heads leads to an empty list:
recursive_predicate([],....).
```

   The recursion with heads removal starts with a `[H|T]` list, the heads of which are successively removed and processed, until the list is empty. The recursion occurs between the list `[H|T]` (in the head of the rule) and the tail of the list `T` (in the recurred predicate in the rule body).

2. Adding - as heads - successive elements, generated by some constraints, to a list which is initially entirely or partially empty. This is continued until some special list is generated, as shown in the following example:

```
recursive_predicate(Tail_of_list,....):-
        % The head is determined and added:
        determine_the_head(Head),
        ........................
        recursive_predicate([Head|Tail_of_list],....).
```

```
        % Adding heads leads to some Special_list:
recursive_predicate(Special_list,,....).
```

The recursion with adding heads start with an entirely or partially empty `Tail_of_list`, to which are successively added `Heads` generated by some constraints, until the list is some `Special_list`. The recursion occurs between the `Tail_of_list` (in the head of the rule) and the *head-added-list* `[Head|Tail_of_list]` in the recurred predicate in the rule body.

The important thing to remember is that only heads may be removed from a list, and only heads me be added to a list.

This is illustrated by program `2_4_reversal.pl` that reverses the order of list elements using two private predicates:

1. `my_reverse(Initial_list, Reversed_list)`

2. `my_reverse(Initial_list, Reversed_list,`
   `Accumulator_of_reversed_list)`

The name `my_reverse` was chosen to distinguish it from the built-in `reverse/2`, which does exactly the same job.

The program is removing successively heads from the the `Initial_list` and adding the removed heads successively to the initially empty list named `Accumulator_of_reversed_list`. When the `Initial_list` is empty (i.e. when all its elements have been transferred in reverse order to the `Accumulator_of_reversed_list`), the `Reversed_list` is unified with the accumulator.

The program looks like this:

```
/*1*/      top:-
/*2*/               my_reverse([a,b,c,d],Reversed_list),
/*3*/               write(Reversed_list).

/*4*/      my_reverse(Initial_list,Reversed_list):-
/*5*/               my_reverse(Initial_list,Reversed_list,[]).
/*6*/      my_reverse([],A,A).

/*7*/      my_reverse([H|T],Reversed_list,A):-
/*8*/               my_reverse(T,Reversed_list,[H|A]).
```

The message generated is:

```
Reversed list = [d, c, b, a]
```

The program uses two predicates with the same name but different arity (`my_reverse/2` and `my_reverse/3`), which is perfectly O.K. Different arities make the names distinguishable to the compiler.

The basic rule is in lines `/*7*/` and `/*8*/`: there the head `H` of the initial list `[H|T]` is removed from this list and added as head to the list `A`, resulting in `[H|A]`.

The use of accumulator deserves some comments. In *Prolog/CLP* accumulators are *artificial variables*[14] that allow to write so-called *tail-recursive* rules, i.e. rules with the head calling itself at the end of the body; the rule in lines `/*7*/` and `/*8*/` is just a trivial example of a tail-recursive rule. Tail-recursion is the most parsimonious type of recursion as far as stack space is concerned: it does not need any stack at all.

2.1.10 Generating lists

To do anything in *Prolog/CLP*, lists have to be used. Lists are usually generated from data sets. This can be done using the `findall/3` built-in with following mode structure:

```
findall(?Term, +Goal, -List)
```

where `List` is the list of all values of `Term` for which `Goal` is satisfied. Consider the following example:

The *Backyard Used Car* company has widely advertised an attractive sale of the following second-hand but relatively new and well-kept models produced by the renowned *Clunker Motors* Company: *Clunker SUV*, *Clunker Great Tour*, *Clunkerlac*, *Clunker Family*, *Clunkerdes*, *Clunker Electric* and *Clunker Green*. The details are given by Table 2.7.

The aim is to determine the mean price of all cars and the mean mileage of cars costing less than 1900 and not of red color and not older than 2006. To

[14] Artificial in this sense that they do not correspond to any of the original problem variables.

Model	Mileage	Year	Price	Color
Clunker SUV	29000	2008	1500	Black
Clunker Great Tour	60000	2009	1900	Green
Clunkerlac	47000	2007	1200	Champagne
Clunker Family	38000	2009	2200	Blue
Clunkerdes	46000	2008	3100	Silver
Clunker Electric	75000	2005	1100	Red
Clunker Green	52000	2006	1300	Silver

Table 2.7: Second-hand car sale data

use this data in a *Prolog* program, a private predicate

```
offer(Model, Mileage, Year, Price, Color)
```

is defined. The program `2_5_clunkers.pl` shows the usage of *findall/3* to extract the needed information from the data:

```
/*1*/  top:-
/*2*/      mean_price_for_all_cars,
/*3*/      mean_mileage_for_selected_cars.

/*4*/  mean_price_for_all_cars:-
/*5*/      findall(Price,offer(_,_,_,Price,_),List),
/*6*/      writeln("List of prices for all cars":List),
/*7*/      length(List, N),
/*8*/      Sum is sum(List),
/*9*/      MeanPrice is Sum/N,
/*10*/     writeln("Mean car price":MeanPrice),nl.

/*11*/ mean_mileage_for_selected_cars:-
/*12*/     findall(Mileage,selected_cars(Mileage),List),
/*12*/     writeln("List of mileage for cars costing less than 1900 and "),
                writeln("not of red color and not older than 2006":List),
/*14*/     length(List, N),
/*15*/     Sum is sum(List),
/*16*/     MeanMileage is Sum/N,
/*17*/     writeln("Mean mileage for cars costing less than 1900 and "),
                writeln("not of red color and not older than 2006":MeanMileage).

/*18*/ selected_cars(Mileage):-
/*19*/     offer(_,Mileage,Year,Price,Color),
/*20*/     Year > 2006,
```

```
/*21*/     Price < 1900,
/*22*/     Color \== "Red".

/*23*/     offer("Clunker SUV", 29000, 2008, 1500, "Black").
/*24*/     offer("Clunker Great Tour", 60000, 2009, 1900, "Green").
/*25*/     offer("Clunkerlac", 47000, 2007, 1200, "Champagne").
/*26*/     offer("Clunker Family", 38000 , 2009, 2200, "Blue").
/*27*/     offer("Clunkerdes", 46000, 2008, 3100, "Silver").
/*28*/     offer("Clunker Electric", 75000, 2005, 1100, "Red").
/*29*/     offer("Clunker Green", 52000, 2006, 1300, "Silver").
```

The program generates following messages:

```
List of prices for all cars : [1500, 1900, 1200, 2200, 3100, 1100, 1300]
Mean car price : 1757.14285714286

List of mileage for cars costing less than 1900 and
             not of red color and not older than 2006 : [29000, 47000]
Mean mileage for cars costing less than 1900 and
             not of red color and not older than 2006 : 38000.0
```

Notice the presence of anonymous variables in line `/*5*/`, due to the circumstance that we need only values of the `Price` variable.

Readers familiar with database languages may notice that *Prolog* is also an elegant language for database queries, equivalent to a powerful subset of *SQL*.

2.1.11 Controlling backtracking with 'cut'

Backtracking in *Prolog* is "automatic". That means each time the search for solution needs backtracking (i.e. each time some predicate fails), *Prolog* backtracks. This is both advantageous and disadvantageous: the advantage consists of relieving programmers from coding backtracking into *Prolog* programs; the disadvantage is that backtracking may sometimes be not quite desirable because it increases the time to get a solution or generates partial solutions of no interest. This can be avoided using the built-in `!/0` referred to as *cut*. The properties of `cut` are summarized in Figure 2.3, were black arrows denote possible backtracking. Program `2_6_playing_with_cut.pl` illustrates all usages of `cut/0`:

```
/*1*/  top:-
/*2*/       a.
/*3*/  a :-
```

```
/*4*/      b, write("   The Prolog compiler did not call the 'cut'."),nl,
/*5*/      write("   Thanks to that, because the first clause for 'b' "),nl,
/*6*/      write("   could not be fulfilled, 'a' is true "),nl,
/*7*/      write("   because the second clause for 'b' was fulfilled."),nl,nl.
/*8*/  a :-
/*9*/      write("We are here 4!"),nl,
/*10*/     write("   Because the Prolog compiler called the 'cut' in"),nl,
/*11*/     write("   the first clause for 'b', it is not possible to "),nl,
/*12*/     write("   call the second clause for 'b'. The first clause for "),nl,
/*13*/     write("   'a' thus remains unfulfilled. However, the second clause "),nl,
/*14*/     write("   for 'a' is fulfilled, because it can be called anyway."),nl,nl.
```

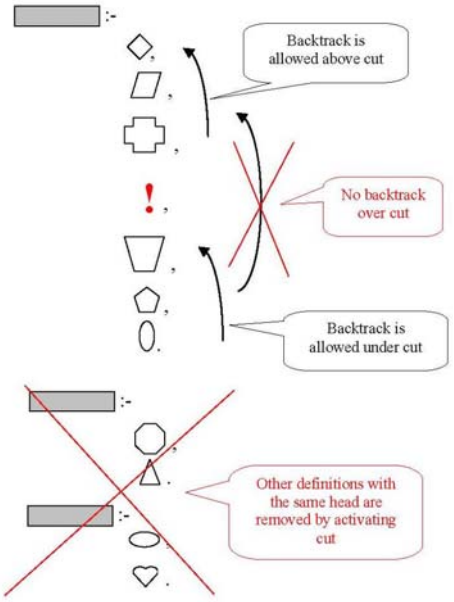


Figure 2.3: Properties of cut (!/0)

```
/*15*/ b :-
/*16*/     c(X),
/*17*/     d(X),
/*18*/     !,
```

```
/*19*/     e(Y),
/*20*/     f(Y),
/*21*/     g(X).

/*22*/ b:-
/*23*/     write(" We are here 0!"),nl.

/*24*/ c(1).

/*25*/ c(2):-
/*26*/     write("We are here 1!"),nl,
/*27*/     write("   Backtrack is possible before the 'cut'!"),nl,nl.

/*28*/ c(3).

/*29*/ d(2).

/*30*/ e(1).

/*31*/ e(2) :-
/*32*/     write("We are here 2!"),nl,
/*33*/     write("   Backtrack is possible after the 'cut'!"),nl,nl.

/*34*/ f(2) :-
/*35*/     write("We are here 3!"),nl,
/*36*/     write("   However, no backtrack is possible from below the place "),nl,
/*37*/     write("   cut has been placed in the first clause for 'b' up above "),nl,
/*38*/     write("   the place 'cut' has been placed in this clause."),nl,nl.

/*39*/ g(3).
```

The message while lines `/*25*/`, `/*26*/` and `/*27*/` are present:

```
We are here 1!
   Backtrack is possible before the 'cut'!

We are here 2!
   Backtrack is possible after the 'cut'!

We are here 3!
   However, no backtrack is possible from below the place
   cut has been placed in the first clause for 'b' up above
   the place 'cut' has been placed in this clause.

We are here 4!
   Because the Prolog compiler called the 'cut' in
   the first clause for 'b', it is not possible to
```

```
   call the second clause for 'b'. The first clause for
   'a' thus remains unfulfilled. However, the second clause
   for 'a' is fulfilled, because it can be called anyway.
```

The message while lines `/*25*/`, `/*26*/` and `/*27*/` are removed:

```
We are here 0!
   The  Prolog  compiler did not call the 'cut'.
   Thanks to that, because the first clause for 'b'
   could not be fulfilled, 'a' is true
   because the second clause for 'b' was fulfilled.
```

So `cut/0` is another (besides `fail/0`) explicitly procedural operator we need to make partially declarative programs to work.

2.1.12 Lameness of Prolog's logic

The Reader has perhaps already noticed that there is rather little logic in *Prolog*, considering the massive stock of knowledge covered by the name *logic*. What's more, this little logic used in *Prolog* is sometimes strangely twisted to account for the fact that *Prolog* (and *CLP*) programs are running on single processors, that process the program clauses and the body predicates sequentially in time, from programs top to programs bottom, and from body left to body right.

Consider the rule structure. The sequence of predicates in the rule's body had been referred to as *conjunction*. As we know from logic, conjunction is commutative; that is changing the order of conjuncted arguments does not change the logical value of the conjunction. However, this does not always hold for rules. The program `2_5_clunkers.pl` gives a good opportunity to demonstrate this limitation of logic as used in *Prolog*. Suppose the line `/*5*/` was put after line `/*8*/`. The program compiles but when queried produces the message:

```
"instantiation fault in (0, _347, _355)".
```

The reason is obvious: predicates in the rules body are (because of the single processor limitation) tested in the order they appear, from left to right. Therefore we can't use (in lines `/*7*/` and `/*8*/`) data for the ungrounded variable `List`: it has not yet been grounded by the moved predicate from line `/*5*/`.

Hence in *Prolog* it is up to the program designer to put the body predicates in proper order.

2.2 Configuration problems

2.2.1 Configuring a 3-element system

To build some system, elements belonging to three classes are needed: a single A-class element, a single B-class element, and a single C-class element. Each class contains a number of different types of elements:

- A-class elements may be of type `a1`, `a2`, and `a3`;
- B-class elements may be of type `b1`, `b2`, `b3`, and `b4`;
- C-class elements may be of type `c1` and `c2`,

with different prices (in *Monetary Units*, MU):

- `a1` price is 1900; `a2` price is 750; `a3` price is 900;
- `b1` price is 300; `b2` price is 500; `b3` price is 450; `b4` price is 600;
- `c1` price is 700; `c2` price is 850,

and different compatibility restrictions:

- `c1` is not compatible with `a2`;
- `b2` is not compatible with `c2`;
- `c2` is not compatible with `b3`;
- `b4` is not compatible with `a2`;
- `b3` is not compatible with `a1`;
- `a3` is not compatible with `b3`;

There are two problems to be solved: (1) determine all configurations consisting of three compatible elements `A`, `B`, and `C` with overall price not larger than 2100 MU[15], (2) determine all optimum configurations consisting of three compatible

[15] This is an FS-type problem.

elements A, B, and C with overall minimum price[16]. Such problems are *generic*, which means they are representatives of a group of concrete problems of similar logical structure, like configuring a lunch menu, configuring a leisure outfit, configuring computer hardware.

2.2.2 Exhaustive search

The most naive approach to solving the configuring problem is *exhaustive search*: it amounts to generating consecutively all states (A, B , C) of the state space and only then testing whether they satisfy all constraints of the problem. This is done notwithstanding the fact that with *luck* the feasible solution may be found for the first state generated and with *no luck* it may be found after generating some substantial number of states. However, with *bad luck* it may be found for the last state generated, and to play safe, the entire state space has to be tested. For the configuration example $3 \times 4 \times 2 = 24$ tests have to be performed. Assuming further that A is grounded first, B next, and C last, the search may be depicted for - the sake of compatibility with backtracking search - by the search tree from Figure 2.4.

Exhaustive search starts (according to the assumption made) with variable A bounded to a1, variable B bounded to b1, and variable C bounded to c1. The resulting configuration (a1,b1,c1) is too expensive, so the next state is generated and so on. The system may be configured in 24 ways (that's the dimension of the state space) from which 13 configuration contain non-compatible elements, and 7 configurations costs more than the threshold price of 2100. Following four configurations are feasible:

- Configuration(a2, b1, c2) priced at 1900;
- Configuration(a3, b1, c1) priced at 1900;
- Configuration(a3, b1, c2) priced at 2050;
- Configuration(a3, b2, c1) priced at 2100.

The exhaustive search may be performed by program 2_7_conf_es.pl:

```
/*1*/ top:-
/*2*/     assert(upper_price_limit(2100)),
/*3*/     upper_price_limit(Upper_price_limit),
```

[16]This is an OS-type problem.

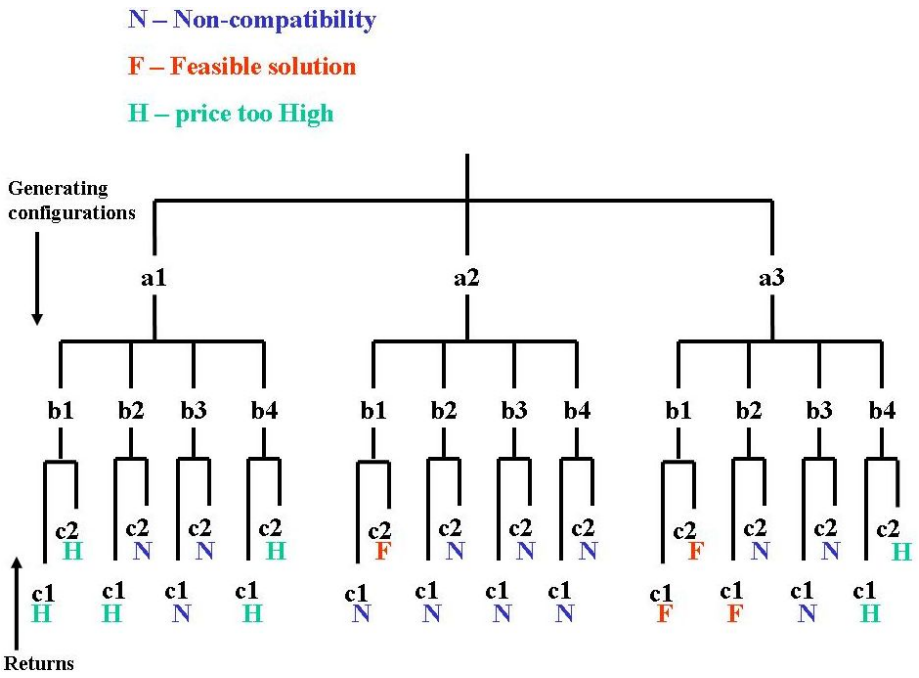


Figure 2.4: Search tree for *exhaustive search*

```
/*4*/     configuration(Upper_price_limit).

/*5*/ configuration(Upper_price_limit):-
/*6*/     member(A,[a1,a2,a3]),
/*7*/     member(B,[b1,b2,b3,b4]),
/*8*/     member(C,[c1,c2]),
/*9*/     not(incompatibility([A,B])),
/*10*/    not(incompatibility([A,C])),
/*11*/    not(incompatibility([B,C])),
/*12*/    price(A,Price_of_A),
/*13*/    price(B,Price_of_B),
/*14*/    price(C,Price_of_C),
/*15*/    Total_price is Price_of_A+Price_of_B+Price_of_C,
/*16*/    Upper_price_limit>=Total_price,
/*17*/    write("Configuration("),write(A),write(","),
/*18*/    write(B),write(","),write(C),write(")"),
/*19*/    write(" priced at "),write(Total_price),
```

```
/*20*/    nl,fail.

/*21*/ configuration(Upper_price_limit):-
/*22*/    write("Those are all configurations "),
/*23*/    write("priced at no more than "),
/*24*/    write(Upper_price_limit),write(".").

/*25*/ price(a1,1900). price(a2,750). price(a3,900).
/*26*/ price(b1,300). price(b2,500). price(b3,450).
/*27*/ price(b4,600). price(c1,700). price(c2,850).
/*28*/ incompatible(c1,a2).
/*29*/ incompatible(b2,c2).
/*30*/ incompatible(c2,b3).
/*31*/ incompatible(b4,a2).
/*32*/ incompatible(b3,a1).
/*33*/ incompatible(a3,b3).

/*34*/ incompatibility([X,Y]):-incompatible(X,Y).
/*35*/ incompatibility([X,Y]):-incompatible(Y,X).
```

The message generated by the program is:

```
Configuration(a2,b1,c2) priced at 1900
Configuration(a3,b1,c1) priced at 1900
Configuration(a3,b1,c2) priced at 2050
Configuration(a3,b2,c1) priced at 2100
Those are all configurations  priced at no more than 2100.
```

It should be noted that:
1) The domains for variables `A`, `B` and `C` have been declared using the `member/2` predicate in lines `/*6*/`, `/*7*/` and `/*8*/`. The predicates from these lines always generate a full state `(A,B,C)` that is next tested for compatibility and price. The number of such states is `24`.
2) Because being incompatible is a commutative relation, instead of defining it once more for changed order of arguments, the predicate `incompatibility/1` has been introduced to take care of this.
3) The number of facts stating incompatibility is less than would be the number of facts stating compatibility. Therefore in lines `/*9*/`, `/*10*/` and `/*11*/` negated incompatibility is tested.
3) The built-in predicate `fail/0` is a predicate that always fails. It is used to force backtracking in order to find all solutions.

2.2.3 Backtracking search

The poor performance of exhaustive search is due to testing constraints only for *complete states*. It is obvious that some constraints may be tested earlier, for *contracted states*. This has been shown on Figure 2.5.

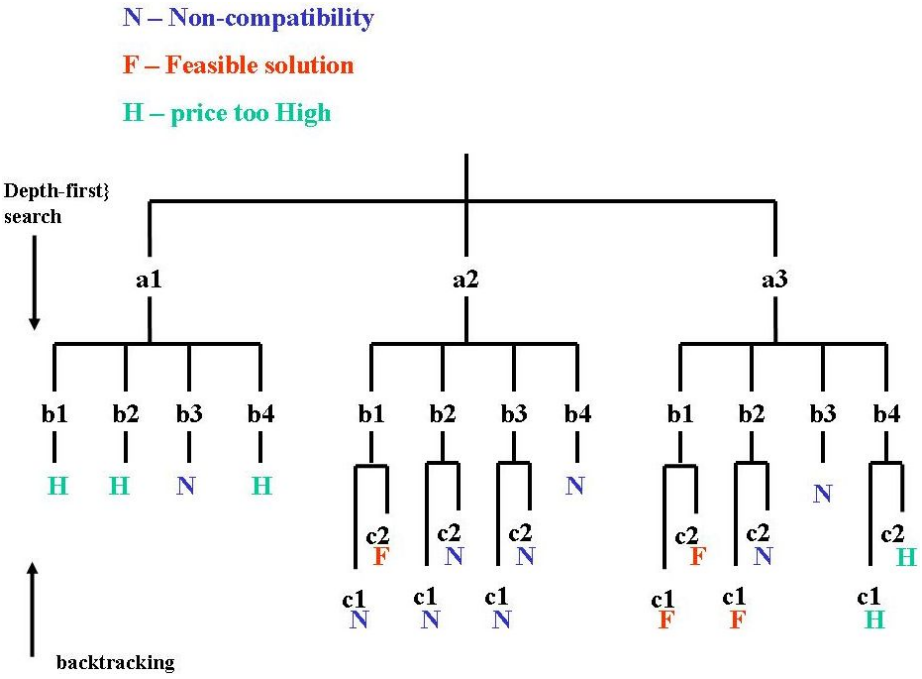


Figure 2.5: Search tree for *depth-first search with standard backtracking*

As can be seen from Figure 2.5, variable `A` is first grounded to `a1`. Because this is not the only possibility of grounding `A` (that may be grounded also to `a2` and `a3`), so a choice point for `A` has to be created. Next, variable `B` is grounded to `b1` while another choice point is created for `B` that may be grounded to `b2`, `b3`, and `b4` as well. The contracted state `(A,B) = (a1,b1)` corresponds to a partial configuration that costs more than the threshold price, so there is no point in going deeper in the search tree: backtracking is initiated consisting of:

- degrounding the last bounded variable `B = b1`;

- returning upward to the nearest choice point which is for `B`;
- because the choice point contains the untested value `b2`, variable `B` is grounded to `b2`;
- the choice point for `B` is retained with `b2` being removed from the set of untested values;
- the contracted state `(A,B) = (a1,b3)` corresponds to a partial configuration that contains incompatible elements. Therefore another backtracking is initiated.

A return may sometimes reach higher in the search tree, and therefore more grounded variables may be degrounded. E.g. for the contracted state `(A,B) = (a2,b4)` non-compatibility appears, so - as before - there is no reason in going deeper in the search tree: however, this time there is no choice point for `B` because all values for `B` have been tested. Therefore both grounded variables `A = a2` and `B = b4` have to be degrounded and a return to choice point for `A`, accompanied with restoring all domain values for `B` and `C`, has to be made. It results in grounding `A = a3`. This is not violating any constraints. So next `B` is grounded to `b1`; the contracted state `(A,B) = (a3,b1)` is satisfying all constraints so far and therefore `C` is grounded to `c1`; the full state `(A,B,C) = (a3,b1,c1)` is a feasible solution: the `(a3,b1,c1)` configuration fulfills all constraint. The discussed backtracking search for a feasible configuration is performed by program `2_8_conf_bs.pl`:

```
/*1*/  top:-
/*2*/     assert(upper_price_limit(2100)),
/*3*/     upper_price_limit(Upper_price_limit),
/*4*/     configuration(Upper_price_limit).

/*5*/  configuration(Upper_price_limit):-
/*6*/     member(A,[a1,a2,a3]),
/*7*/     price(A,Price_of_A),
/*8*/     Price_of_A =< Upper_price_limit,
/*9*/     member(B,[b1,b2,b3,b4]),
/*10*/    not(incompatibility([A,B])),
/*11*/    price(B,Price_of_B),
/*12*/    Price_of_AB is Price_of_A+Price_of_B,
/*13*/    Price_of_AB =< Upper_price_limit,
/*14*/    member(C,[c1,c2]),
/*15*/    not(incompatibility([A,C])),
/*16*/    not(incompatibility([B,C])),
/*17*/    price(C,Price_of_C),
```

```
/*18*/    Total_price is Price_of_A+Price_of_B+Price_of_C,
/*16*/    Total_price =< Upper_price_limit,
/*17*/    write("Configuration("),write(A),write(","),
/*18*/    write(B),write(","),write(C),write(")"),
/*19*/    write(" priced at "),write(Total_price),
/*20*/    nl,fail.


/*21*/ configuration(Upper_price_limit):-
/*22*/    write("Those are all configurations "),
/*23*/    write("priced at no more than "),
/*24*/    write(Upper_price_limit),write(").


/*25*/ price(a1,1900). price(a2,750). price(a3,900).
/*26*/ price(b1,300). price(b2,500). price(b3,450).
/*27*/ price(b4,600). price(c1,700). price(c2,850).


/*28*/ incompatible(c1,a2).
/*29*/ incompatible(b2,c2).
/*30*/ incompatible(c2,b3).
/*31*/ incompatible(b4,a2).
/*32*/ incompatible(b3,a1).
/*33*/ incompatible(a3,b3).


/*34*/ incompatibility([X,Y]):- incompatible(X,Y),!.
/*35*/ incompatibility([X,Y]):- incompatible(Y,X),!.
```

The message generated by the program is:

```
Configuration(a2,b1,c2) priced at 1900
Configuration(a3,b1,c1) priced at 1900
Configuration(a3,b1,c2) priced at 2050
Configuration(a3,b2,c1) priced at 2100
Those are all configurations priced at no more than 2100.
```

This is a good place to point at the effectiveness of depth-first backtracking search as compared with exhaustive search: for the latter the search tree had 24 leaves and therefore 24 backtrackings had to be made, whereas for the former there are 18 leaves and that many backtrackings to be performed. For larger state-spaces the savings due to depth-first backtracking are most often relatively larger.

2.3 Optimum configuration problems

2.3.1 Branch-and-bound for optimum configuration

Quite often we are interested in finding only an[17] optimum configuration for which the configuration price (considered to be the *objective function*) is minimized. This can be done by a slight modification of the `2_7_conf_es.pl` program that transforms it into a *branch-and-bound* search program. *Branch-and-bound* performs also depth-first search with backtracking, but does it differently, as shown in Figure 2.6.

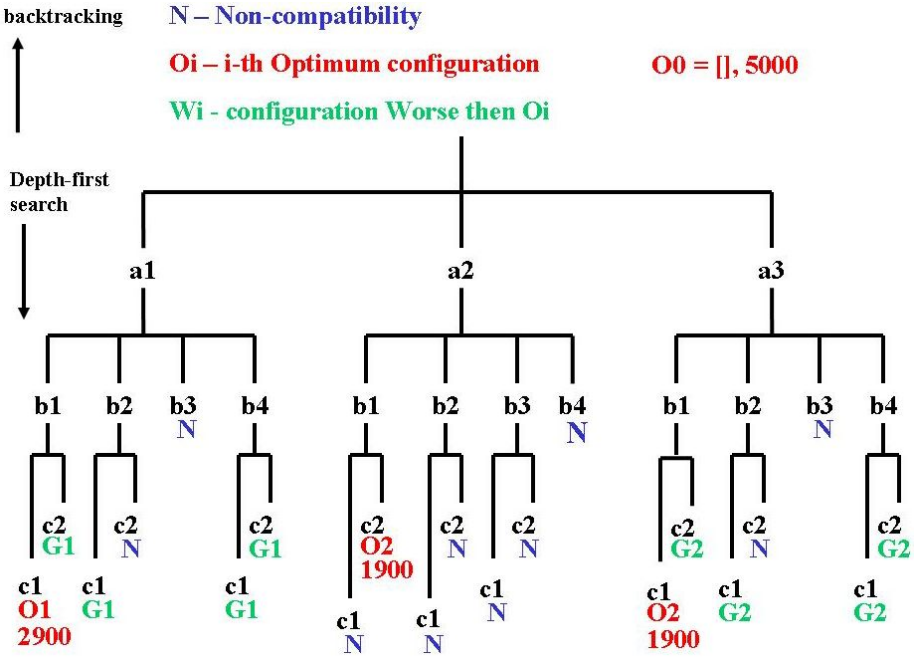


Figure 2.6: Search tree for *branch-and-bound* search

The idea of *branch-and-bound* (which is a general method for finding optimum

[17]There may be more than one optimum solution; Therefore we speak about *an* optimum solution rather than *the* optimum solution.

solutions for combinatorial optimization problems) may be described as follows:

1. A provisional lower bound for the objective function and associated optimum configuration is declared . This has been done by invoking a dynamic data base `optimum_configuration(Configuration,Price)` and asserting into it the initial lower bound, e.g. as `optimum_configuration([], 5000)`. I.e. the initial provisional optimum configuration is an empty one but quite expensive;

2. Next a depth-first search is started with constraints handled similarly as for depth-first backtracking search, but additionally:

   - all contracted states, for which the objective function is already larger than for the provisional lower bound, are handled like unsatisfied constraint, i.e. result in backtracking while the provisional lower bound remains unchanged;
   - all full states, for which the objective function is smaller or equal than for the provisional lower bound, are used to update this bound, which is followed by backtracking in order to search for (perhaps) a yet better configuration.

3. The sequence of those steps is repeated for all branches of the search tree.

This most simple version of *branch-and-bound* will be further referred to as `standard`. It is built into program `2_9_conf_opt.pl`:

```
/*1*/ top:-
/*2*/      assert(optimum_configuration([],5000)),
/*3*/      fail.

/*4*/ top:-
/*5*/     member(A,[a1,a2,a3]),
/*6*/     member(B,[b1,b2,b3,b4]),
/*7*/     not(incompatibility([A,B])),
/*8*/     price(A,Price_A),
/*9*/     price(B,Price_B),
/*10*/    optimum_configuration(_,Smallest_price_so_far),
/*11*/    Price_AB is Price_A+Price_B,
/*12*/    Price_AB<Smallest_price_so_far,
/*13*/    member(C,[c1,c2]),
/*14*/    not(incompatibility([A,C])),
/*15*/    not(incompatibility([B,C])),
/*16*/    price(C,Price_C),
```

```
/*17*/    Price is Price_AB+Price_C,
/*18*/    update_optimum_configuration([A,B,C],Price),
/*19*/    fail.

/*20*/ top:-
/*21*/    write("The least expensive configuration is: "),nl,
/*22*/    optimum_configuration([A,B,C],Price),
/*23*/    write("Configuration("),write(A),write(","),
/*24*/    write(B),write(", "),write(C),write(")"),
/*25*/    write(" priced at "),write(Price), nl,
/*26*/    fail.

/*27*/ top:-
/*28*/    write("Those are all optimum configurations.").

/*29*/ update_optimum_configuration([A,B,C],Price):-
/*30*/    optimum_configuration(_,Smallest_price_so_far),
/*31*/    Smallest_price_so_far > Price,
/*32*/    retract_all(optimum_configuration(_,_)),
/*33*/    assert(optimum_configuration([A,B,C],Price)),!.

/*34*/ update_optimum_configuration([A,B,C],Price):-
/*35*/    optimum_configuration(_,Smallest_price_so_far),
/*36*/    Smallest_price_so_far = Price,
/*37*/    assert(optimum_configuration([A,B,C],Price)),!.

/*38*/ update_optimum_configuration(_,Price):-
/*39*/    optimum_configuration(_,Smallest_price_so_far),
/*40*/    Smallest_price_so_far<Price,!.

/*41*/ price(a1,1900). price(a2,750). price(a3,900).
/*42*/ price(b1,300). price(b2,500). price(b3,450).
/*43*/ price(b4,600). price(c1,700). price(c2,850).

/*44*/ incompatible(c1,a2).
/*45*/ incompatible(b2,c2).
/*46*/ incompatible(c2,b3).
/*47*/ incompatible(b4,a2).
/*48*/ incompatible(b3,a1).
/*49*/ incompatible(a3,b3).

/*50*/ incompatibility([X,Y]):- incompatible(X,Y),!.
/*51*/ incompatibility([X,Y]):- incompatible(Y,X),!.
```

The message generated by the program is:

```
The least expensive configuration is:
```

```
Configuration(a2,b1, c2) priced at 1900
Configuration(a3,b1, c1) priced at 1900
Those are all optimum configurations.
```

2.4 Assignment problems

2.4.1 Golfers

Problems with negative information (i.e. stating that something is not true) may be quite cumbersome to solve, even if the negative information is scarce. A good illustration of such problems is given by the following example[18]:

A foursome of golfers (Fred, Joe, Bob, Tom) is standing at a tee, in a line from left to right. Each golfer wears different colored pants:

- one is wearing red pants;
- the golfer to Fred's immediate right is wearing blue pants;
- Joe is second in line;
- Bob is wearing plaid pants;
- Tom isn't in position one or four, and he isn't wearing the hideous orange pants.

In what order are the four golfers standing at the tee, and what color are each golfer's pants? This is a good opportunity to demonstrate ECL^iPS^e muscles!

The modeling starts with defining a number of private predicates. They may be the following:

- `conditions(Golfers_position,Golfers_name, Color_of_golfers_pants)`
- `all_positions_are_different(Position_1,Position_2,Position_3,Position_4)`
- `all_colors_are_different(Color_1, Color_2, Color_3,Color_4)`

[18]This FS-type problem has been first formulated and solved using the *Jess* Rule Engine for the Java Platform, see [Friedman-Hill-03].

- `wearing_red_pants(Position_1, Pants_color_for_golfer_on_position_1, Position_2, Pants_color_for_golfer_on_position_2, Position_3, Pants_color_for_golfer_on_position_3, Position_4, Pants_color_for_golfer_on_position_4)`
- `blue_pants_right_on_Fred(Position_1, Pants_color_for_golfer_on_position_1, Position_2, Pants_color_for_golfer_on_position_2, Position_3, Pants_color_for_golfer_on_position_3, Position_4, Pants_color_for_golfer_on_position_4)`
- `colors(colors_name)`
- `position(positions_number).`

They are used in the `2_10_golfers.pl` program :

```
/*1*/   top:-

   % Fred is standing somewhere (position P1) and has pants of some color (color C1):
/*2*/       conditions(P1,"Fred",C1),

   % 3)Joe is second in line (position P2, color of pants C2):
/*3*/       conditions(P2, "Joe",C2),
/*4*/       P2 is 2,

   % 4)Bob is wearing plaid pants (and stands at position P4):
/*5*/       conditions(P4, "Bob", C4),
/*6*/       C4 = plaid,

   % 5)Tom isn't in position one or four, and he isn't
   % wearing the hideous orange pants (stands at position P3 and has pants of color C3) :
/*7*/       conditions(P3,"Tom",  C3),
/*8*/       C3 \==  orange,
/*9*/       P3 =\= 2,
/*10*/      P3 =\= 4,

/*11*/      all_positions_are_different(P1, P2, P3, P4),
/*12*/      all_colors_are_different(C1, C2, C3, C4),

   % 1)someone is wearing red pants:
/*13*/      wearing_red_pants(P1, C1, P2, C2, P3, C3, P4, C4),

   % 2)the golfer to Fred's immediate right is wearing blue pants:
/*14*/      blue_pants_right_on_Fred(P1, C1, P2, C2, P3, C3, P4, C4),

/*15*/      write("Fred is in position "),write(P1),
                    write(" and wears "),write(C1), write(" pants."),nl,
/*16*/      write("Joe is in position "), write(P2),
                    write(" and wears "),write(C2), write(" pants."),nl,
/*17*/      write("Tom is in position "), write(P3),
                    write(" and wears "),write(C3), write(" pants."),nl,
```

```
/*18*/      write("Bob is in position "), write(P4),
                    write(" and wears "),write(C4), write(" pants."),nl,nl,fail.

/*19*/    top:-
/*20*/      write("That is all!"),nl.

/*21*/      conditions(Number,_,Color) :-
/*22*/      position(Number),
/*23*/      color(Color).

/*24*/    wearing_red_pants(_,C1,_,_,_,_,_,_) :-
/*25*/      C1 = red,!.
/*26*/    wearing_red_pants(_,_,_,C2,_,_,_,_) :-
/*27*/      C2 = red,!.
/*28*/    wearing_red_pants(_,_,_,_,_,_,C3,_,_) :-
/*29*/      C3 = red,!.
/*30*/    wearing_red_pants(_,_,_,_,_,_,_,_,C4) :-
/*31*/      C4 = red,!.

/*32*/    blue_pants_right_on_Fred(P1,_,P2,C2,_,_,_,_,_) :-
/*33*/      P2 is P1 + 1,
/*34*/      C2 = blue,!.
/*35*/    blue_pants_right_on_Fred(P1,_,_,_,_,P3,C3,_,_) :-
/*36*/      P3 is P1 + 1,
/*37*/      C3 = blue,!.
/*38*/    blue_pants_right_on_Fred(P1,_,_,_,_,_,_,P4,C4) :-
/*39*/      P4 is P1 + 1,
/*40*/      C4 = blue,!.

/*41*/    all_positions_are_different(X1, X2, X3, X4) :-
/*42*/      X1 =\= X2, X1 =\= X3, X1 =\= X4,
/*43*/      X2 =\= X3, X2 =\= X4, X3 =\= X4.

/*44*/    all_colors_are_different(X1, X2, X3, X4) :-
/*45*/      X1 \== X2, X1 \== X3, X1 \== X4,
/*46*/      X2 \== X3, X2 \== X4, X3 \== X4.

/*47*/      color(orange).
/*48*/      color(blue).
/*49*/      color(red).
/*50*/      color(plaid).

/*51*/      position(1).
/*52*/      position(2).
/*53*/      position(3).
/*54*/      position(4).
```

The solution is as follows:

```
Fred is in position 1 and wears orange pants.
Joe is in position 2 and wears blue pants.
Tom is in position 3 and wears red pants.
Bob is in position 4 and wears plaid pants.
```

2.4.2 Three cubes

Determining attributes for items described by different types of attributes is sometimes problem-ridden. In the following example three cube sizes should be determined while different cube attributes are disclosed such as number and color[19].

The three cubes are of different sizes (small, large, medium), of different colors (black,grey, white) and have different numbers (1,2,3). It is known that:

(1) The large cube is brighter than the medium cube;
(2) The small cube has number 2;
(3) The number of the black cube is greater than the number on the white cube;
(4) The size of cube with number 3 is smaller than the size of the grey cube.

What are the sizes, colors and numbers of all cubes? To solve the problem the following private predicates are defined:

- `cube(Color, Size, Number)`,
- `smaller_size(Smaller_size, Larger_size)`,
- `brighter(Brighter_color, Darker_color)`.

The program solving the puzzle (`2_11_three_cubes.pl`) is as follows:

```
/*1*/ top:-
   %(1) The large cube is brighter than the medium cube:
/*2*/      cube(Color_of_large_cube,large,Number_of_large_cube),
/*3*/      cube(Color_of_medium_cube,medium,Number_of_medium_cube),
/*4*/      brighter(Color_of_large_cube,Color_of_medium_cube),

   %(2) The small cube has number 2:
/*5*/      cube(Color_of_small_cube,small,2),
```

[19] This is an FS-type problem.

```
   %(3) The number of the black cube is greater than the number on the white cube:
/*6*/      cube(black,_,Number_of_black_cube),
/*7*/      cube(white,_,Number_of_white_cube),
/*8*/      Number_of_black_cube > Number_of_white_cube,

   %(4) The size of cube with number 3 is smaller than the size of the grey cube:
/*9*/     cube(_,Size_3, 3),
/*10*/    cube(grey,Size_of_grey_cube,Number_of_grey_cube),
/*11*/    smaller_size(Size_3,Size_of_grey_cube),

  %  The numbers are different:
/*12*/    Number_of_grey_cube =\= Number_of_white_cube,
/*13*/    Number_of_grey_cube =\= Number_of_black_cube,
/*14*/    Number_of_white_cube =\= Number_of_black_cube,
/*15*/    2 =\= Number_of_large_cube,
/*16*/    2 =\= Number_of_medium_cube,
/*17*/    Number_of_large_cube=\=Number_of_medium_cube,

  %  The colors are different:
/*18*/    Color_of_large_cube\==Color_of_medium_cube,
/*19*/    Color_of_large_cube\==Color_of_small_cube,
/*20*/    Color_of_small_cube\==Color_of_medium_cube,

/*21*/   writeln("Color of_large_cube": Color_of_large_cube),
/*22*/    writeln("Number of_large_cube": Number_of_large_cube),nl,
/*23*/    writeln("Color of_medium_cube": Color_of_medium_cube),
/*24*/    writeln("Number of_medium_cube": Number_of_medium_cube),nl,
/*25*/    writeln("Color of_small_cube": Color_of_small_cube),
/*26*/    writeln("Number of_small_cube": "2"),nl.

/*27*/ cube(Color,Size,Number):-
/*28*/    member(Color,[black,grey, white]),
/*29*/    member(Size,[small, large, medium]),
/*30*/    member(Number,[1,2,3]).

/*31*/ smaller_size(small, large).
/*32*/ smaller_size(small, medium).
/*33*/ smaller_size(medium, large).

/*34*/ brighter(white,grey).
/*35*/ brighter(white,black).
/*36*/ brighter(grey,black).
```

The program generates following solution:

```
  Color of_large_cube : grey
  Number of_large_cube : 1
```

```
Color of_medium_cube : black
Number of_medium_cube : 3

Color of_small_cube : white
Number of_small_cube : 2
```

2.4.3 Who is the killer?

A substantial difficulty while modeling problems in *Prolog* (or *CLP*) is the design of relevant private predicates. This is best seen for the next example, where the private predicates chosen are far from obvious. They are of course not the only choice that may be used for modeling the problem.

The following criminal puzzle[20] has to be solved[21]:
Mike has been murdered. Alex, Ben and Colin are the only suspects. While interrogated:
Alex said he is innocent, Ben was Mike's friend but Colin hated Mike.
Ben said that he was out of town on the day of the murder, besides he didn't even know Mike.
Colin said he is innocent but he saw Alex and Ben with Mike just before the murder.

Who killed Mark assuming that all except possibly the murderer are telling the truth? The suspects' statements are formalized using the private predicate:

```
statements_of_suspect(List_of_statements ).
```

The question is answered by program `2_12_who_killed.pl` that uses the well-known Sherlock Holmes principle: *connect facts in a consistent system and the solution follows.*

```
/*1*/ top:-
/*2*/    find_murderer.

/*3*/ statements_of_Alex([innocent("Alex"),friends("Ben","Mike"),
                           hates("Colin","Mike")]).
```

[20] This example is from http://www.binding-time.co.uk/whodunit.html
[21] This is an FS-type problem.

```
/*4*/ statements_of_Ben([alibi("Ben"),did_not_know("Ben","Mike")]).
/*5*/ statements_of_Colin([innocent("Colin"),together("Colin","Mike"),
                           together("Ben","Mike"),together("Alex","Mike")]).

/*6*/ find_murderer:-
/*7*/     statements_of_Alex(Statements_of_Alex),
/*8*/     statements_of_Ben(Statements_of_Ben),
/*9*/     statements_of_Colin(Statements_of_Colin),
/*10*/    consistent_statements(Statements_of_Ben,Statements_of_Colin),
/*11*/    inconsistent_statements(Statements_of_Alex,Statements_of_Ben),
/*12*/    inconsistent_statements(Statements_of_Alex,Statements_of_Colin),
/*13*/    write("Alex is the murderer."),nl,!.

/*14*/ find_murderer:-
/*15*/    statements_of_Alex(Statements_of_Alex),
/*16*/    statements_of_Ben(Statements_of_Ben),
/*17*/    statements_of_Colin(Statements_of_Colin),
/*18*/    consistent_statements(Statements_of_Alex,Statements_of_Colin),
/*19*/    inconsistent_statements(Statements_of_Alex,Statements_of_Ben),
/*20*/    inconsistent_statements(Statements_of_Ben,Statements_of_Colin),
/*21*/    write("Ben is the murderer."),nl,!.

/*22*/ find_murderer:-
/*23*/    statements_of_Alex(Statements_of_Alex),
/*24*/    statements_of_Ben(Statements_of_Ben),
/*25*/    statements_of_Colin(Statements_of_Colin),
/*26*/    consistent_statements(Statements_of_Alex,Statements_of_Ben),
/*27*/    inconsistent_statements(Statements_of_Alex,Statements_of_Colin),
/*28*/    inconsistent_statements(Statements_of_Ben,Statements_of_Colin),
/*29*/    write("Colin is the murderer."),nl,!.

/*30*/ consistent_statements(Statement_1,Statement_2):-
/*31*/    not(inconsistent_statements(Statement_1,Statement_2)).

/*32*/ inconsistent_statements(Statement_1,Statement_2):-
/*33*/    cartesian_product(Statement_1,Statement_2,Cartesian_product),
/*34*/    test_pairwise_inconsistency(Cartesian_product).

/*35*/ cartesian_product([], _, []).
/*36*/ cartesian_product([H|T], L, M) :-
/*37*/    generate_pairs(H,L,M1),
/*38*/    cartesian_product(T, L, M2),
/*39*/    append(M1, M2, M).

/*40*/ generate_pairs(_, [], []).
/*41*/ generate_pairs(A, [B|L], [[A,B]|N] ) :-
/*42*/ generate_pairs(A, L, N).

/*43*/ test_pairwise_inconsistency([[H1,H2]|T]):-
```

```
/*44*/    not(inconsistent_pairs(H1,H2)),
/*45*/    test_pairwise_inconsistency(T).
/*46*/ test_pairwise_inconsistency([[H1,H2]|_]):-
/*47*/    inconsistent_pairs(H1,H2),
/*48*/    !.

/*49*/ inconsistent_pairs(P1,P2):-
/*50*/    inconsistency([P1,P2]).
/*51*/ inconsistent_pairs(P1,P2):-
/*52*/    inconsistency([P2,P1]).

/*53*/ inconsistency([hates("Ben","Mike"),friends("Ben","Mike")]).
/*54*/ inconsistency([friends("Ben","Mike"),did_not_know("Ben","Mike")]).
/*55*/ inconsistency([together("Ben","Mike"),did_not_know("Ben","Mike")]).
/*55*/ inconsistency([friends("Colin","Mike"),hates("Colin","Mike")]).
/*56*/ inconsistency([innocent("Alex"),guilty("Alex")]).
/*57*/ inconsistency([innocent("Colin"),guilty("Colin")]).
/*58*/ inconsistency([alibi("Ben"),together("Ben","Mike")]).
/*59*/ inconsistency([alibi("Ben"),guilty("Ben")]).
/*60*/ inconsistency([alibi("Colin"),together("Colin","Mike")]).
```

The solution is:

```
Ben is the murderer.
```

2.4.4 Placing queens - defining variables

The queens placement problem[22] is a favorite AI benchmark: it aims at finding all placements of `N` queens on an $N \times N$ chessboard in a way that no single queen must be able to attack the other[23]. As often happens in *Prolog*, the way variables are defined is crucial for the search effectiveness. For an 8×8 chessboard, variables are defined by the list:

```
[X1,X2,...,Xi,...,X8]
```

where `Xi` is the number of the chessboard row, for which the queen is placed in the `i`th column. This definition alone satisfies two constraints:

1. No two queens will ever appear in the same column because each list position is unique.

[22] This is an FS-type problem.

[23] Advanced cases of the benchmark operate for chessboards accommodating hundreds of queens.

2. No two queens will ever appear in the same row provided the 8-tuple `X1,X2,...,X8` is equal to a *permutation* of the 8-tuple `1,2,3,4,5,6,7,8`.

2.4.5 Exhaustive search for queens

To program exhaustive search for queens, following private predicate are introduced:

- `eight_queens([X1,X2,...,X8])` with argument given by the list of queens is the main predicate.
- `permutations(Permutation_List,Initial_List)`, which calculates consecutive permutations of the initial list `[1,2,3,4,5,6,7,8]`.
- `save([New_queen_to_be_placed|List_of_queens_already_placed])`, which is fulfilled if the new queen to be placed is not attacking any queen on the list of already placed queens.
- `no_attack(New_Queen_to_be_placed,List_of_queens_already_placed)` that initiates the checks of conflicts between the `New_Queen_to_be_placed` and the `List_of_queens_already_placed`.
- `no_attack(New_Queen_to_be_placed,List_of_queens_already_placed, Shift_of_New_Queen_to_be_placed_on_the_diagonal)` that actually checks for the absence of conflicts for feasible shifts of the new queen to consecutive columns along the upward and downward diagonal, starting with shift `1`.

The exhaustive search generates consecutively all permutations of the 8-tuple `1,2,3,4,5,6,7,8`, and next checks, whether it corresponds to a safe placement. This is done by the `2_13_queens_es.pl` program:

```
/*1*/  top:-
/*2*/      all_solutions.

/*3*/  eight_queens([X1,X2,X3,X4,X5,X6,X7,X8]):-
/*4*/      permutations([X1,X2,X3,X4,X5,X6,X7,X8],[1,2,3,4,5,6,7,8]),
/*5*/      safe([X1,X2,X3,X4,X5,X6,X7,X8]).

/*6*/  permutations([],[]).
/*7*/  permutations([X|Xs],Ls):-
/*8*/      remove(X,Ls,Rs),
/*9*/      permutations(Xs,Rs).
```

```
/*10*/ remove(X,[X|Xs],Xs).
/*11*/ remove(X,[Y|Ys],[Y|Rs]):-
/*12*/    remove(X,Ys,Rs).

/*13*/ safe([]).
/*14*/ safe([X|Xs]):-
/*15*/    no_attack(X,Xs),
/*16*/    safe(Xs).

/*17*/ no_attack(X,Xs):-
/*18*/    no_attack(X,Xs,1).

/*19*/ no_attack(_,[],_).
/*20*/ no_attack(X,[Y|Ys],Nb):-
/*21*/    X=\=Y-Nb,
/*22*/    X=\=Y+Nb,
/*23*/    Nb1 is Nb+1,
/*24*/    no_attack(X,Ys,Nb1).

/*25*/ all_solutions:-
/*26*/    eight_queens(X),
/*27*/    write(X),nl,
/*28*/    fail.
/*29*/ all_solutions:-
/*30*/    write("That's all!").
```

There are 92 placements, from which only the first and last two are presented:

```
[1, 5, 8, 6, 3, 7, 2, 4]
[1, 6, 8, 3, 7, 4, 2, 5]
.........................
[8, 3, 1, 6, 2, 5, 7, 4]
[8, 4, 1, 3, 6, 2, 7, 5]
That's all!
```

The last but one placement is shown in Figure 2.7.

The exhaustive search tree for 8 trees is just too large to be presented. Instead a smaller exhaustive search tree for 4 queens is shown in Figure 2.8.

2.4.6 Backtracking search for queens

Exhaustive search has an obvious shortcoming discussed already in Sections 1.2 and 2.2.2. Assume that already the placement of the first two queens is unsafe.

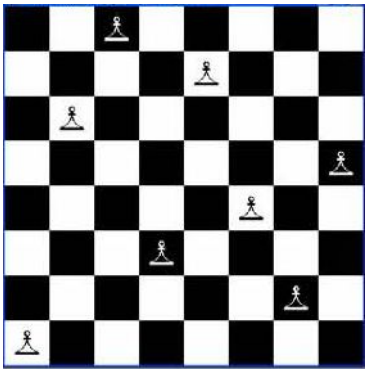


Figure 2.7: Last but one placement of 8 queens

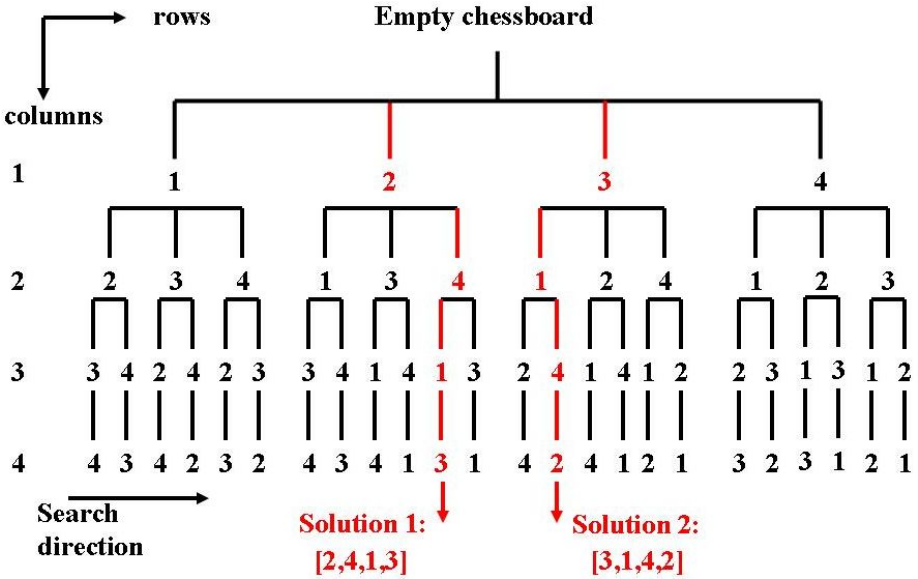


Figure 2.8: Exhaustive search tree for 4 queens

Then, instead canceling the last placement and returning to the nearest safe placement, the placement of queens is continued, and only after all queens have been placed, the safety of the placement is checked.

Exhaustive search could be improved upon by a following search strategy: let the list `[x1,x2,..xi]` corresponds to a safe placement of the first `i` queens. Another queen is added to the list and a safety check is performed. If the placement remains safe, yet another queen is added. If the safety check fails, a return is initiated to such previous placement, for which some untested queen choice is still possible. Such search strategy, recognized as *depth-first search with standard backtracking*, is performed by the program `2_13_queens_bs.pl`. The search may be made yet more effective by noticing that the used modeling of placements defined by the list:

`[X1,X2,...,Xi,...,X8]`

where `Xi` is the number of the chessboard row, for which the queen is placed in the `i`th column, has yet another important benefits. It is, by its very nature, fulfilling two constraints:

1. No two queens will ever be placed in the same column, because any `Xi` occupies the unique `i`th position in the list.
2. No two queens will ever be placed in the same row, because the value of any `Xi` is uniquely determined from a list of integers `[1,2,3,4,5,6,7,8]`.

So search for safe placements has to be done only along the upward and downward diagonal of the chessboard.

To program depth-first search with backtracking for queens, following private predicate are introduced:

- ```
  queens(List_of_queens_added_to_queens_placed,
            List_of_queens_already_placed,
                    List_of_available_queens)
  ```
  that is extracting queens from the `List_of_available_queens` using variables from the `List_of_queens_added_to_queens_placed` and testing safety for the chosen queen to be added. Only if the new placement would be safe, the chosen queen is actually added to the `List_of_queens_already_placed`.
- `no_attack/2` has been defined in Section 2.4.5.
- `no_attack/3` has been defined in Section 2.4.5.
  ```

- `remove(Head,[Head|Tail],Tail)` that removes the `Head` of the list `[Head|Tail]` returning `Tail`.

The program `2_14_queens_bs.pl` is as follows:

```
/*1*/    top:-
/*2*/         all_solutions.

/*3*/    eight_queens(X):-
/*4*/         queens(X,[],[1,2,3,4,5,6,7,8]).

/*5*/    queens([],_,[]).
/*6*/    queens([X|Xs],Placed,List_of_available_queens):-
/*7*/         remove(X,List_of_available_queens,New_list_of_available_queens),
/*8*/         no_attack(X,Placed),
/*9*/         queens(Xs,[X|Placed],New_list_of_available_queens).

/*10*/   remove(X,[X|Xs],Xs).
/*11*/   remove(X,[Y|Ys],[Y|Rs]):-
/*12*/        remove(X,Ys,Rs).

/*13*/   no_attack(X,Placed):-
/*14*/        no_attack(X,Placed,1).
/*15*/   no_attack(_,[],_).

/*16*/   no_attack(X,[Y|Ys],Nb):-
/*17*/        X=\=Y-Nb,
/*18*/        X=\=Y + Nb,
/*19*/        Nb1 is Nb + 1,
/*20*/        no_attack(X,Ys,Nb1).

/*21*/   all_solutions:-
/*22*/        eight_queens(X),
/*23*/        write(X),nl,
/*24*/        fail.
/*25*/   all_solutions:-
/*26*/        write("That's all!").
```

The message generated by this program is the same as for the `2_13_queens_es.pl` program.

The recursive beauty of the definitions for `no_attack/3` and `queens/3` is worth contemplating for a while. The role of variable `X` for determining the queen to be added is worth noting: if the `no_attack/2` predicate in line `/*8*/` fails, backtracking is performed to line `/*7*/` where a new value `X` is picked from

the `List_of_available_ queens`. The new queens row position `X` is checked against consecutively placed queens for consecutively shifting columns alongside the upward and downward diagonal in line `/*8*/`. Notice also in line `/*9*/` how the approved `X` is added as head of a list of queens already placed.

The same reason as given for exhaustive search makes it impossible to picture the depth-first backtracking search tree for `/*8*/` queens. Instead a more simple case for 4 queens is illustrated by Figure 2.9. Obviously, depth-first backtracking search is again more effective than exhaustive search: instead of 24 leaves, the tree has now only 18 leaves.

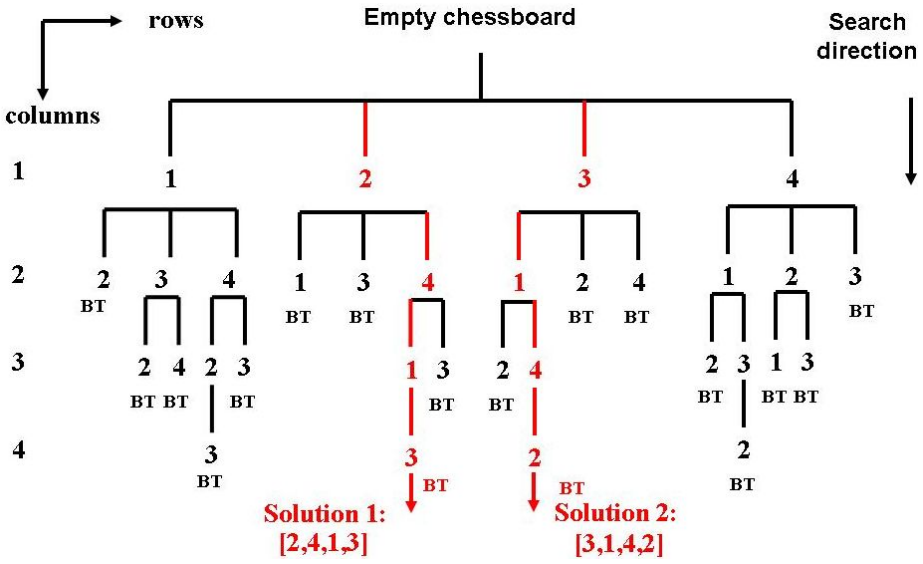


Figure 2.9: Depth-first backtracking search for 4 queens.

Additionally an animation of search for this search tree is shown in Figures 2.10 and 2.11, where the abbreviation `BT` means *BackTrack*.

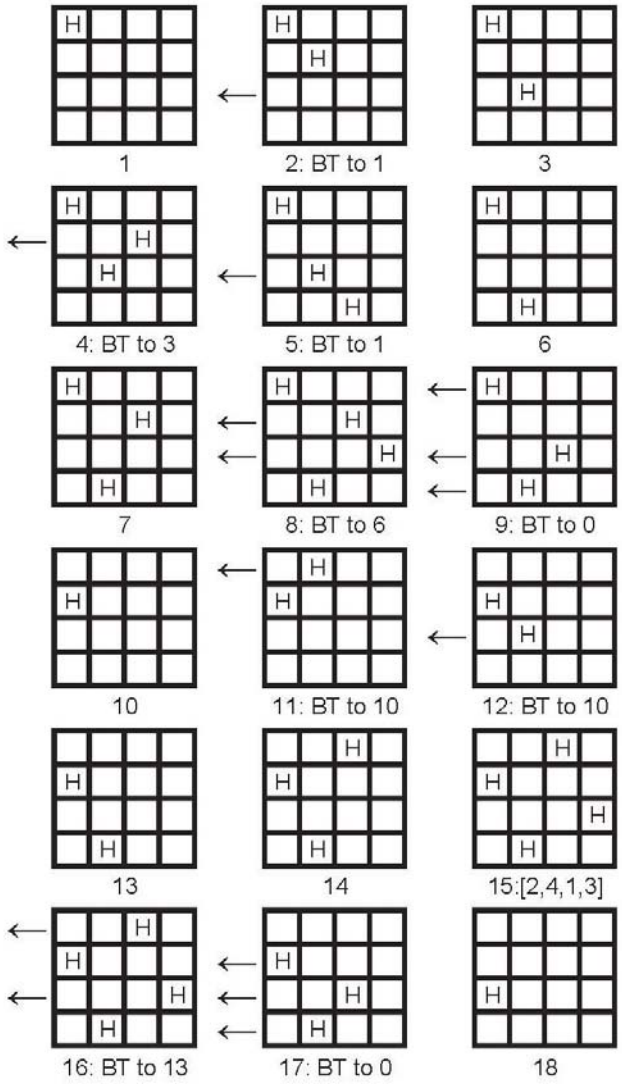


Figure 2.10: Animation of search for 4 queens search tree, part 1

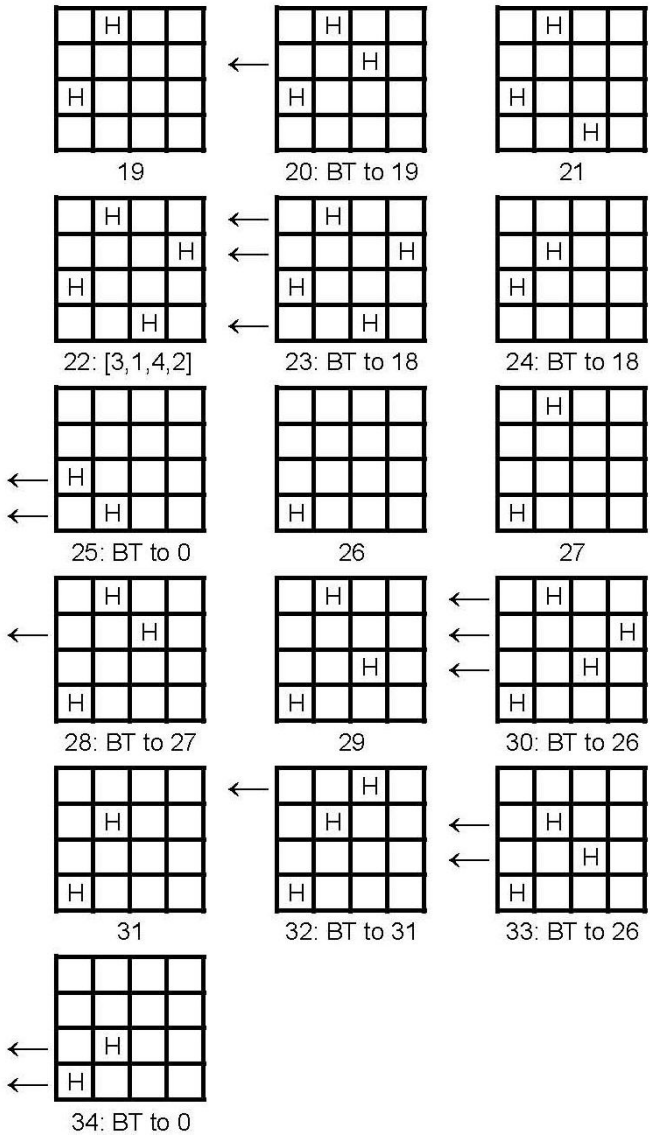


Figure 2.11: Animation of search for 4 queens search tree, part 2

2.4.7 Examination - backtracking search

Quite often puzzles are saturated with *negative knowledge* i.e. knowledge about what should not be done. Such puzzles present no special problem to *Prolog* as shown by the following problem:

An examination room has 17 places arranged as shown in Table 2.8.

	M1	M2	M3	M4
M5	M6	M7	M8	M9
M10	M11	M12	M13	M14
		M15	M16	M17

Table 2.8: Examination room layout

This room will be used for a written examination taken by 17 students who are expected to solve problems in one of four different examination papers, numbered `1`, `2`, `3` and `4`. The teaching staff wants to secure themselves against cheating. So students writing the same paper had to be completely isolated from each other - so much so that their places were not adjacent in any way (horizontally, vertically or at corners). How to distribute the papers among places to achieve this? Could it be done if there were only three different examination papers[24]?

The first question is answered by program `2_15_exzamination.pl`:

```
/*0*/   top:-
/*1*/       L=[1,2,3,4],
/*2*/       member(M1,L),   /*3*/   member(M2,L),
/*4*/       member(M3,L),   /*5*/   member(M4,L),
/*6*/       member(M5,L),   /*7*/   member(M6,L),
/*8*/       member(M7,L),   /*9*/   member(M8,L),
/*10*/      member(M9,L),   /*11*/  member(M10,L),
/*12*/      member(M11,L),  /*13*/  member(M12,L),
/*14*/      member(M13,L),  /*15*/  member(M14,L),
/*16*/      member(M15,L),  /*17*/  member(M16,L),
/*18*/      member(M17,L),

/*19*/      M1 =\= M2,        /*20*/      M1 =\= M5,
/*21*/      M1 =\= M6,        /*22*/      M1 =\= M7,
/*23*/      M2 =\= M6,        /*24*/      M2 =\= M7,
```

[24]This is an FS-type problem.

```
/*25*/      M2 =\= M3,        /*26*/      M2 =\= M8,
/*27*/      M3 =\= M7,        /*28*/      M3 =\= M8,
/*29*/      M3 =\= M9,        /*30*/      M3 =\= M4,
/*31*/      M4 =\= M8,        /*32*/      M4 =\= M9,
/*33*/      M5 =\= M6,        /*34*/      M5 =\= M10,
/*35*/      M5 =\= M11,       /*36*/      M6 =\= M10,
/*37*/      M6 =\= M11,       /*38*/      M6 =\= M7,
/*39*/      M6 =\= M12,       /*40*/      M7 =\= M11,
/*41*/      M7 =\= M12,       /*42*/      M7 =\= M8,
/*43*/      M7 =\= M13,       /*44*/      M8 =\= M12,
/*45*/      M8 =\= M13,       /*46*/      M8 =\= M14,
/*47*/      M8 =\= M9,        /*48*/      M9 =\= M13,
/*49*/      M9 =\= M14,       /*50*/      M10 =\= M11,
/*51*/      M11 =\= M15,      /*52*/      M11 =\= M12,
/*53*/      M12 =\= M15,      /*54*/      M12 =\= M16,
/*55*/      M12 =\= M13,      /*56*/      M13 =\= M15,
/*57*/      M13 =\= M16,      /*58*/      M13 =\= M17,
/*59*/      M13 =\= M14,      /*60*/      M14 =\= M16,
/*61*/      M14 =\= M17,      /*62*/      M15 =\= M16,
/*63*/      M16 =\= M17,
/*64*/      write("   "),write(M1),write(", "), write(M2),
              write(", "),write(M3),write(", "), write(M4),nl,
/*65*/      write(M5),write(", "),write(M6),write(", "), write(M7),
              write(", "),write(M8),write(", "), write(M9),nl,
/*66*/      write(M10),write(", "),write(M11),write(", "), write(M12),
              write(", "), write(M13),write(", "), write(M14),nl,
/*67*/      write("      "),write(M15),write(", "), write(M16),
              write(", "),write(M17), nl.
```

```
One possible solution is as follows:

   1, 2, 1, 2
2, 3, 4, 3, 4
4, 1, 2, 1, 2
      3, 4, 3
```

It took quite a long time (203 seconds on a na 2.0 GHz notebook running under Windows XP). In Section 3.7.4 another more efficient way to solve the problem as *CLP* problem will be presented.

It is easy to show that for three different examination papers there is no feasible assignment of papers to places: it suffices to change the lists in lines `/*2*/`,...,`/*18*/` for `[1,2,3]` to get the message `No`. The discussed problem is a demonstration of the famous *four color theorem*, which states that the minimum number of colors needed to color any planar map (or nodes of a corresponding planar graph), in a way that all adjacent colors are different, is `four`[25].

2.4.8 Paradoxes in Prolog

A paradox is a self-contradictory or counter-intuitive statement or argument in logic. Most often it cannot be true but also cannot be false. Consider the famous Bertrand Russel *barber paradox*: a small-town barber is ordered to shave all those male inhabitants, and those only, who do not shave themselves. The question is, may the barber shave himself?

It can be shown that whatever does the barber, the imposed order is violated:

- if he shaves himself then, as a *shaving himself male inhabitant*, he should not be shaved by the barber, i.e. by himself;
- if he does not shave himself then, as a *not shaving himself male inhabitant*, he should be shaved by the barber i.e. by himself.

So we have a *vicious circle*: it results because the barber is also this small-town inhabitant. Any barber coming from a neighborhood town could easily shave himself or use the services provided by the small-town barber.

Any attempt to solve this paradox using *Prolog* is bound to lead to stack overflow no matter how large the stack. This is illustrate by program `2_16_barber.pl`:

```
/*1*/   top:-
/*2*/         shaves(barber,barber).

/*3*/   shaves(barber,X):-
/*4*/         not(shaves(X,X)).
```

[25] It is interesting to know that this apparently simple theorem resisted a long series of attempts to prove it using mathematics. It finally succumbed to a computer-assisted proof (sort of exhaustive search), which demonstrated the non-existence of plenary graphs that would need five colors to color them in the sense of the theorem, see [Lines-92]. The mathematicians weren't quite happy about it.

The message generated is:

```
Overflow of the local/control stack!
You can use the "-l kBytes" (LOCALSIZE) option to have a larger stack.
Peak sizes were: local stack 40384 kbytes, control stack 90688 kbytes
```

The stack overflow is caused because to satisfy the predicate `shaves(barber, barber)`, the negated predicate `not(shaves(barber,barber))` has to be satisfied, but to achieve this the predicate `shaves(barber,barber)` has to fail, and a backtrack occurs to line `/*3*/`, etc. etc., the viciousness is there. What's more, any backtrack to the predicate `shaves(barber,barber)` is accompanied by saving some information on the internal *Prolog* stack that sooner or latter is overfilled. The advice provided automatically for such cases about increasing the stack sizes is - for the discussed situation - entirely inadequate.

It may be added that the rule in lines `/*3*/` and `/*4*/` is not a recursive *Prolog* rule, because it is not defined between a list (in the head of the recursive rule) and the tail of this list (in the body of the recursive rule).

Because *Prolog* (exactly like human reasoning) is powerless against vicious circles resulting from paradoxes, they should be avoided exactly like avoiding division by `0`. A consolations is provided by the circumstance that properly and completely defined real-world problems are free of vicious circles. E.g. if the problem was: 1)The barber shaves himself, and 2)The barber shaves all remaining small-town male inhabitants who do not shave themselves, then - contrasted with intellectual puns - no vicious circle will appear.

2.4.9 How to become your own grandfather?

Nicklaus Wirth in his popular textbook [Wirth-75] presented the following story (sometimes attributed also to Mark Twain) of a man complaining about the wretchedness of his life[26]:

I married a widow with a grown daughter. My father, who visited us frequently, fell in love with the daughter and took her as his wife. This made my father my adopted son, and my adopted daughter became my stepmother.

After a year my wife gave birth to a son, who became the adopted brother

[26]This is an FS-type problem.

of my father and at the same time my uncle, since he was my stepmother's brother.

But my father's wife, i.e. my adopted daughter, also gave birth to a son. So this was my brother and also my grandson, since he was the son of my daughter.

This meant I'd married my grandmother, since she was the mother of my mother. As my wife's husband, I was also her adopted grandson.

Our friends say that I am my own grandfather, Is it true?

Let's prove it using *Prolog* under assumption that an *adopted* family relation is to be treated as a *normal* one, e.g. *adopted daughter* = *daughter*, *adopted brother* = *brother*, etc. The following order of arguments is assumed for private predicates:

```
father(Father,Son),
mother(Mother,Son/Daughter),
grandfather(Grandfather,Grandson),
grandmother(Grandmother,Grandson),
brother(Father,Brother_1,Brother_2),
uncle(Father,Uncle,Nephew).
```

The program (`2_17_grandfather.pl`) is as follows:

```
/*1*/      top:-

  % My and my_wifes son is the adopted brother of my father:
/*2*/              brother(_,my_father,my_and_my_wifes_son),

  % My and my_wifes son is my uncle:
/*3*/              uncle(my_and_my_wifes_son,my_father,me),

  % Son of my adopted_daughter is my brother:
/*4*/              brother(_,son_of_my_adopted_daughter,me),

  % Son of my adopted_daughter is my grandson:
/*5*/              grandfather(me,son_of_my_adopted_daughter),

  % my wife is my grandmother;
/*6*/              grandmother(my_wife,me),

  % I am my own grandfather:
/*7*/              grandfather(me,me),nl,
```

```
/*8*/             write("Everything is O.K.").

/*9*/     grandfather(Grandfather,Grandson):-
/*10*/            father(Grandfather,Grandfathers_son),
/*11*/            father(Grandfathers_son,Grandson).

/*12*/   grandmother(Grandmother,Grandson):-
/*13*/            mother(Grandmother,Grandmothers_daughter),
/*14*/            mother(Grandmothers_daughter,Grandson).

  % I am the son of my father:
/*15*/     father(my_father,me).

  % my father is my adopted son:
/*16*/     father(me,my_father).

  % I am the father of my wife's son:
/*17*/     father(me,my_and_my_wifes_son).

  % my father is the father of the son of my adopted_daughter:
/*18*/     father(my_father,son_of_my_adopted_daughter).

  % My adopted_daughter is my stepmother:
/*19*/     mother(my_adopted_daughter,me).

  % My wife is the mother of my adopted daughter:
/*20*/     mother(my_wife,my_adopted_daughter).

/*21*/     brother(Father,Brother_1,Brother_2):-
/*22*/            father(Father,Brother_1),
/*23*/            father(Father,Brother_2).

/*24*/     uncle(Father,Uncle,Nephew):-
/*25*/            brother(_,Father,Uncle),
/*26*/            father(Uncle,Nephew).
```

The message generated is

```
Everything is O.K.
```

meaning that all postulated family relations are true.

2.4.10 Using conditional predicates

The basic conditional built-in:

```
+Condition -> +Then ; +Else.
```

has following properties:

- First `Condition` is called and if it succeeds any further solutions of `Condition` are cut and `Then` is called. `Else` is never called in this case regardless of the outcome of `Then`.
- If `Condition` fails, `Else` is called. In this case, `Then` is never called.

In *Prolog* programs for `Else` often stands `True` with obvious meaning. The predicate may be used to simplify some *Prolog* programs as shown by the example:

Andrew, Barbara and Christopher have decided to attend some extracurricular lectures. Each one choose a different lecture, in different days, on different hours, namely:
1) Andrew will attend the lecture by Professor Paul.
2) Tuesdays lecture does not start at 2:00 p.m.
3) The lecture on "Knowledge engineering" does not start at 5:30 p.m.
4) Thursdays lecture start at 3:45 p.m.
5) Christopher will attend the lecture on "Econometric models".
6) Barbara would like to attend the Tuesday lecture.
7) The lecture on "Artificial Intelligence" is delivered in Building D3.
8) Wednesdays lecture are delivered in Room 104.
9) Professor Smith is not delivering the lecture "Econometric models".
10)Professor Jones is not delivering his lecture in Room K2.
A program that determines who will attend which lecture, where, when and delivered by whom, is given by `2_25_lectures.pl`:

```
/*1*/  top:-
/*2*/          students(Name_1,Name_2,Name_3),
/*3*/          lectures(Lecture_1,Lecture_2,Lecture_3),
/*4*/          professors(Professor_1,Professor_2,Professor_3),
/*5*/          rooms(Room_1,Room_2,Room_3),
/*6*/          days(Day_1,Day_2,Day_3),
/*7*/          hours(Hour_1,Hour_2,Hour_3),

/*8*/          constraints(Name_1,Lecture_1,Professor_1,Room_1,Day_1,Hour_1),
/*9*/          constraints(Name_2,Lecture_2,Professor_2,Room_2,Day_2,Hour_2),
/*10*/         constraints(Name_3,Lecture_3,Professor_3,Room_3,Day_3,Hour_3),

/*11*/         write(Name_1),write(" will attend a lecture on "),write(Lecture_1),
               write(" by Professor "),write(Professor_1), write(" in Room "),
               write(Room_1), write(" on "), write(Day_1), write(" at "),
               write(Hour_1), nl,
```

```
/*12*/         write(Name_2),write(" will attend a lecture on "),write(Lecture_2),
               write(" by Professor "),write(Professor_2), write(" in Room "),
               write(Room_2), write(" on "), write(Day_2), write(" at "),
               write(Hour_2),nl,

/*13*/         write(Name_3),write(" will attend a lecture on "),write(Lecture_3),
               write(" by Professor "),write(Professor_3),  write(" in Room "),
               write(Room_3), write(" on "), write(Day_3), write(" at "),
               write(Hour_3),nl.

/*14*/ students(Name_1,Name_2,Name_3):-
/*15*/         name(Name_1),
/*16*/         name(Name_2),
/*17*/         name(Name_3),
/*18*/         all_different(Name_1,Name_2,Name_3).

/*19*/ lectures(Lecture_1,Lecture_2,Lecture_3):-
/*20*/         lecture(Lecture_1),
/*21*/         lecture(Lecture_2),
/*22*/         lecture(Lecture_3),
/*23*/         all_different(Lecture_1,Lecture_2,Lecture_3).

/*24*/ professors(Professor_1,Professor_2,Professor_3):-
/*25*/         professor(Professor_1),
/*26*/         professor(Professor_2),
/*27*/         professor(Professor_3),
/*28*/         all_different(Professor_1,Professor_2,Professor_3).

/*29*/ rooms(Room_1,Room_2,Room_3):-
/*30*/         room(Room_1),
/*31*/         room(Room_2),
/*32*/         room(Room_3),
/*33*/         all_different(Room_1,Room_2,Room_3).

/*34*/ days(Day_1,Day_2,Day_3):-
/*35*/         day(Day_1),
/*36*/         day(Day_2),
/*37*/         day(Day_3),
/*38*/         all_different(Day_1,Day_2,Day_3).

/*39*/ hours(Hour_1,Hour_2,Hour_3):-
/*40*/         hour(Hour_1),
/*41*/         hour(Hour_2),
/*42*/         hour(Hour_3),
/*43*/         all_different(Hour_1,Hour_2,Hour_3).

/*44*/ constraints(Name,Lecture,Professor,Room,Day,Hour):-
/*45*/         (  (Name == "Andrew")-> Professor = "Paul"
/*46*/         ; true   ),
```

```
/*47*/         (  (Day == "Tuesday")-> Hour \== "2:00 p.m."
/*48*/         ; true   ),

/*49*/         (  ( Lecture == "Knowledge Engineering")->  Hour \== "5:30 p.m."
/*50*/         ; true   ),

/*51*/         (  ( Day == "Thursday")->  Hour = "3:45 p.m."
/*52*/         ; true   ),

/*53*/         (  ( Name == "Christopher")->  Lecture = "Econometric Models"
/*54*/         ; true   ),

/*55*/         (  ( Name == "Barbara") ->  Day = "Tuesday"
/*56*/         ; true   ),

/*57*/         (  ( Lecture == "Artificial Intelligence") ->  Room = "D3"
/*59*/         ; true   ),

/*59*/         (  ( Day == "Wednesday") -> Room \== "104"
/*60*/         ; true   ),

/*61*/         (  ( Professor == "Smith") ->  Lecture \== "Econometric Models"
/*62*/         ; true   ),

/*63*/         (  ( Professor == "Jones") -> Room \== "K2"
/*64*/         ; true   ).

/*65*/ all_different(Variable_1,Variable_2,Variable_3):-
/*66*/         Variable_1 \== Variable_2,
/*67*/         Variable_1 \== Variable_3,
/*68*/         Variable_2 \== Variable_3.

/*69*/ name("Andrew").
/*70*/ name("Barbara").
/*71*/ name("Christopher").

/*72*/ lecture("Knowledge Engineering").
/*73*/ lecture("Econometric Models").
/*74*/ lecture("Artificial Intelligence").

/*75*/ professor("Paul").
/*76*/ professor("Smith").
/*77*/ professor("Jones").

/*78*/ room("D3").
/*79*/ room("104").
/*80*/ room("K2").
```

```
/*81*/ day("Tuesday").
/*82*/ day("Wednesday").
/*83*/ day("Thursdays").

/*84*/ hour("2:00 p.m.").
/*85*/ hour("5:30 p.m.").
/*86*/ hour("3:45 p.m.").
```

The solution is:

```
Andrew will attend a lecture on Knowledge Engineering by
   Professor Paul in Room K2 on Wednesday at 2:00 p.m.
Barbara will attend a lecture on Artificial Intelligence by
   Professor Smith in Room D3 on Tuesday at 5:30 p.m.
Christopher will attend a lecture on Econometric Models by
   Professor Jones in Room 104 on Thursdays at 3:45 p.m.
```

2.5 Sequencing problems

2.5.1 Farmer-wolf-goat-cabbage

This popular puzzle is a nice example of finding trajectories in the state space:

A farmer is standing on the west side of the river and with him are a wolf, a goat and a cabbage. In the river there is a small boat. The farmer wants to cross the river with all the three items that are with him. There are no bridges and in the boat there is only room for the farmer and one item. However, the crossings are danger-ridden:

- If the farmer leaves the goat with the cabbages alone on the same side of the river, the goat will eat the cabbages.
- If the farmer leaves the wolf and the goat on the same side of the river, the wolf will eat the goat.

Only the farmer can separate the wolf from the goat and the goat from the cabbage. How can the farmer cross the river with all three items, without one eating the other[27]?

[27]This OST-type problem is attributed to Alcuin of York (730 - 804), an English scholar,

The first thing needed is to define a *state* that accumulates all data needed to properly determine the next move. The state of the system *farmer-wolf-goat-cabbage* is given by declaring their whereabouts, see Figure 2.12. While crossing the river no state may appear twice.

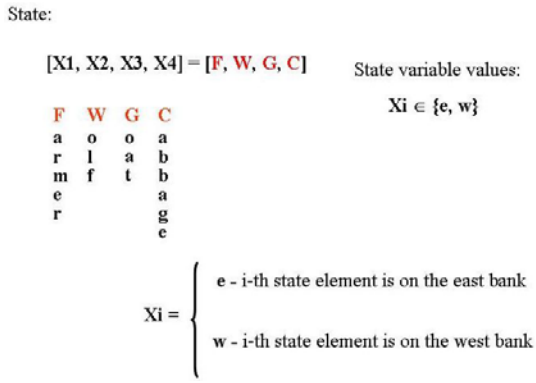


Figure 2.12: State of the system *farmer-wolf-goat-cabbage*

The solution is given by program `2_18_fwgc.pl`:

```
/*1*/  top:-
/*2*/         cross_the_river(state(w,w,w,w),state(e,e,e,e)).

/*3*/  cross_the_river(Initial_state,Final_state):-
/*4*/         feasible_crossing(Initial_state,Final_state,
                  [Initial_state],Final_sequence),nl,
/*5*/         reverse(Final_sequence,Final_sequence_r),
/*6*/         write_feasible_crossing(Final_sequence_r),
/*7*/         fail.
/*8*/  cross_the_river(_,_):- nl, write("Those are all solutions!").

/*9*/  feasible_crossing(Current_state,Final_state,
                    Final_sequence_accu,Final_sequence):-
/*10*/         crossing(Current_state,Next_state),
/*11*/         not(unsafe(Next_state)),
```

ecclesiastic, poet, mathematician and teacher from York, Northumbria. He wrote a textbook *Propositiones ad Acuendos Juvenes* (in English: *Problems to Sharpen the Young*) containing 53 puzzles, some of them of the "river crossing" type.

```
/*12*/         not(member(Next_state,Final_sequence_accu)),
/*13*/         feasible_crossing(Next_state,Final_state,
                  [Next_state|Final_sequence_accu],Final_sequence).

/*14*/  feasible_crossing(Final_state,Final_state,
                  Final_sequence,Final_sequence):- !.

  % Farmer and wolf change river bank,
  % goat and cabbage stay put in their places:
/*15*/ crossing(state(X,X,Go,Ca),state(Y,Y,Go,Ca)):-
/*16*/        opposite_banks(X,Y).

  % Farmer and goat change river bank,
  % wolf and cabbage stay put in their places:
/*17*/ crossing(state(X,W,X,Ca),state(Y,W,Y,Ca)):-
/*18*/        opposite_banks(X,Y).

  % Farmer and cabbage change river bank,
  % wolf and goat stay put in their places:
/*19*/ crossing(state(X,W,Go,X),state(Y,W,Go,Y)):-
/*20*/        opposite_banks(X,Y).

  % Farmer only changes river bank,
  % wolf, goat and cabbage stay put in their places:
/*21*/ crossing(state(X,W,Go,Ca),state(Y,W,Go,Ca)):-
/*22*/        opposite_banks(X,Y).

  % Wolf and goat cannot be left with no farmers supervision:
/*23*/ unsafe( state(Y,X,X,_) ):-
/*24*/        opposite_banks(Y,X).

  % Goat and cabbage cannot be left with no farmers supervision:
/*25*/ unsafe( state(Y,_,X,X) ):-
/*26*/        opposite_banks(Y,X).

/*27*/ opposite_banks(w,e).
/*28*/ opposite_banks(e,w).

/*29*/ write_feasible_crossing([H1,H2|T]) :-
/*30*/        write_crossing(H1,H2),
/*31*/        write_feasible_crossing([H2|T]).
/*32*/  write_feasible_crossing([_|[]]):-
/*33*/    writeln("All safely crossed the river.").

/*34*/ write_crossing(state(X,W,G,C), state(Y,W,G,C)):-
/*35*/        translate(X,X_translated),
/*36*/        translate(Y,Y_translated),
/*37*/        write("Farmer moves from "),write(X_translated),write(" to "),
```

```
                write(Y_translated),write("."),nl.

/*38*/ write_crossing(state(X,X,G,C), state(Y,Y,G,C)):-
/*39*/        translate(X,X_translated),
/*40*/        translate(Y,Y_translated),
/*41*/        write("Farmer moves with wolf from "),write(X_translated),
                write(" to "),write(Y_translated),write("."),nl.

/*42*/ write_crossing(state(X,W,X,C), state(Y,W,Y,C)) :-
/*43*/        translate(X,X_translated),
/*44*/        translate(Y,Y_translated),
/*45*/        write("Farmer moves with goat from "),write(X_translated),
                write(" to "),write(Y_translated),write("."),nl.

/*46*/ write_crossing(state(X,W,G,X), state(Y,W,G,Y)) :-
/*47*/        translate(X,X_translated),
/*48*/        translate(Y,Y_translated),
/*49*/        write("Farmer moves with cabbage from "),write(X_translated),
                write(" to "),write(Y_translated),write("."),nl.

/*50*/ translate(w,"west bank").
/*51*/ translate(e,"east bank").
```

There are two solutions to this problem. The first one is:

```
Farmer moves with goat from west bank to east bank.
Farmer moves from east bank to west bank.
Farmer moves with wolf from west bank to east bank.
Farmer moves with goat from east bank to west bank.
Farmer moves with cabbage from west bank to east bank.
Farmer moves from east bank to west bank.
Farmer moves with goat from west bank to east bank.
All safely crossed the river,
```

depicted on Figure 2.13.

As can be seen, the farmer must first take the goat across the river. He then returns and picks up the wolf. He leaves the wolf off and takes the goat back across the river with him. Then he leaves the goat at the starting point and takes the cabbage over to where the wolf is. He returns and picks up the goat, and then lands where the wolf and the cabbage are.

The second solution is:

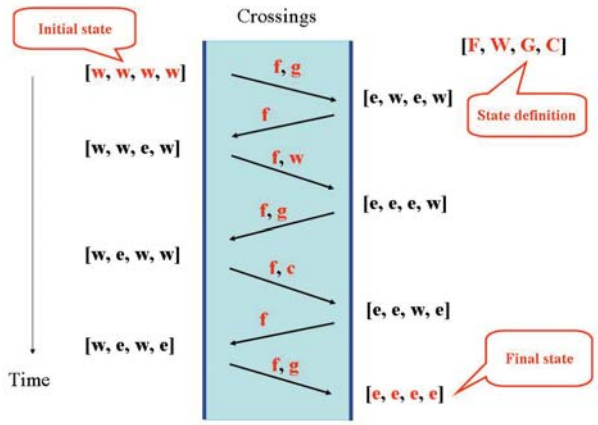


Figure 2.13: First solution river crossings for farmer, wolf, goat and cabbage

```
Farmer moves with goat from west bank to east bank.
Farmer moves from east bank to west bank.
Farmer moves with cabbage from west bank to east bank.
Farmer moves with goat from east bank to west bank.
Farmer moves with wolf from west bank to east bank.
Farmer moves from east bank to west bank.
Farmer moves with goat from west bank to east bank.
All safely crossed the river,

    Those are all solutions!
```

It is depicted on Figure 2.14.

This time the farmer also starts with taking the goat across the river. He then returns and picks up the cabbage. He leaves the cabbage off and takes the goat back across the river with him. Then he leaves the goat at the starting point and takes the wolf over to where the cabbage is. He returns and picks up the goat, and then lands where the wolf and the cabbage are.

The program contains an interesting feature: the number of crossings is *implicitly* minimized by demanding that no state ever appears twice. This is

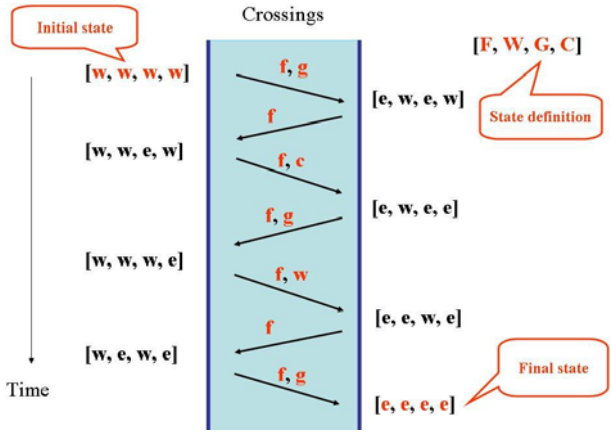


Figure 2.14: Second solution river crossings for farmer, wolf, goat and cabbage

done in line `/*12*/`: any new state `Next_State` may not belong to the list `Final_sequence_accu` of states already accumulated. Unfortunately, this feature is only a fortuitous heuristics that just works for the example discussed, but is not of general nature and does not work for all conceivable optimum state trajectory problems.

2.5.2 Missionaries and cannibals

The missionaries and cannibals problem is a more complicated state trajectory determination problem. It is usually stated as follows:
Three missionaries and three cannibals must cross a river using a boat that can carry at most two people. The crossings must be done so that if on any bank missionaries are present, they cannot be outnumbered by cannibals. Otherwise the cannibals would eat the missionaries[28]. The state of the system is defined in Figure 2.15.

The problem is solved by program `2_19_mac.pl`. Its basic private predicates

[28]Obviously, this is a politically incorrect puzzle. Those who object to its incorrectness may easily formulate a politically correct version: if on any bank missionaries are present, they must be outnumbered by cannibals, because otherwise the missionaries would convert the cannibals.

State: [Nm, Nc, Bl]

Nm – number of missionaries on left river bank

Nc – number of canibals on left river bank

Bl – boat location

State variable values:

Nm ∈ {0, 1, 2, 3}

Nc ∈ {0, 1, 2, 3}

Bl ∈ {blb, brb},

bl(r)b – boat on left (right) bank

Figure 2.15: State of the system *missionaries-cannibals*

are:

```
cross_the_river(Inital_state,Final_state,Boat_location)
feasible_crossing(Initial_state,Final_state,
          Path_accumulator,Path,Boat_location)
```

The program is:

```
/*1*/    top:-
/*2*/          assert(counter(0)), % for counting solutions
/*3*/          cross_the_river(state(3,3,blb), state(0,0,brb),blb).

/*4*/    cross_the_river(Inital_state,Final_state,Boat_location):-
/*5*/          feasible_crossing(Inital_state,Final_state,[Inital_state],
                  Crossings,Boat_location), nl,
/*6*/          write_feasible_crossing(Crossings),
/*7*/          fail.
/*8*/    cross_the_river(_,_,_):-
/*9*/          write("Those are all solutions.").

/*10*/   feasible_crossing(Present_state,Final_state,Accu_of_crossings,
                       Crossings,Boat_location_before):-
/*11*/          crossing(Present_state,Next_state,Boat_location_before),
/*12*/          check_feasability(Next_state,Accu_of_crossings),
/*13*/          change_boat_location(Boat_location_before,Boat_location_after),
/*14*/          feasible_crossing( Next_state,Final_state,
                       [Next_state|Accu_of_crossings],Crossings,Boat_location_after).
```

```
  % Final state reached:
/*15*/   feasible_crossing(Final_state,Final_state,Crossings,Crossings,_):- nl,
/*16*/           enumerate,
/*17*/           counter(Number_of_solutions),
/*18*/           write("Solution number "),write(Number_of_solutions),write(":"),nl,
/*19*/           write("Crossings = "),write(Crossings),nl.

  % A single missionary moves to the right bank:
/*20*/   crossing(state(X,K,_),state(Y,K,brb),blb):-
/*21*/           Y is X-1.

  % Two missionaries move to the right bank:
/*22*/   crossing(state(X,K,_),state(Y,K,brb),blb):-
/*23*/           Y is X-2.

  % A single cannibal moves to the right bank:
/*24*/   crossing(state(M,X,_),state(M,Y,brb),blb):-
/*25*/           Y is X-1.

  % Two cannibals move to the right bank:
/*26*/   crossing(state(M,X,_),state(M,Y,brb),blb):-
/*27*/           Y is X-2.

  % A missionary and a cannibal move to the right bank:
/*28*/   crossing(state(X,X1,_),state(Y,Y1,brb),blb):-
/*29*/           Y is X-1,
/*30*/           Y1 is X1-1.

  % Two missionaries move to the left bank:
/*31*/   crossing(state(X,K,_),state(Y,K,blb),brb):-
/*32*/           Y is X+2.

  % A single missionary moves to the left bank:
/*33*/   crossing(state(X,K,_),state(Y,K,blb),brb):-
/*34*/           Y is X+1.

  % A single cannibal moves to the left bank:
/*35*/   crossing(state(M,X,_),state(M,Y,blb),brb):-
/*36*/           Y is X+1.

  % Two cannibals move to the left bank:
/*37*/   crossing(state(M,X,_),state(M,Y,blb),brb):-
/*38*/           Y is X+2.

  % A missionary and a cannibal move to the left bank:
/*39*/   crossing(state(X,X1,_),state(Y,Y1,blb),brb):-
/*40*/           Y is X+1,
```

```
/*41*/         Y1 is X1+1.

/*42*/   check_feasability(S1,Crossings):-
/*43*/         not(unsafe(S1)),
/*44*/         not(unfeasible(S1)),
/*45*/         not(member(S1,Crossings)).

/*46*/   change_boat_location(brb,blb).
/*47*/   change_boat_location(blb,brb).

  % On the left bank the cannibals will be in majority:
/*48*/   unsafe(state(M,K,_)):-
/*49*/         M>0,
/*50*/         M<K.

  % On the right bank the cannibals will be in majority:
/*51*/   unsafe(state(M,K,_)):-
/*52*/         M<3,
/*53*/         K<M.

/*54*/   unfeasible( state(M,_,_)):-
/*55*/         M<0.

/*56*/   unfeasible(state(_,K,_)):-
/*57*/         K<0.

/*58*/   unfeasible(state(M,_,_)):-
/*59*/         M>3.

/*60*/   unfeasible(state(_,K,_)):-
/*61*/         K>3.

/*62*/   write_feasible_crossing([H1,H2|T]):-
/*63*/         write_crossing(H1,H2),
/*64*/         write_feasible_crossing([H2|T]).
/*65*/   write_feasible_crossing([_|[]]) :-
/*66*/         writeln("All safely crossed the river.").

/*67*/   write_crossing(state(X,K,_),state(Y,K,_)):-
/*68*/         Y is X+1,
/*69*/         write("A missionary moved from left bank to right bank."),nl.

/*70*/   write_crossing( state(M,X,_),state(M,Y,_)):-
/*71*/         Y is X+1,
/*72*/         write("A cannibal moved from left bank to right bank."),nl.

/*73*/   write_crossing(state(X,K,_),state(Y,K,_)):-
/*74*/         Y is X+2,
```

```
/*75*/         write("Two missionaries moved from left bank to right bank."),nl.

/*76*/   write_crossing(state(M,X,_),state(M,Y,_)):-
/*77*/         Y is X+2,
/*78*/         write("Two cannibals moved from left bank to right bank."),nl.

/*79*/   write_crossing(state(X,X1,_),state(Y,Y1,_)):-
/*80*/         Y is X+1,
/*81*/         Y1 is X1+1,
/*82*/         write("A missionary and a cannibal moved from left bank to "),
                       write("right bank."),nl.

/*83*/   write_crossing(state(X,K,_),state(Y,K,_)):-
/*84*/         Y is X-1,
/*85*/         write("A missionary moved from right bank to left bank."),nl.

/*86*/   write_crossing(state(M,X,_),state(M,Y,_)):-
/*87*/          Y is X-1,
/*88*/         write("A cannibal moved from right bank to left bank."),nl.

/*89*/   write_crossing(state(X,K,_),state(Y,K,_)):-
/*90*/         Y is X-2,
/*91*/         write("Two missionaries moved from right bank to left bank."),nl.

/*92*/   write_crossing(state(M,X,_),state(M,Y,_)):-
/*93*/         Y is X-2,
/*94*/         write("Two cannibals moved from right bank to left bank."),nl.

   /*95*/   write_crossing(state(X,X1,_),state(Y,Y1,_)) :-
/*96*/         Y is X-1,
/*97*/         Y1 is X1-1,
/*98*/         write("A missionary and a cannibal moved from right bank to "),
                       write("left bank."),nl.

/*99*/   enumerate:-
/*100*/         retract(counter(Old)),
/*101*/   New is Old  1, +
/*102*/   assert(counter(New)).
```

The problem has 4 solutions:

```
Solution number 1:
Crossings = [state(0, 0, brb), state(1, 1, blb), state(0, 1, brb),
             state(0, 3, blb), state(0, 2, brb), state(2, 2, blb),
             state(1, 1, brb), state(3, 1, blb), state(3, 0, brb),
             state(3, 2, blb), state(3, 1, brb), state(3, 3, blb)]
A missionary and a cannibal moved from left bank to right bank.
A missionary moved from right bank to left bank.
```

```
Two cannibals moved from left bank to right bank.
A cannibal moved from right bank to left bank.
Two missionaries moved from left bank to right bank.
A missionary and a cannibal moved from right bank to left bank.
Two missionaries moved from left bank to right bank.
A cannibal moved from right bank to left bank.
Two cannibals moved from left bank to right bank.
A cannibal moved from right bank to left bank.
Two cannibals moved from left bank to right bank.
All safely crossed the river.


Solution number 2:
Crossings = [state(0, 0, brb), state(0, 2, blb), state(0, 1, brb),
             state(0, 3, blb), state(0, 2, brb), state(2, 2, blb),
             state(1, 1, brb), state(3, 1, blb), state(3, 0, brb),
             state(3, 2, blb), state(3, 1, brb), state(3, 3, blb)]
Two cannibals moved from left bank to right bank.
A cannibal moved from right bank to left bank.
Two cannibals moved from left bank to right bank.
A cannibal moved from right bank to left bank.
Two missionaries moved from left bank to right bank.
A missionary and a cannibal moved from right bank to left bank.
Two missionaries moved from left bank to right bank.
A cannibal moved from right bank to left bank.
Two cannibals moved from left bank to right bank.
A cannibal moved from right bank to left bank.
Two cannibals moved from left bank to right bank.
All safely crossed the river.


Solution number 3:
Crossings = [state(0, 0, brb), state(1, 1, blb), state(0, 1, brb),
              state(0, 3, blb), state(0, 2, brb), state(2, 2, blb),
              state(1, 1, brb), state(3, 1, blb), state(3, 0, brb),
              state(3, 2, blb), state(2, 2, brb), state(3, 3, blb)]
A missionary and a cannibal moved from left bank to right bank.
A missionary moved from right bank to left bank.
Two cannibals moved from left bank to right bank.
A cannibal moved from right bank to left bank.
Two missionaries moved from left bank to right bank.
A missionary and a cannibal moved from right bank to left bank.
Two missionaries moved from left bank to right bank.
A cannibal moved from right bank to left bank.
Two cannibals moved from left bank to right bank.
A missionary moved from right bank to left bank.
A missionary and a cannibal moved from left bank to right bank.
All safely crossed the river.
```

```
Solution number 4:
Crossings = [state(0, 0, brb), state(0, 2, blb), state(0, 1, brb),
             state(0, 3, blb), state(0, 2, brb), state(2, 2, blb),
             state(1, 1, brb), state(3, 1, blb), state(3, 0, brb),
             state(3, 2, blb), state(2, 2, brb), state(3, 3, blb)]
Two cannibals moved from left bank to right bank.
A cannibal moved from right bank to left bank.
Two cannibals moved from left bank to right bank.
A cannibal moved from right bank to left bank.
Two missionaries moved from left bank to right bank.
A missionary and a cannibal moved from right bank to left bank.
Two missionaries moved from left bank to right bank.
A cannibal moved from right bank to left bank.
Two cannibals moved from left bank to right bank.
A missionary moved from right bank to left bank.
A missionary and a cannibal moved from left bank to right bank.
All safely crossed the river.
```

Those are all solutions.
Solution 1 is depicted on Figure 2.16.

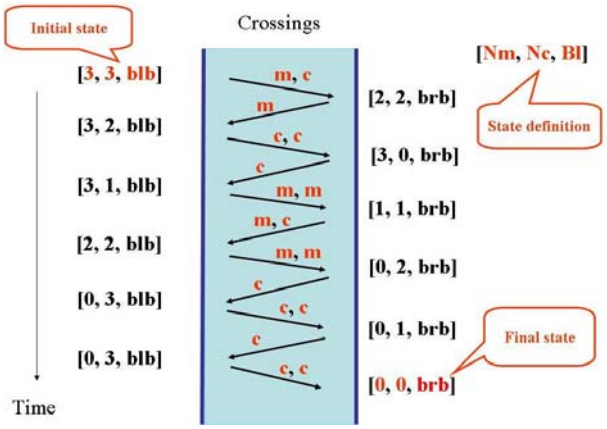


Figure 2.16: River crossings for missionaries and canibals by solution 1

The general idea of *missionaries and canibals* is the same as for *farmer-wolf-goose-cabbage*: to any sequence of states already visited (and present in the accumulator) a new feasible state is added till the new state is equal to the final

state. As in Section 2.5.1, the number of river crossings is minimized *implicitly* by demanding in line `/*39*/` that no state may appear twice. As before, we are fortunate that this heuristic leads to an optimum solution. Because of the large number of solutions a counter has been added (see lines `/*2*/`, `/*16*/`, `/*17*/`, `/*99*/-/*102*/`) to take care of the numbering.

2.5.3 Towers of Hanoi

Recursion was used for many predicates so far. *Prolog* people just love recursion because of its succinctness and calculating power. A particularly convincing argument for its virtues is given by the program solving the *Towers of Hanoi* puzzle[29]. This puzzle is due to the French mathematician Edouard Lucas (1842-1891). Lucas assumed the presence of three rods, onto any of them a number of holed disks of different sizes can slide. The puzzle starts with the disks in a stack of ascending order of diameters on one rod with the largest disk at the bottom.

The goal is to move the entire stack disk-wise to another rod while fulfilling the following constraints:

1. Only one disk may be moved at a time.
2. Each move consists of taking the top disk from some rod and sliding it onto another rod.
3. No disk may be slid on top of a smaller disk.

Let's consider the general case of `N` disks. To move `N` disks from their initial left rod to their final right rod, it is necessary:

1. Move $N-1$ disks from the left rode to the middle rod using the final rod as intermediary. Assume that it done in T_{N-1} steps.
2. Move the last disk from the left rode to the right rode. Altogether $T_{N-1}+1$ steps are needed.

Now the situation is similar to the initial one, before step `1` was taken; the difference is that now $N-1$ disks have to be moved from the middle rod to the right rod using as intermediary rod left. This can be done also in T_{N-1} steps. All moves needed thus $T_N = 2T_{N-1}+1$ steps. The difference equation:

$$T_N = 2T_{N-1} + 1$$

[29] This is an FST-type problem.

has a solution equal to $T_N = 2^N - 1$ which can be proved using mathematical induction[30].

A program solving the Tower of Hanoi puzzle (`2_20_hanoi.pl`) is as follows:

```
/*1*/    top:-
/*2*/          write(" Declare number of disks: "),nl,
/*3*/          read_token(Number, integer),
/*4*/          write(Number),nl,
/*5*/          hanoi(Number).

/*6*/    hanoi(N) :-
/*7*/          move(N,"Left","Middle","Right").

  %  Move a  single disk directly from
 %/*  position "Left" to position "Right":
/*8*/    move(1,A,_,C) :-
/*9*/          command(A,C).

  % In order to move N disks from
  % position "Left" to position "Right":
/*10*/ move(N,A,B,C) :-
/*12*/        N1 is N-1,
  % move N-1 disks from position
  % "Left" to position "Middle"
  % using position "Right":
/*13*/        move(N1,A,C,B),
  % move the last disk from position
  % "Left" to position  "Right":
/*14*/        command(A,C),
  % move N-1 disks from position
  % "Middle" to position "Right"
  % using position "Left":
/*15*/        move(N1,B,A,C).

/*16*/ command(Position_1,Position_2):-
/*17*/        write("Move disk from position "),write(Position_1),
/*18*/        write(" to position "),write(Position_2),write(". "),nl.
```

The dialogue and message generated is as follows:

```
Declare number of disks:  3
Move disk from position Left to position Right.
Move disk from position Left to position Middle.
Move disk from position Right to position Middle.
```

[30]For $N = 0$ is $T(0) = 0$. If for $N-1$ is $T(N-1) = 2^{N-1} - 1$, then for $T(N) = 2T_{N-1} + 1 = 2(2^{N-1} - 1) + 1 = 2^N - 1$.

```
Move disk from position Left to position Right.
Move disk from position Middle to position Left.
Move disk from position Middle to position Right.
Move disk from position Left to position Right.
```

Unusual in this program is the double recursion: `move(N,A,B,C)` from line `/*11*/` is defined by `move(N1,A,C,B)` from line `/*13*/` and `move(N1,B,A,C)` from line `/*15*/`. Figure 2.17 shows the moves for 3 disks.

Lucas is supposed to enrich his 8-disc puzzle by a Tower of Brahma legend, which states that Brahma, at the beginning of time, ordered a group of monks to move `64` golden discs between `3` diamond rods as described by the puzzle. According to the legend, when the last move is completed, the end of the world will occur. This legend gives justice to Lucas' understanding of computational complexity: assuming the monks will make a move each second, the moving of a `64`-disk stack will take $2^{64} - 1 = 18446744073709551615$ seconds, i.e. roughly `584` billion years to complete. Let us not forget that the estimate age of the universe is roughly `13.7` billion years.

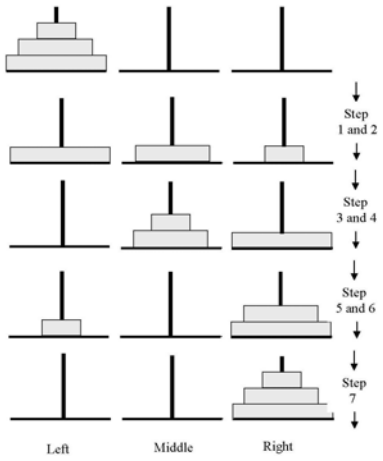


Figure 2.17: Tower of Hanoi solution for 3 disks

2.6 Optimum sequencing problems

2.6.1 A simple maze

The program `2_21_maze.pl` finds the shortest path (measured by the number of passed cells) from cell `(0,0)` to cell `6,6)` for the maze from Figure 2.18[31]. Only horizontal and vertical transitions between cells are feasible. To find the shortest path, the *branch-and-bound* method used for finding optimum configurations (see 2.3.1) has been applied.

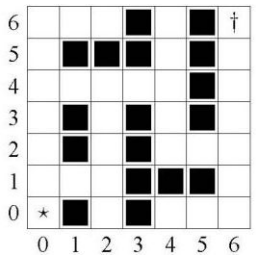


Figure 2.18: A simple maze

The program `2_21_maze.pl` is as follows:

```
/*1*/  top:-
/*2*/       assert(shortest_path([[]],80)),
/*3*/       maze.

/*4*/       from_to([0,0],[0,1]).     /*5*/       from_to([0,1],[0,2]).
/*6*/       from_to([0,2],[0,3]).     /*7*/       from_to([0,3],[0,4]).
/*8*/       from_to([0,4],[0,5]).     /*9*/       from_to([0,5],[0,6]).
/*10*/      from_to([0,6],[1,6]).     /*11*/      from_to([1,6],[2,6]).
/*12*/      from_to([0,4],[1,4]).     /*13*/      from_to([1,4],[2,4]).
/*14*/      from_to([2,4],[3,4]).     /*15*/      from_to([3,4],[4,4]).
/*16*/      from_to([0,1],[1,1]).     /*17*/      from_to([1,1],[2,1]).
/*18*/      from_to([2,1],[2,2]).     /*19*/      from_to([2,2],[2,3]).
/*20*/      from_to([2,3],[2,4]).     /*21*/      from_to([4,4],[4,5]).
/*22*/      from_to([4,5],[4,6]).     /*23*/      from_to([4,4],[4,3]).
/*24*/      from_to([4,3],[4,2]).     /*25*/      from_to([4,2],[5,2]).
/*26*/      from_to([5,2],[6,2]).     /*27*/      from_to([6,2],[6,1]).
/*28*/      from_to([6,1],[6,0]).     /*29*/      from_to([6,0],[5,0]).
/*30*/      from_to([5,0],[4,0]).     /*31*/      from_to([6,2],[6,3]).
```

[31] This is an OST-type problem.

```
/*32*/      from_to([6,3],[6,4]).     /*33*/      from_to([6,4],[6,5]).
/*34*/      from_to([6,5],[6,6]).

/*35*/ transition(A,B):-
/*36*/      from_to(A,B).
/*37*/ transition(A,B):-
/*38*/      from_to(B,A).

/*39*/ maze:-
/*40*/      path([[6,6]],Present_solution),
/*41*/      length(Present_solution,Present_length),
/*42*/      update_shortest(Present_solution, Present_length),
/*43*/      fail.
/*44*/ maze:-
/*45*/      shortest_path(Final_solution,Final_length),
/*46*/      write("Final_solution = "),write(Final_solution),nl,
/*47*/      write("Final_length ="),write(Final_length),nl,nl,
/*48*/      fail.
/*49*/ maze:-
/*50*/      write("Those are all solutions of minimum length."),nl.

/*51*/ path([Present_state|Path_covered],Final_solution):-
/*52*/      transition(Present_state,Next_state),
/*53*/      not(member(Next_state,Path_covered)),
/*54*/      path([Next_state,Present_state|Path_covered],Final_solution).
/*55*/ path([[0,0]|Path_covered],[[0,0]|Path_covered]).

/*56*/ update_shortest(Present_solution,Present_length):-
/*57*/      shortest_path(_,Final_length),
/*58*/      Present_length<Final_length,
/*59*/      retractall(shortest_path(_,_)),
/*60*/      assert(shortest_path(Present_solution,Present_length)),!.
/*61*/ update_shortest(Present_solution, Present_length):-
/*62*/     shortest_path(_,Final_length),
/*63*/     Present_length=Final_length,
/*64*/     assert(shortest_path(Present_solution, Present_length)),!.
/*65*/ update_shortest(_,Present_length):-
/*66*/     shortest_path(_,Final_length),
/*67*/     Present_length>Final_length,!.
```

The solution is:

```
Final solution = [[0, 0], [0, 1], [0, 2], [0, 3], [0, 4], [1, 4], [2, 4],
[3, 4], [4, 4], [4, 3], [4, 2], [5, 2], [6, 2], [6, 3], [6, 4],
[6, 5], [6, 6]]
```

```
Final_Length =17

Final solution = [[0, 0], [0, 1], [1, 1], [2, 1], [2, 2], [2, 3], [2, 4],
[3, 4], [4, 4], [4, 3], [4, 2], [5, 2], [6, 2], [6, 3], [6, 4],
[6, 5], [6, 6]]
Final_Length =17
```

Those are all solutions of minimum length.

The domain declaration is once again implicit and given by all facts `from-to/2`. The *state* is obviously given by cell coordinates (*horizontal,vertical*).

2.6.2 Mine field

More complicated maze problems are given by *mine fields*, for which a path that minimizes the overall danger is to be found[32]. For the simple mine field from Figure 2.19 with dangers declared in cells, the least dangerous path from cell `(0,0)` to cell `(3,3)` is to be found, assuming danger being additive.

3	3	3	1	1
2	3	3	1	3
1	3	3	4	3
0	1	1	1	1
	0	1	2	3

Figure 2.19: A simple mine field

The state of the mine field is - as for the maze - given by cell coordinates (*horizontal, vertical*). The cell contain values of `Danger` associated with transiting the cell. The "dangers" do not belong to the state because they do not influence the moves to be made. Only vertical or horizontal moves are allowed.

The `Overall_danger` is the sum of `Dangers` transited cells. The corresponding program `2_22_mine_field.pl` is as follows:

[32] This is an OST-type problem.

```
/*1*/  top:-
/*2*/       assert(safest_path([[]],50)),
/*3*/       mine_field.

/*4*/       from_to([0,0],[0,1]).% columns, from bottom to top
/*5*/       from_to([0,1],[0,2]).
/*6*/       from_to([0,2],[0,3]).

/*7*/       from_to([1,0],[1,1]).
/*8*/       from_to([1,1],[1,2]).
/*9*/       from_to([1,2],[1,3]).

/*10*/      from_to([2,0],[2,1]).
/*11*/      from_to([2,1],[2,2]).
/*13*/      from_to([2,2],[2,3]).

/*14*/      from_to([3,0],[3,1]).
/*15*/      from_to([3,1],[3,2]).
/*16*/      from_to([3,2],[3,3]).

/*17*/      from_to([0,0],[1,0]). % rows, from left to riught
/*18*/      from_to([1,0],[2,0]).
/*19*/      from_to([2,0],[3,0]).

/*20*/      from_to([0,1],[1,1]).
/*21*/      from_to([1,1],[2,1]).
/*22*/      from_to([2,1],[3,1]).

/*23*/      from_to([0,2],[1,2]).
/*24*/      from_to([1,2],[2,2]).
/*25*/      from_to([2,2],[3,2]).

/*26*/      from_to([0,3],[1,3]).
/*27*/      from_to([1,3],[2,3]).
/*28*/      from_to([2,3],[3,3]).

/*29*/  transition(A,B):-
/*30*/      from_to(A,B).
/*31*/  transition(A,B):-
/*32*/      from_to(B,A).

/*33*/      danger([0,0],1).
/*34*/      danger([0,1],3).
/*35*/      danger([0,2],3).
/*36*/      danger([0,3],3).

/*37*/      danger([1,0],1).
/*38*/      danger([1,1],3).
/*39*/      danger([1,2],3).
```

```
/*40*/      danger([1,3],3).

/*41*/      danger([2,0],1).
/*42*/      danger([2,1],4).
/*43*/      danger([2,2],1).
/*44*/      danger([2,3],1).

/*45*/      danger([3,0],1).
/*46*/      danger([3,1],3).
/*47*/      danger([3,2],3).
/*48*/      danger([3,3],1).

/*49*/  mine_field:-
/*50*/      path([[3,3]],Path),
/*51*/      overall_danger(Path,Overall_danger),
/*52*/      update_safest(Path,Overall_danger),
/*53*/      fail.
/*54*/  mine_field:-
/*55*/      safest_path(Path,Overall_danger),
/*56*/      write("Safest path = "),write(Path),nl,
/*57*/      write("Overall danger = "),write(Overall_danger),nl,nl,
/*58*/      fail.
/*59*/  mine_field:-
/*60*/      write("Those are all solutions of minimum overall danger."),nl.

/*61*/  path([Present_state|Path_covered],Path):-
/*62*/      transition(Present_state,Next_state),
/*63*/      not(member(Next_state,Path_covered)),
/*64*/      path([Next_state,Present_state|Path_covered],Path).
/*65*/  path([[0,0]|Path_covered],[[0,0]|Path_covered]).

/*66*/  overall_danger([H|T],N):-
/*67*/      overall_danger([H|T],N,0).
/*68*/  overall_danger([],N,N).
/*69*/  overall_danger([H|T],N,A):-
/*70*/      danger(H,NN),
/*71*/      A_New is A+NN,
/*72*/      overall_danger(T,N,A_New).

/*73*/  update_safest(Path, Overall_danger):-
/*74*/      safest_path(_,Present_Danger),
/*75*/      Present_Danger>Overall_danger,
/*76*/      retractall(safest_path(_,_)),
/*77*/      assert(safest_path(Path,
                 Overall_danger)),!.
/*78*/  update_safest(Path, Overall_danger):-
```

```
/*79*/      safest_path(_,Present_Danger),
/*80*/      Present_Danger=Overall_danger,
/*81*/      assert(safest_path(Path, Overall_danger)),!.
/*82*/  update_safest(_,Overall_danger):-
/*83*/      safest_path(_,Present_Danger),
/*84*/      Present_Danger<Overall_danger,!.
```

The message is:

```
Safest path = [[0, 0], [1, 0], [2, 0], [2, 1], [2, 2], [2, 3], [3, 3]]
Overall danger = 10

Those are all solutions of minimum overall danger.
```

2.6.3 Hampton Court maze

Sometimes the distances of the mazes paths are not known. The number of forks in the final path may then be minimized. How to do it is shown by a program solving the famous Hampton Court Maze. This is how Jerome K. Jerome in his book *Three Man in a Boat (To Say Nothing of the Dog)* described what happened to somebody trying to find a way out of the famous hedge maze at Hampton Court near London:

"Harris asked me if I'd ever been in the maze at Hampton Court. He said he went in once to show somebody else the way. He had studied it up in a map, and it was so simple that it seemed foolish - hardly worth the twopence charged for admission.

Figure 2.20: Hampton Court maze

Harris said he thought that map must have been got up as a practical joke, because it wasn't a bit like the real thing, and only misleading. It was a country cousin that Harris took in. He said:

"We'll just go in here, so that you can say you've been, but it's very simple. It's absurd to call it a maze. You keep on taking the first turning to the right. We'll just walk round for ten minutes, and then go and get some lunch."

They met some people soon after they had got inside, who said they had been there for three-quarters of an hour, and had had about enough of it. Harris told them they could follow him, if they liked; he was just going in, and then should turn round and come out again. They said it was very kind of him, and fell behind, and followed.

They picked up various other people who wanted to get it over, as they went along, until they had absorbed all the persons in the maze. People who had given up all hopes of ever getting either in or out, or of ever seeing their home and friends again, plucked up courage at the sight of Harris and his party, and joined the procession, blessing him. Harris said he should judge there must have been twenty people, following him, in all; and one woman with a baby, who had been there all the morning, insisted on taking his arm, for fear of losing him.

Harris kept on turning to the right, but it seemed a long way, and his cousin said he supposed it was a very big maze.

"Oh, one of the largest in Europe," said Harris.

"Yes, it must be," replied the cousin, "because we've walked a good two miles already."

Harris began to think it rather strange himself, but he held on until, at last, they passed the half of a penny bun on the ground that Harris's cousin swore he had noticed there seven minutes ago. Harris said: "Oh, impossible!" but the woman with the baby said, "Not at all," as she herself had taken it from the child, and thrown it down there, just before she met Harris. She also added that she wished she never had met Harris, and expressed an opinion that he was an impostor. That made Harris mad, and he produced his map, and explained his theory.

"The map may be all right enough," said one of the party, "if you know whereabouts in it we are now."

Harris didn't know, and suggested that the best thing to do would be to go back to the entrance, and begin again. For the beginning again part of it there was not much enthusiasm; but with regard to the advisability of going back to the entrance there was complete unanimity, and so they turned, and trailed after Harris again, in the opposite direction. About ten minutes more passed, and then they found themselves in the center.

Harris thought at first of pretending that that was what he had been aiming at; but the crowd looked dangerous, and he decided to treat it as an accident.

Anyhow, they had got something to start from then. They did know where they were, and the map was once more consulted, and the thing seemed simpler than ever, and off they started for the third time.

And three minutes later they were back in the center again.

After that, they simply couldn't get anywhere else. Whatever way they turned brought them back to the middle. It became so regular at length, that some of the people stopped there, and waited for the others to take a walk round, and come back to them. Harris drew out his map again, after a while, but the sight of it only infuriated the mob, and they told him to go and curl his hair with it. Harris said that he couldn't help feeling that, to a certain extent, he had become unpopular.

They all got crazy at last, and sang out for the keeper, and the man came and climbed up the ladder outside, and shouted out directions to them. But all their heads were, by this time, in such a confused whirl that they were incapable of grasping anything, and so the man told them to stop where they were, and he would come to them. They huddled together, and waited; and he climbed down, and came in.

He was a young keeper, as luck would have it, and new to the business; and when he got in, he couldn't find them, and he wandered about, trying to get to them, and then he got lost. They caught sight of him, every now and then, rushing about the other side of the hedge, and he would see them, and rush to get to them, and they would wait there for about five minutes, and then he would reappear again in exactly the same spot, and ask them where they had been.

They had to wait till one of the old keepers came back from his dinner before they got out."

In order to model the maze, it has to be presented as shown in Figure 2.21, which clearly defines the coordinates of all forks.

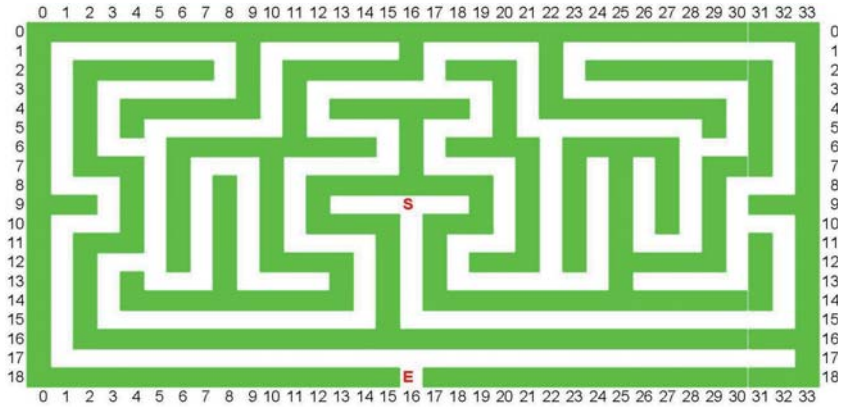


Figure 2.21: Hampton Court Maze coordinates

The program `2_23_hampton_court.pl` determines the shortest (by the number of crossed forks) path from the court center (marked with S - Start) to its entrance (marked E):

```
/*1*/  top:-
/*2*/       assert(shortest_path([[]],80)),
/*3*/       maze.

  %  The maze model is of the form:
  %  'from_to(Fork_coordinates, Adjacent_fork_coordinates)':
/*4*/       from_to([18,16],[17,16]).
/*5*/       from_to([17,16],[17,32]).
/*6*/       from_to([17,16],[6,5]).
/*7*/       from_to([6,5],[1,15]).
/*8*/       from_to([6,5],[12,5]).
```

```
/*9*/       from_to([12,5],[13,12]).
/*10*/      from_to([12,5],[3,17]).
/*11*/      from_to([3,17],[5,22]).
/*12*/      from_to([3,17],[13,22]).
/*13*/      from_to([13,22],[7,24]).
/*14*/      from_to([13,22],[5,22]).
/*15*/      from_to([5,22],[6,28]).
/*16*/      from_to([6,28],[7,26]).
/*17*/      from_to([6,28],[10,30]).
/*18*/      from_to([10,30],[13,26]).
/*19*/      from_to([10,30],[9,16]).

/*20*/  transition(A,B):-
/*21*/      from_to(A,B).
/*22*/  transition(A,B):-
/*23*/      from_to(B,A).

/*24*/  maze:-
/*25*/      path([[18,16]],Present_solution),
/*26*/      length(Present_solution,Present_length),
/*27*/      update_shortest(Present_solution,Present_length),
/*28*/      fail.

/*29*/  maze:-
/*30*/      shortest_path(Final_solution,Final_length),
/*31*/      write("The shortest path is"),write(Final_solution),nl,
/*32*/      fail.

/*33*/  maze:-
/*34*/      write("That's all!"),nl.

/*35*/  path([Present_state|Path_covered],Final_solution):-
/*36*/      transition(Present_state,Next_state),
/*37*/      not(member(Next_state,Path_covered)),
/*38*/      path([Next_state,Present_state|Path_covered],Final_solution).
/*39*/      path([[9,16]|Path_covered],[[9,16]|Path_covered]).

/*40*/  update_shortest(Present_solution,Present_length):-
/*41*/      shortest_path(_,Final_length),
/*42*/      Present_length<Final_length,
/*43*/      retractall(shortest_path(_,_)),
/*44*/      assert(shortest_path(Present_solution,Present_length)),!.

/*45*/  update_shortest(Present_solution, Present_length):-
/*46*/      shortest_path(_,Final_length),
/*47*/      Present_length=Final_length,
/*48*/      assert(shortest_path(Present_solution, Present_length)),!.

/*49*/  update_shortest(_,Present_length):-
```

```
/*50*/      shortest_path(_,Final_length),
/*51*/      Present_length>Final_length,!.
```

The message is:

```
/*10*/ The shortest path is:
/*10*/ [[9,16], [10,30], [6,28], [5,22], [3,17], [12,5], [6,5], [17,16], [18,16]]
/*10*/ It's length (measured by the number of crossed forks) is 9
/*10*/ That's all!
```

The optimum path is shown in the lower part of Figure 2.22.

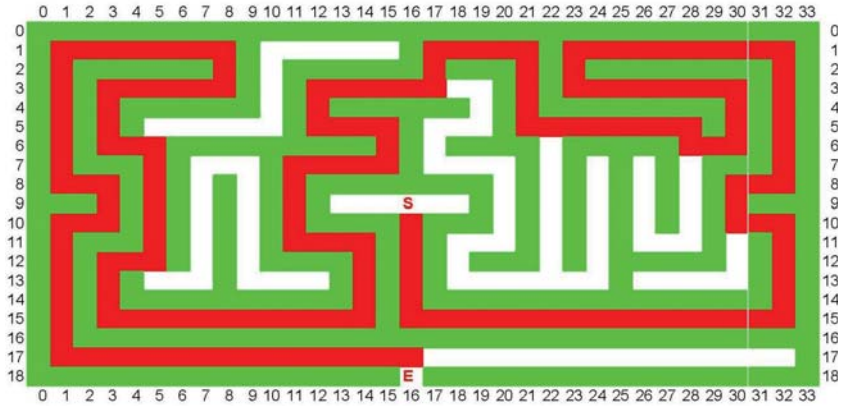


Figure 2.22: Hampton Court Maze solution

2.6.4 Water jugs problem

There are many water jugs problems. The one chosen is concerned with three jugs of capacity 8, 5 and 3 liters. Neither has any measuring markers on it. The 8-liter jug is filled with water. How can this water be used to fill the remaining two jugs exactly with four liters each while using only the three jugs that have no measuring markers, and minimizing the number of pourings[33]?

[33]This is an OST-type problem.

Defining the state of the jugs as:

```
state(Water_in_8_litre_jug, Water_in_5_litre_jug,
      Water_in_3_litre_jug),
```

and using the built-in `length(?List,?N)`, the solution is given by program `2_24_three_jugs.pl`:

```
/*1*/  top:-
/*2*/          Initial_state = state(8,0,0),
/*3*/          pour(Initial_state,Sequence_of_states),
/*4*/          length(Sequence_of_states, N),
/*5*/          assert(sequence_of_states(N,Sequence_of_states)),
/*6*/          fail.
/*7*/  top:-
/*8*/          assert(shortest_sequence_of_states(20,[])),
/*9*/          optimize.

/*10*/   pour(Initial_state,Sequence_of_states):-
/*11*/         pour(Initial_state,[Initial_state],Sequence_of_states).

/*12*/   pour(State,Accumulator,Sequence_of_states):-
/*13*/         state_transition(State,Next_state),
/*14*/         not(member(Next_state,Accumulator)),
/*15*/         pour(Next_state,[Next_state|Accumulator],Sequence_of_states).
/*16*/   pour(Final_state,Accumulator,Accumulator):-
/*17*/         final_state(Final_state),!.

/*18*/ final_state(state(4,4,0)).

 % Possible pourings:
 % pouring(From_jug_A, To_jug_B,
 % With_limit_for_B, New_filling_of_A, New_filling_of_B):

  % pouring from jug 1 to 2,  2 may contain no more than 5 liters:
/*19*/   state_transition(state(X,Y,Z),state(K,L,Z)):-
/*20*/         pouring(X,Y,5,K,L).

 % pouring from jug 2 to 1,  1 may contain no more than 8 liters:
/*21*/   state_transition(state(X,Y,Z),state(K,L,Z)):-
/*22*/         pouring(Y,X,8,L,K).

 % pouring from jug 1 to 3,  3 may contain no more than 3 liters:
/*23*/   state_transition(state(X,Y,Z),state(K,Y,M)):-
/*24*/         pouring(X,Z,3,K,M).

  % pouring from jug 3 to 1,  1 may contain only 8 liters:
/*25*/   state_transition(state(X,Y,Z),state(K,Y,M)):-
/*26*/         pouring(Z,X,8,M,K).
```

```
 % pouring from jug 2 to 3, w 3 may contain no more than 3 liters:
/*27*/   state_transition(state(X,Y,Z),state(X,L,M)):-
/*28*/         pouring(Y,Z,3,L,M).

 % pouring from jug 3 to 2, w 2 may contain no more than 5 liters:
/*29*/   state_transition(state(X,Y,Z),state(X,L,M)):-
/*30*/         pouring(Z,Y,5,M,L).

/*31*/   pouring(X,Y,LimitY,K,L):-
/*32*/         check(X,Y,LimitY),
/*33*/         !,
/*34*/         NX is X - 1,
/*35*/         NY is Y + 1,
/*36*/         pouring(NX,NY,LimitY,K,L).
/*37*/   pouring(X,Y,_,X,Y).

/*38*/   check(X,Y,Limit):-
/*39*/        X > 0,
/*40*/        Y < Limit,!.

/*41*/   optimize:-
/*42*/         optimum_sequence_of_states,
/*43*/         shortest_sequence_of_states(N,Sequence_of_states),
/*44*/         reverse(Sequence_of_states, Reversed_sequence),
/*45*/         write("Optimum_solution : "),nl,
/*46*/         write(Reversed_sequence),nl,
/*47*/         write("Number of pourings: "),write(N).

/*48*/   optimum_sequence_of_states:-
/*49*/         sequence_of_states(X,Trajectory_X),
/*50*/         shortest_sequence_of_states(Y,Trajectory_Y),
/*51*/         update(X,Y,Trajectory_X,Trajectory_Y),
/*52*/         fail.
/*53*/   optimum_sequence_of_states.

/*54*/   update(X,Y,Trajectory_X,Trajectory_Y):-
/*55*/         X < Y,
/*56*/         !,
/*57*/         retract(shortest_sequence_of_states(Y,Trajectory_Y)),
/*58*/          assert(shortest_sequence_of_states(X,Trajectory_X)).
/*59*/   update(_,_,_,_).
```

The message generated is:

```
  Optimum_solution :
  [state(8,0,0), state(3,5,0), state(3,2,3), state(6,2,0),
   state(6,0,2), state(1,5,2), state(1,4,3), state(4,4,0)]
```

```
Number of pourings: 8
```

The process of filling the three jugs is shown in Figure 2.23. Measuring markers on jugs in Figure 2.23 have to illustrate the fillings, but are not used to control the fillings.

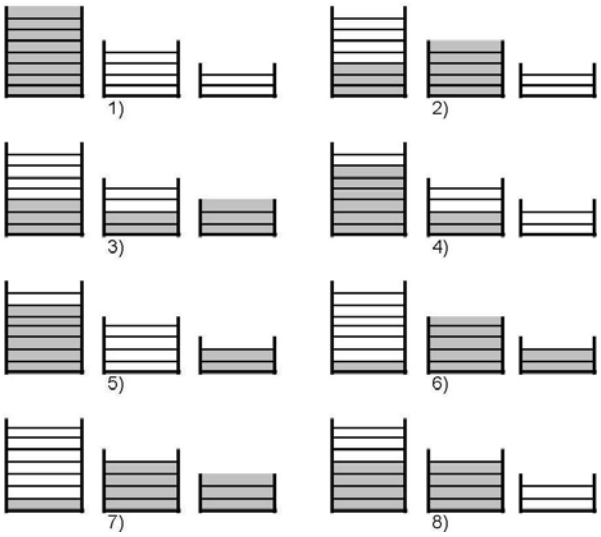


Figure 2.23: Filling of three jugs

2.7 Exercises

Domains

The domain declarations in *Prolog* programs are usually done implicitly and sometimes hidden in strange places. Determine the variable domains for all *Prolog* examples from the present chapter.

Fibonacci numbers

Leonardo Fibonacci (c. 1170 – c. 1250) was an eminent mathematician and mathematics teacher in the Republic of Pisa (now being part of Italy). He is famous because of the attempt to model the growth of rabbit popu-

lations, rabbits being at his time a widely craved source of meet and fur. He assumed that a newly-born pair of rabbits of both genders are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits; assuming further that rabbits never die and a mating pair always produces one new pair every month from the second month on, the number of pairs of the rabbit population increase in a month by month basis as follows:

`0,1,1,2,3,5,8,13,21,34,55,89,144,...`,

Denoting the number of rabbit pairs on the beginning of the `n`th month-long period by F_n, the process may be described by the double recursion:

$$F_n = F_{n-1} + F_{n-2}$$

where: $F_0 = 0, \quad ,F_1 = 1$.

Write a program for calculating Fibonacci numbers that is not tail-recursive, and another one that is tail-recursive.

Girl friends

John has five girl friends: 1)Ann is blonde, 27 years old, is a Doctor of Medicine, is married, has two children, a boy and a girl, likes swimming, 2)Beverly is blonde, 20 years old, is a student, single, no children, likes cooking, 3)Colette is brunette, 24 years old, housewife, married, no children, likes acting, 4)Diana is blonde, 21 years old, a secretary, divorced, one child - a girl, likes being entertained, 5)Edna is blonde, 25 years old, a nurse, divorced, no children, likes classical music. Use `findal/3` to establish data of all those girl friends that are not divorced, not older than 24, and like a non-sporting activity.

Games

At the local games evening, four lads were competing in the Scrabble and chess competitions. Liam beat Mark in chess, James came third and the 16 year old won. Liam came second in Scrabble, the 15 year old won, James beat the 18 year old and the 19 year old came third. Kevin is 3 years younger than Mark. The person who came last in chess, came third in Scrabble and only one lad got the same position in both games. Write a program to determine the ages of the lads and the positions in the two games.

Musical recital

At a musical recital five students (John, Kate, Larry, Mary and Nick) performed five musical pieces, two by Bach, two by Mozart and one by Vivaldi. There were three violinists and two pianists. Each student performed only one piece, and played only one instrument. Find the order of the students, their respective instruments and the composer, with the following conditions: 1. The composers were not played consecutively. Vivaldi was played last and Mozart was played first. 2. There was one piano piece that was played between two violin pieces, and two violin pieces between the first and last piano piece. 3. There were no piano pieces by Mozart. 4. Kate played third. 5. Nick played the piano, and immediately followed John, who played a piece by Mozart. 6. Mary did not play a piece by Vivaldi.

Master classes [34]

The great mezzo-soprano Flora Nebbiacorno has retired from the international opera stage, but she still teaches master classes regularly. At a recent class, her five students were one soprano, one mezzo-soprano, two tenors, and one bass. (The first two voice types are women's, and the last two are men's). Their first names are Chris, J.P., Lee, Pat, and Val – any of which could belong to a man or a woman – and their last names are Kingsley, Robinson, Robinson (the two are unrelated but have the same last name), Ulrich, and Walker. Write a program to find the order in which these five sang for the class, identifying each by full name and voice type, provided that:

1. The first and second students were, in some order, Pat and the bass. 2. The second and third students included at least one tenor. 3. Kingsley and the fifth student (who isn't named Robinson) were, in some order, a mezzo-soprano and a tenor. 4. Neither the third student, whose name is Robinson, nor Walker has the first name of Chris. 5. Ulrich is not the bass or the mezzo-soprano. 6. Neither Lee or Val (who wasn't third) is a tenor. 7. J.P. wasn't third, and Chris wasn't fifth. 8. The bass isn't named Robinson.

Jam making contest

At the recent inter-departmental jam making contest, four lucky candidates took part to make the juiciest strawberry jam. The ages of the contestants were 14, 17, 20, 22. As it happens the person who came last

[34] This exercise is from http://brownbuffalo.sourceforge.net/

was the oldest, whereas Stuart was three years older than the person who came second. James was neither the oldest nor the youngest and Kev finished ahead of the 17 year old, but didn't win. John was also unlucky this time and didn't win either. Write a program to determinate who finished where and how old they are.

Bridge meeting

Four ladies meet each week on Thusday to play bridge. On each meeting they decide what everyone has to bring for the next meeting. 1. Mrs. Andrew will bring chocolate cake. 2. Neither Mrs. Brown, nor Viven, nor Ann Clark will bring cookies. 3. Rachel, who is not from Davidson's family, will bring coffee. 4. Mary will not bring the wine. Write a program to determine the whole name of each lady and what is she supposed to bring next week.

Two jugs

You are given two jugs, a 4-gallon one and a 3-gallon one. Neither has any measuring markers on it. There is a tap that can be used to fill the jugs with water. Write a program to determine how can you get exactly 2 gallons of water into the 4-gallon jug.

Ships

There are 5 ships in a port[35]. 1. The Greek ship leaves at six and carries coffee. 2. The ship in the middle has a black chimney. 3. The English ship leaves at nine. 4. The French ship with a blue chimney is to the left of a ship that carries coffee. 5. To the right of the ship carrying cocoa is a ship going to Marseille. 6. The Brazilian ship is heading for Manila. 7. Next to the ship carrying rice is a ship with a green chimney. 8. A ship going to Genoa leaves at five. 9. The Spanish ship leaves at seven and is to the right of the ship going to Marseille. 10. The ship with a red chimney goes to Hamburg. 11. Next to the ship leaving at seven is a ship with a white chimney. 12. The ship on the border carries corn. 13. The ship with a black chimney leaves at eight. 14. The ship carrying corn is anchored next to the ship carrying rice. 15. The ship to Hamburg leaves at six.

Write a program to determine which ship goes to Port Said and which ship carries tea.

[35]This exercise is from http://www.mathsisfun.com/puzzles

River crossing 1

Four adventurers (Alex, Brook, Chris and Dusty) need to cross a river in a small canoe[36]. The canoe can only carry 100 kg. Alex weighs 90 kg, Brook weighs 80 kg, Chris weighs 60 kg and Dusty weighs 40 kg, and they have 20 kg of supplies. Write a program showing how do they get across.

River crossing 2

Three humans and three monkeys (one big, two small) need to cross a river. But there is only one boat, and it can only hold two bodies (regardless of their size), and only the humans or the big monkey are strong enough to row the boat. Furthermore, the number of monkeys can never outnumber the number of humans on the same side of the river, or the monkeys will attack the humans. Write a program to demonstrate how can all six get across the river without anyone getting hurt.

River crossing 3

There is a family on one side of the river: 1. Father 2. Mother 3. Son 4. Daughter 5. Maid 6. Dog They need to get to the other side of the river. Only 1 small boat is available to bring them across. The boat is big enough for only 2 people OR 1 person + dog. Here's the tricky part: * Only Father, Mother and Maid knows how to row the boat. At all times, * Father cannot be alone with the Son, without the Mother, or else he will hit the Son. * Mother cannot be alone with the Daughter, without the Father, or else she will slap the Daughter * Maid MUST be with the Dog, or else the Dog will bite anyone in sight. Write a program for the family of 6 to get across the river, without getting hit, slapped or bitten.

River crossing 4

Three couples AA, BB and CC (the gents Andrew, Basil and Charles and the corresponding ladies Ann, Barbara and Celine) had to cross a river in a small boat that held only two people that. No husband would leave his wife in the company of another man unless he himself was present. Besides there are additional personal constraints which should not be violated:
- Andrew should not row alone because he is afraid of the river;
- Ann cannot row because of her advanced pregnancy;
- Barbara cannot row because her arm is broken;
- all other people could row;
- Andrew and Charles should not row together because the hate each

[36]This exercise is from http://www.mathsisfun.com/puzzles

other;
- for the same reason Andrew and Charles should not remain by themselves on the same river side.

Write a program for the couples o get across the river without jealousy arising, and no personal constraint being violated.

Liars

It is known only one character is telling the truth. Mr. April says that Mr. May tells lies. Mr. May says that Mr. June tells lies. Mr. June says that both Mr. April and Mr. May tell lies. Write a program which determines who is telling the truth.

Pets

At a recent Pets Anonymous reunion, the attendees were discussing which pets they had recently owned. James used to have a dog. The person who used to own a mouse now owns a cat, but the person who used to have a cat does not have a mouse. Kevin has now or used to have a dog, I can't remember which. Becky has never owned a mouse. Only one person now owns the pet they previously had. Rebecca said very little throughout the meeting and nobody mentioned the hamster. Write a program to determine who owns which pet and what they used to own.

Snail racing

After the recent Brain-Bashers snail racing contest, the four contestants were congratulating each other. Only one snail wore the same number as the position it finished in. Alfred's snail wasn't painted yellow nor blue, and the snail who wore 3, that was painted red, beat the snail who came in third. Arthur's snail beat Anne's snail, whereas Alice's snail beat the snail who wore 1. The snail painted green, Alice's, came second and the snail painted blue wore number 4. Anne's snail wore number 1. Write a program to work out who's snail finished where, its number and the color it was painted.

Professions

Messrs Butcher, Baker, Carpenter and Plumber have met for the first time after college graduation. No-one is currently, nor ever has been in the same profession as their name and on-one has had the same profession twice. Charlie has never been a carpenter and Mr Butcher in now a plumber. Dave used to be a butcher, whereas Mr Brian Baker never has.

Mr Plumber is not called Eddie and Mr Carpenter did not used to be a butcher. Write a program to determine the full names of each of the attendees, along with their current and previous profession.

Pre-Olympic Rehearsal

At last month's Pre-Olympic Rehearsal, four top athletes competed in two qualifying 400 meter races. As the results were expected to be mislaid, various notes were taken to ensure the accuracy of the overall placing: No-one finished both races in the same position. John beat Mr Donald in both races. Steve Curtail came third in the second race and Dave came last in the first race. In the second race, Mr Arnold won and Mr Bowler came last. In the first race, Steve beat Kev, but Kev beat John. Write a program to determine who finished where in each of the races.

Nine students

Alex, Bret, Chris, Derek, Eddie, Fred, Greg, Harold and John are nine students who live in a three storey building, with three rooms on each floor. A room in the West wing, one in the center, and one in the East wing. If you look directly at the building, the left side is West and the right side is East. Each student is assigned exactly one room. Write a program to find where each of their room is, provided : 1. Harold does not live on the bottom floor. 2. Fred lives directly above John and directly next to Bret (who lives in the West wing). 3. Eddie lives in the East wing and one floor higher than Fred. 4. Derek lives directly above Fred. 5. Greg lives directly above Chris.

Wine barrels

A man, who recently passed away, was the owner of a winery. In his will, he left 21 barrels (seven of which are filled with wine, seven of which are half full, and seven of which are empty) to his three sons. However, the wine and barrels must be split so that each son has the same number of full barrels, the same number of half-full barrels, and the same number of empty barrels. Note that there are no measuring devices handy. Write a program that determines how can the barrels and wine be evenly divided.

Greetings

Kent and Hannah invited some of their friends at a dinner. Some friends arrived with their spouses while some arrived alone. Each guest greeted with every of the two hosts and with each other guest. When two men greeted each other there were handshaking. When two women greeted

each other there were kissing. The same was true when a man and a woman greeted each other. It is known 6 handshakes and 12 kisses have been done in total. Write a program to determine how many guests arrived at the dinner, how many of them were in couples and how many of them were alone? Obviously, when two guests arrived as a couple they didn't greet each other.

Politically correct missionaries and cannibals

Modify program `2_19_mac.pl` so as to meet the criterium of political correctness presented by the footnote to Section 2.5.2.

Art theft

After a local art theft, six suspects were being interviewed. Below is a summary of their statements:

Alan said: It wasn't Brian. It wasn't Dave. It wasn't Eddie.
Brian said: It wasn't Alan. It wasn't Charlie. It wasn't Eddie.
Charlie said: It wasn't Brian. It wasn't Freddie. It wasn't Eddie.
Dave said: It wasn't Alan. It wasn't Freddie. It wasn't Charlie.
Eddie said: It wasn't Charlie. It wasn't Dave. It wasn't Freddie.
Freddie said: It wasn't Charlie. It wasn't Dave. It wasn't Alan

Police know that exactly four of them told one lie each and all of the other statements are true. From this information write a program to determine who committed the theft.

Competition

Five friends were competing for jobs in the Huge International Corporation. After all interviews and examinations the results were presented to the competitors. A bystander watching the friends overheard that:

- Art sadly confessed he has not been ranked on the first position;
- Ben admitted he has been ranked as third after Carl
- Art added that Carl has not been ranked second;
- Ben added that Ed was neither the first nor the last in the ranking;
- Dusty admitted he was ranked just after Art.

Does the bystander has enough information to rank all five friends? Write a suitable program.

One more maze

For the maze from Figure 2.24 find the shortest path (as measured by the number of path forks) for the dragon to reach and fight the dinosaur.

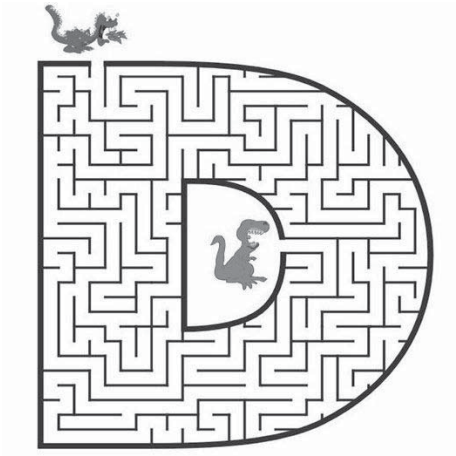


Figure 2.24: Dragon-dinosaur maze

Secret Service delators

Six former Secret Service delators enjoy their retirement living in the same six-floor Apartment House. Each gentleman delator (Al, Bob and Chase) and each lady delator (Debi, Elsa and Fay) live on different floors. The delator family names are Airhead, Zero, Deadbeat, Herd, Flake and Nutter. Write a program to determine the names and family names of all delators and the number of denunciation reports written by each of them, provided that:

- they wrote altogether 280 denunciation reports, each delator at least one report;
- Chase lives one the floor below Flake;
- the family name of Elsa is neither Herd nor Deadbeat;

- Fay lives on a higher floor than Debi, but on a lower floor than Herd.
- Bob lives just above Nutter and just below this fellow who wrote 40 reports.
- Airhead lives neither on the first floor, nor on the six floor.
- Al wrote half the number of reports as the resident from the six floor, who wrote half the number of reports as Zero;
- Debi does not live on the first floor;
- Bob wrote 20 reports more than Zero;
- Zero's name is not Debi;
- Nutter wrote 10 reports less than Airhead.

More uses of conditional predicates

Have a look at those examples from Chapter 2 which have been solved with no use of the basic conditional predicate from Section 2.4.10. Can any of them be solved using the conditional predicate? Design for some of them a program.

Chapter 3

CLP with elementary predicates for feasible solutions

3.1 Elementary predicates

The range of built-ins made available to users is for *CLP* languages much greater and decisively more powerful than for *Prolog*. They may be dichotomized into:

- *Elementary predicates* which are predicates of fundamental functionality over input variables contained at most in one lists. The are made available by `ic` and `branch_and_bound` libraries.
- *Global predicates* which are predicates of advanced functionality over a number of input lists. The are made available by libraries like `ic_global`, `ic_cumulative`, `ic_edge_finder`, `ic_edge_finder3`.

Obviously, the nature and usage of elementary predicates is simpler than of *global predicates*. Elementary predicates form the basic building blocks of CLP programs and their properties as well as the way they are handled deserve close attention. Therefore we start with using them, while leaving the discussion and application of global predicates to latter Sections 4 and 6.

3.2 How CLP languages differ from Prolog?

3.2.1 Basic differences

1. In *Prolog* programs, variable domains were declared *implicitly* by elements of lists scattered in various predicates in various places of the program. *CLP* programs contain, in their top part, *explicitly* declared domains for all variables used in the program.

2. Prolog could handle only variables defined over domains of terms. CLP languages are able to handle variables from a decisive broader range of domains, e.g. integer domains, real domains, symbolic domains.

3. In Prolog constraint propagation was done *via* unification. CLP languages use more efficient constraint propagation methods known as *consistency techniques*.

4. CLP languages use more efficient search method compared with depth first search with standard backtracking. The more efficient methods are among others *forward checking* and *forward checking-looking ahead*.

5. CLP languages integrate the mentioned search methods and constraint propagation techniques into efficient and easy-to-use *search and propagation solvers*.

6. In Prolog programs search is started automatically whenever a query is invoked. For CLP languages search is started by a special predicate (usually *built-in*) that grounds variables in some order, most often named `labeling/1`. The properties of this basic labeling predicate correspond to the rule:

```
labeling([H|T]):-
    indomain(H),
    labeling(T).
labeling([]).,
```

   where the built-in predicate `indomain(List_of_Variables)` grounds the variables from the `List_of_Variables` successively to values from their domain, in such order as they appear in the `List_of_Variables`, from left

to right. This order is sometimes not the most efficient one, so CLP languages (including ECL^iPS^e) makes available a number of search heuristics different from that realized by `labeling/1`, see 3.3.

7. While programming in ECL^iPS^e *Prolog*, no libraries need to be attached to the program. On the other hand, while programming in ECL^iPS^e *CLP*, the program must start with a declaration of needed libraries. The most often needed libraries are the following:

   - The `ic` (`interval constraint`) library that is a hybrid integer/real interval arithmetic[1] constraint solver. Its aim is to make it convenient for programmers to write hybrid solutions to problems, mixing together integer and real constraints and variables. It is the basic library, needed for the majority of problems discussed in chapters 3,..6.
   - The `lib(branch_and_bound)` library that implements a highly parameterized `branch and bound` algorithm, see chapters 5 and 6.
   - The `eplex` library with LP, MIP and quadratic programming solvers, providing also the possibility of interfacing with third-party optimization software.
   - The `ic_global` library that implements a number of global constraints over lists of integer input variables.
   - The `cumulative` library that implements the cumulative scheduling constraint, see Chapter 6.
   - The libraries `ic_edge_finder` and `ic_edge_finder3` that implement stronger versions of the cumulative and disjunctive constraints and cumulative scheduling constraints.
   - The `ic_sets` library that makes available a solver for constraints over the domain of finite sets of integers.
   - The `ic_symbolic` library that makes available a solver for constraints over ordered symbolic domains.

   A detailed presentation of all libraries may be found in the ECL^iPS^e *Constraint Library Manual*, available in the *ECLiPSe Documentation*, see Figure 5.

[1] Interval arithmetic - as contrasted with "normal" arithmetic - deals with arithmetic operations on real-valued intervals. The result of arithmetic interval operations is not given by some set of state variable values, but by some set of state variable intervals. It will be used intensively while discussing constraint solving for continuous variables.

3.2.2 Similarity

The main similarity between *Prolog* and *CLP* is that both infer using *search* and *propagation*

The concepts mentioned will be illustrated by a number of examples, the first one is the queens placement problem.

3.2.3 Queens - CLP approaches

So far two solutions for the queens placement problem were presented:

1. Exhaustive search, for which all possible permutations for
   `[X1,X2,...,Xn] = [1,2,...,n]`
   were consecutively generated and their "safety" was tested.

2. Depth first search with standard backtracking, for which a safe partial placement `[Xj,Xk,...]` was extended by adding another queen and testing the extended placement for safety; if it is safe we proceed with adding yet another queen, if this test fails backtrack is done to the nearest placement for which there is still an untested choice of some queen to be added. Standard backtracking is pruning some branches of the exhaustive search tree, thereby contributing to the efficiency of the search.

However, there are two drawbacks of depth first search with standard backtracking:
1)backtracking is performed only as the result of violating some constraints; 2)„trashing" i.e. repeated failure due to the appearance of similar partial solutions, as shown in Figure 3.1.

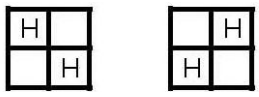


Figure 3.1: Partial queens placement generating *trashing*

Let's try to find a feasible solution for the queen placement problem using elementary constraints and tools available at the ECL^iPS^e platform. This

could be done only by enhancing backtracking search: no CLP language enables exhaustive search, offering only more advanced search methods.

3.2.4 *Forward Checking* for queens

Standard backtracking search may be improved using *Forward Checking*. Its salient feature is to initiate backtracking before some constraint fails, but when this failure will happen in the next search step. Strictly speaking - *Forward Checking* is not only a *search technique*, i.e. a tool for grounding, degrounding and regrounding variables in some order. It combines a search technique with a *consistency-based* constraint propagation technique which is much more effective than Prolog's unification.

Forward Checking is best illustrated using the simple 4 queens placement problem. It is assumed that:

- any queen i has its domain given by a list:
  `[X1,X2,X3,X4]`
  of feasible `Xi` values, denoting the number of the chessboard row, in which the queen is placed in the `i`th column;
- initially all domains are the same and given by `[1, 2, 3, 4]`;
- to some existing safe partial placement [xi,xk,..] a new queen is added to a position determined by her domain; this is just what *Forward Checking* is about - we never place the queen on a position outside her domain, i.e. a position which is not safe;
- adding a new queen is followed by updating the domains of all queens not placed yet. This is a particular case of *constraint propagation*: the constraint introduced by the newly placed queen is propagated across the domains of the remaining queens;
- if some domain happens to be empty, *backtracking* (BT) is performed to this nearest previous placement, for which there exists still non-empty domains for queens not placed yet.

This is illustrated by Figure 3.2 where red $\times$ denote places *off limits* for unplaced queens, i.e. places removed from their domains.

The animation from Figure 3.2 corresponds to the search tree from Figure 3.3.

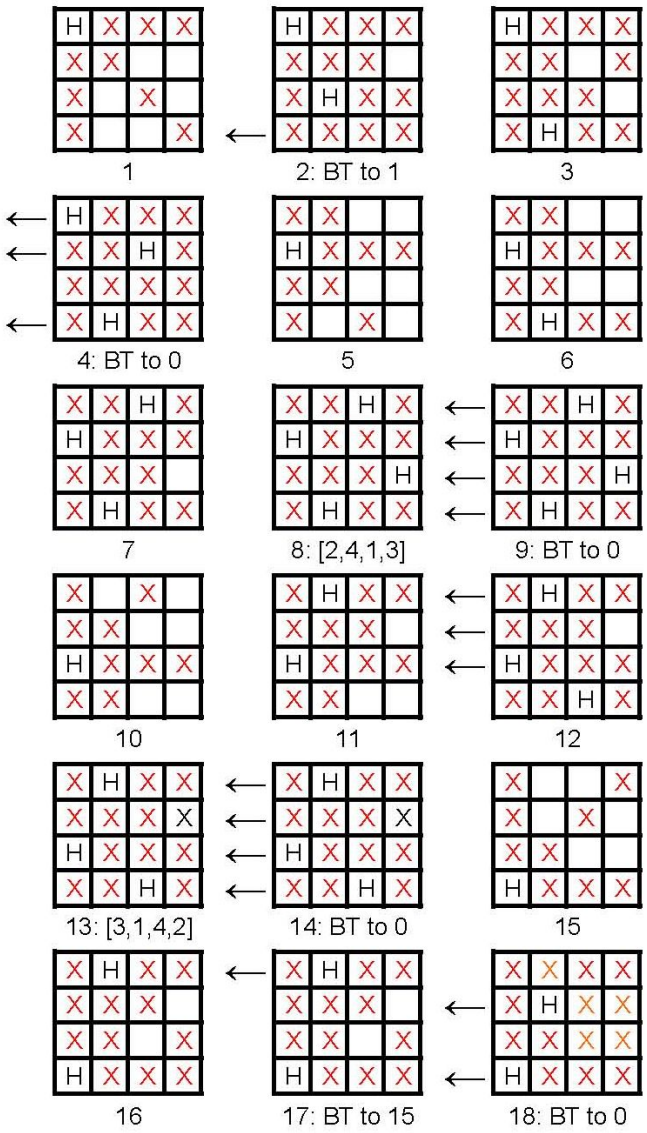


Figure 3.2: *Forward Checking* for four queens

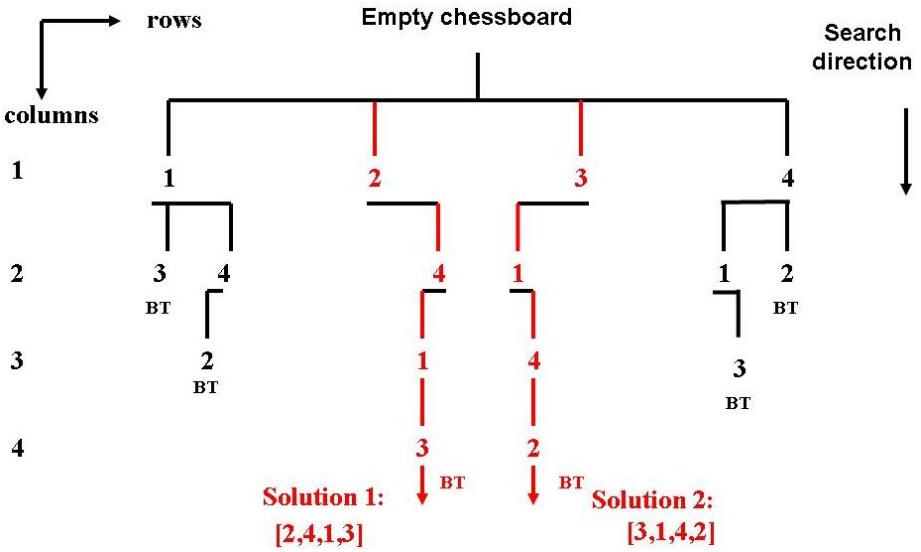


Figure 3.3: Search tree for *Forward Checking* for four queens

It can be seen that, while *Forward Checking*, backtracking is not initiated by violating some constraint, but by the inevitability of a constraint being violated next, this being indicated by the appearance of an empty domain. *Forward Checking* generates a new placement by adding the next queen to the already existing safe placement only if the new queen has a non-empty domain. Otherwise backtracking is performed. The result is that *Forward Checking* is pruning some additional branches of the search tree as compared with depth-first search with standard backtracking, thus increasing search effectiveness.

3.2.5 *Looking Ahead+Forward Checking* for queens

Forward Checking has yet some drawbacks: it is not aware of consequences more remote then the next search step and thus attempts to place queens on places that result in empty domains not in the next step, but in the next plus one step. Such situation is shown in Figure 3.4.

Looking Ahead is practically always used together with *Forward Checking*. It initiates backtracking as soon as the violation of some constraint in the next

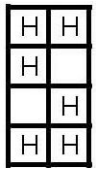


Figure 3.4: A queen placement that invokes *Forward Checking* in vain

plus one search step is to be predicted[2]. This is best illustrated for placing 4 queens, as shown in Figures 3.5 i 3.6.
Notice that:

- *Forward Checking* alone is not testing non-empty domains of queens not placed yet;
- *Looking Ahead + Forward Checking* is testing whether non-empty domains of queens not placed yet contain non-safe placements; if so, backtracking is performed.

To end this Section, some words of consolation are due:

- the ECL^iPS^e user is not expected to deal explicitly with the described backtracking enhancements;
- they are automatically provided by the mere declaration of stating some goal.

The above discussion just aims to give the ECL^iPS^e user some idea about why is it more efficient than *Prolog*.

3.3 Search heuristics

The queen placement problems shows that two decisions influence the search effectiveness. They are answers to following questions:

1. What variable should first be chosen for grounding, what next, what afterwards, etc?

[2]Obviously, this prediction must be "cheaper" in numerical terms than simply testing the state for the next plus one search step.

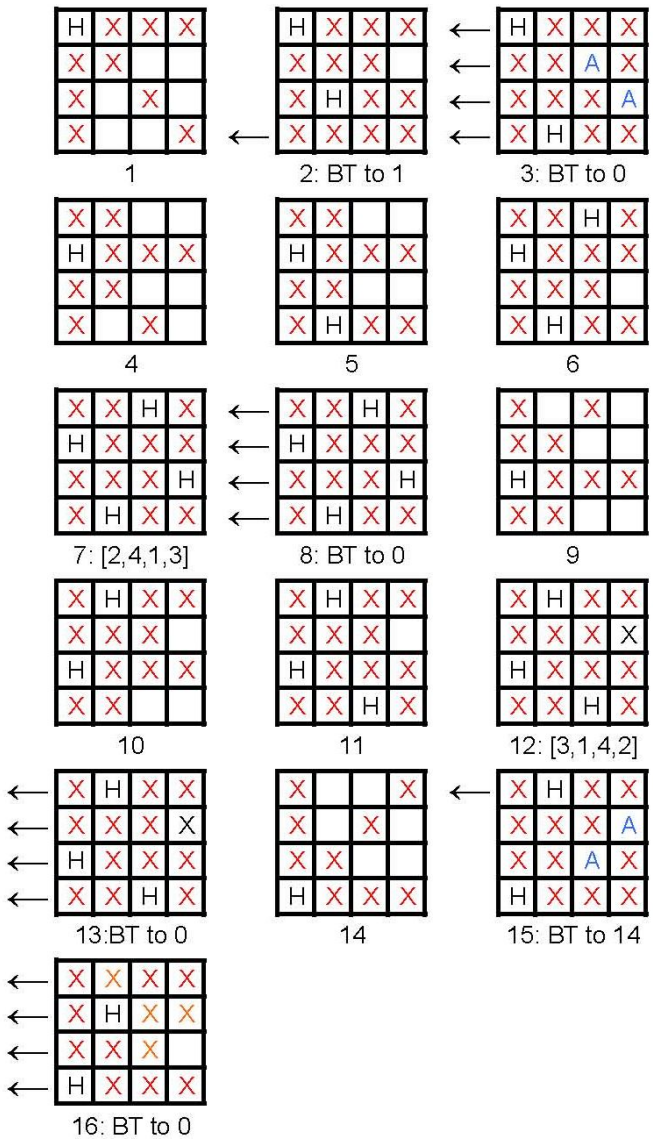


Figure 3.5: *Looking Ahead+Forward Checking* for four queens

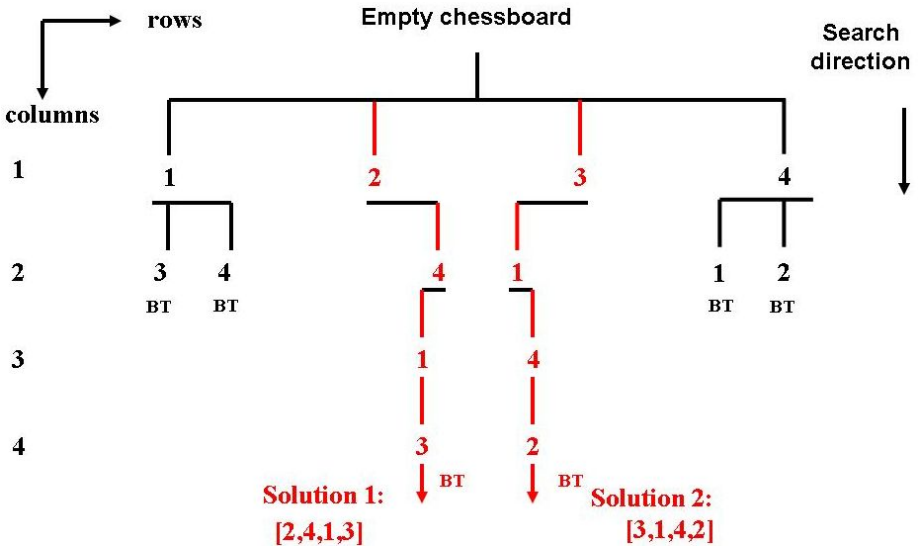


Figure 3.6: Search tree for *Looking Ahead+Forward Checking*for four queens

2. What value (from the domain of the first variable chosen) should be used for grounding, what value (from the domain of the next variable chosen) should be used for grounding, etc? .

For the queen placement examples the variables were chosen starting with the head of the variable list, the head of the tail was chosen next, etc. It should be noticed that this was not the best (in terms of search efficiency) choice. Starting near the "middle" of the list (e.g. choosing first the second variable for grounding), it can be seen from Figures 3.5 and 3.6 that the solution would be obtained with a smaller number of backtracks.

For the queen placement examples the chosen variable was first grounded to the first value from its domain, then to the second value, etc. It should be noticed that this was also not the best choice. While starting near the middle of the domain (e.g. grounding first the variable on value 2), it can be seen from Figures 3.5 and 3.6 that the solution would be obtained with a smaller number of backtracks.

The ways to choose the variable order and value order are covered by an umbrella term *search heuristics*:

1. The order of variable to be grounded depends upon the *variable choice heuristic*.

2. The order of values to which the selected variable is grounded depends upon the *value choice heuristic*.

In Chapter 5 search heuristics for a more advanced search predicate than `labeling/1` will be discussed.

However, it should be emphasized already at this point that there are no means of knowing beforehand which search heuristics to choose for some particular problem. The only feasible approach (if efficiency is of importance for repeatedly using the same program with different data) is by *exhaustively searching* all heuristics made available by ECL^iPS^e.

3.4 Consistency techniques

The name *consistency techniques* covers algorithms dedicated towards making a set of integer variables, defined by names and domains, to fulfill a set of constraints by properly adjusting their domains. This is done be removing from the domains values that are inconsistent. Consistency techniques are used in CLP languages for *constraint propagation*, i.e. for removing inconsistent values from variable domains each time a new constraint is tested. In CLP languages problems with combinatorial constraints (i.e. integer constraints and symbolic constraints) have variables defined by integer domains. There is a broad range of *consistency algorithms*. Their names are derived from constraint graphs, used for binary constraints: their nodes correspond to variables and their domains, their arcs correspond to binary constraints. A detailed discussion of consistency techniques may be found [Tsang-95], [Dechter-03] and [Rossi-06]. Depending upon the number of variables present in a constraint, the following consistency techniques are distinguished:

- *Node consistency* - *NC* for unary constraints;

- *Arc consistency* - *AC* for binary constraints;

- *Path consistency* - *PC* for tenary and higher arity constraints.

Path consistency algorithms are seldom ever used, because path consistency may be expressed in a simpler way. E.g. the case of path consistency for $X = Y + Z$ with corresponding domains D_X, D_Y i D_Z may be presented by a set of unary constraints:

$$X >= min(D_Y) + min(D_Z)$$
$$X <= max(D_Y) + max(D_Z)$$
$$Y >= min(D_X) - max(D_Z)$$
$$Y <= max(D_X) - min(D_Z)$$
$$Z >= min(D_X) - max(D_Y)$$
$$Z <= max(D_X) - max(D_Y)$$

The effectiveness of existing consistency techniques has an important bearing on the methodology of CSP and COP: *they must be modelled using integer variables*. This is sometimes easier said than done, and occasionally may look strange indeed. However, this is something anybody learning CLP has to master.

Constraint propagation in CLP is an autonomous activity: it can sometimes be used for inference purposes with no search.

3.5 Propagating constraints with failure

In ECL^iPS^e programs symbols of arithmetic operations and relations for discrete variables have to be prefixed by `#`. For better understanding of consistency techniques let us consider a simple
example given by program `3_1_domain_0.ecl`[3]:

```
/*1*/   :- lib(ic).

/*2*/   top :-
/*3*/       [X,Y,Z]::1..10,
/*4*/       get_domain(X,PX),
/*5*/       get_domain(Y,PY),
/*6*/       get_domain(Z,PZ),
/*7*/       write("X = "), write(PX),nl,
/*8*/       write("Y = "), write(PY),nl,
```

[3]This is an FS-type problem.

```
/*9*/       write("Z = "), write(PZ),nl,nl,

/*10*/      write("Propagation of constraint  Y < Z results in"),nl,
/*11*/      Y#<Z,
/*12*/      get_domain(X,CX),
/*13*/      get_domain(Y,CY),
/*14*/      get_domain(Z,CZ),
/*15*/      write("X = "), write(CX),nl,
/*16*/      write("Y = "), write(CY),nl,
/*17*/      write("Z = "), write(CZ),nl,nl,

/*18*/      write("Propagation of constraint  X = Y + Z results in:"),nl,
/*19*/      X#=Y+Z,
/*20*/      get_domain(X,DX),
/*21*/      get_domain(Y,DY),
/*22*/      get_domain(Z,DZ),
/*23*/      write("X = "), write(DX),nl,
/*24*/      write("Y = "), write(DY),nl,
/*25*/      write("Z = "), write(DZ),nl,nl,

/*26*/      write("Propagation of constraint  X = Z + 3 results in:"),nl,
/*27*/      X#=Z+3,
/*28*/      get_domain(X,TX),
/*29*/      get_domain(Y,TY),
/*30*/      get_domain(Z,TZ),
/*31*/      write("X = "), write(TX),nl,
/*32*/      write("Y = "), write(TY),nl,
/*33*/      write("Z = "), write(TZ),nl,nl,

/*34*/      write("Propagation of constraint  X > 2+Z results in:"),nl,
/*35*/      X#>2+Z,
/*36*/      get_domain(X,TTX),
/*37*/      get_domain(Y,TTY),
/*38*/      get_domain(Z,TTZ),
/*39*/      write("X = "), write(TTX),nl,
/*40*/      write("Y = "), write(TTY),nl,
/*41*/      write("Z = "), write(TTZ),nl,nl,

/*42*/      write("Propagation of constraint  Y = 2*Z results in:"),nl,
/*43*/      Y#=2*Z,
/*44*/      get_domain(X,SX),
/*45*/      get_domain(Y,SY),
/*46*/      get_domain(Z,SZ),
/*47*/      write("X = "), write(SX),nl,
/*48*/      write("Y = "), write(SY),nl,
/*49*/      write("Z = "), write(SZ).
```

```
The solution is as follows:

      X = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
      Y = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
      Z = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

The initial domains are shown in Figure 3.7.

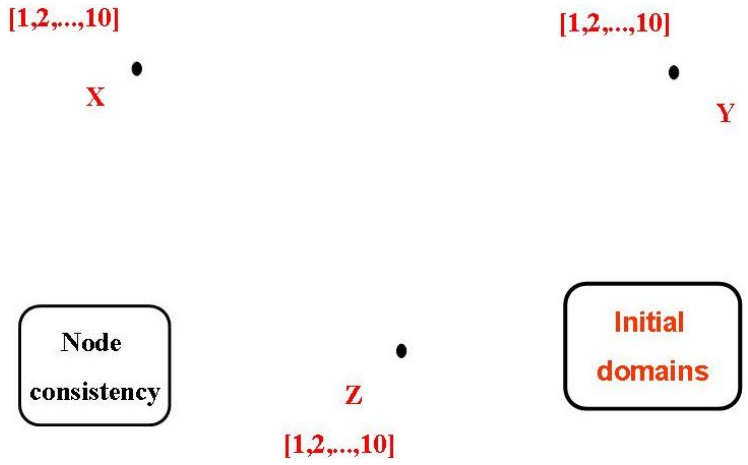


Figure 3.7: Initial domains for variables X, YiZ

```
Propagation of constraint Y < Z results in:
      X = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
      Y = [1, 2, 3, 4, 5, 6, 7, 8, 9]
      Z = [2, 3, 4, 5, 6, 7, 8, 9, 10]
```

Results of this propagation are shown in Figure 3.8.

```
Propagation of constraint X = Y + Z results in:
      X = [3, 4, 5, 6, 7, 8, 9, 10]
      Y = [1, 2, 3, 4, 5, 6, 7, 8]
      Z = [2, 3, 4, 5, 6, 7, 8, 9]
```

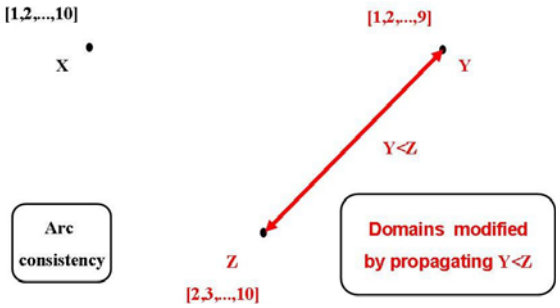


Figure 3.8: Results of successful propagation for $Y < Z$

Results of this propagation are shown in Figure 3.9.

```
Propagation of constraint X = Z + 3 results in:
      X = [5, 6, 7, 8, 9, 10]
      Y = [1, 2, 3, 4, 5, 6]
      Z = [2, 3, 4, 5, 6, 7]
```

Results of this propagation are shown in Figure 3.10.

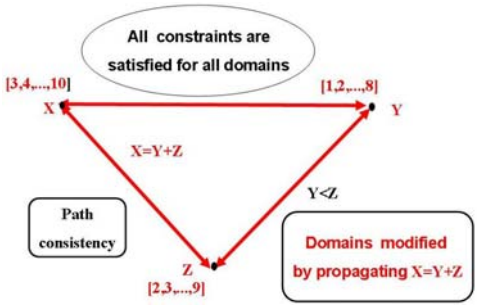


Figure 3.9: Results of successful propagation for $X = Y + Z$

```
Propagation of constraint X > 2+Z results in:
      X = [5, 6, 7, 8, 9, 10]
```

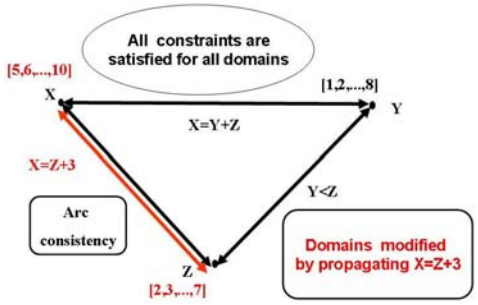


Figure 3.10: Results of successful propagation for $X = Z + 3$

```
Y = [1, 2, 3, 4, 5, 6]
Z = [2, 3, 4, 5, 6, 7]
```

Results of this propagation are shown in Figure 3.11.

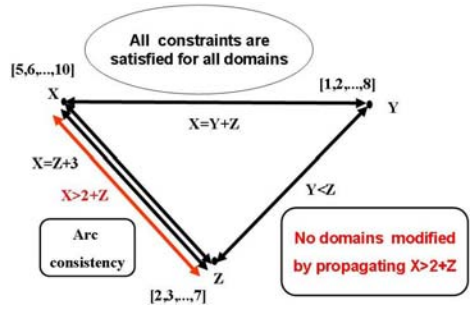


Figure 3.11: Results of successful propagation for $X > 2 + Z$

```
Propagation of constraint  Y = 2*Z results in:

This results in failure: No
```

Results of this propagation are shown in Figure 3.12.

Up to line `/*41*/` the constraint propagation decreases the domain sizes. Line

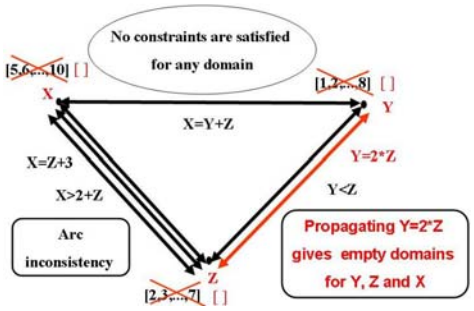


Figure 3.12: Results of unsuccessful propagation for $Y = 2 * Z$

`/*42*/` introduces a constraint inconsistent with this from line `/*10*/`; this results in the domains of `Y` nd `Z` becoming empty. The program ends with failure: the set of inequalities is inconsistent for the declared initial domains.

3.6 Successful propagation of constraints

Constraint propagation *via* consistency techniques is an *incomplete inference method*. However, occasionally propagation alone may procure a unique solution to *CSP*'s. This is illustrated by following programs.

3.6.1 A simple example

Consider the program `3_2_domain_1.ecl`[4]:

```
/*1*/   :- lib(ic).

/*2*/   top :-
/*3*/       [X,Y,Z]::1..10,
/*4*/       get_domain(X,PX),
/*5*/       get_domain(Y,PY),
/*6*/       get_domain(Z,PZ),
/*7*/       write("X = "), write(PX),nl,
/*8*/       write("Y = "), write(PY),nl,
/*9*/       write("Z = "), write(PZ),nl,nl,
```

[4]This is an FS-type problem.

```
/*10/       write("Propagation of constraint X = Y+3 results in:"),nl,
/*10*/      X#=Y+3,
/*11*/      get_domain(X,CX),
/*12*/      get_domain(Y,CY),
/*13*/      get_domain(Z,CZ),
/*14*/      write("X = "), write(CX),nl,
/*15*/      write("Y = "), write(CY),nl,
/*16*/      write("Z = "), write(CZ),nl,nl,

/*17/       write("Propagation of constraint Y < 3 results in:"),nl,
/*18*/      Y#<3,
/*19*/      get_domain(X,DX),
/*20*/      get_domain(Y,DY),
/*21*/      get_domain(Z,DZ),
/*22*/      write("X = "), write(DX),nl,
/*23*/      write("Y = "), write(DY),nl,
/*24*/      write("Z = "), write(DZ),nl,nl,

/*25/       write("Propagation of constraint X > 2+Z results in:"),nl,
/*26*/      X#>2+Z,
/*27*/      get_domain(X,TX),
/*28*/      get_domain(Y,TY),
/*29*/      get_domain(Z,TZ),
/*30*/      write("X = "), write(TX),nl,
/*31*/      write("Y = "), write(TY),nl,
/*32*/      write("Z = "), write(TZ),nl,nl,

/*33/       write("Propagation of constraint Y = 2*Z results in:"),nl,
/*33*/      Y#=2*Z,
/*34*/      get_domain(X,SX),
/*35*/      get_domain(Y,SY),
/*36*/      get_domain(Z,SZ),
/*37*/      write("X = "), write(SX),nl,
/*38*/      write("Y = "), write(SY),nl,
/*39*/      write("Z = "), write(SZ),nl.
```

The solution is as follows:

```
X = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
Y = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
Z = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

Propagation of constraint X = Y+3 results in:
X = [4, 5, 6, 7, 8, 9, 10]
Y = [1, 2, 3, 4, 5, 6, 7]
Z = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

```
Propagation of constraint Y < 3 results in:
X = [4, 5]
Y = [1, 2]
Z = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

Propagation of constraint X > 2+Z results in:
X = [4, 5]
Y = [1, 2]
Z = [1, 2]

Propagation of constraint Y = 2*Z results in:
X = [5]
Y = [2]
Z = [1]
```

Below there are some more examples for which constraint propagation alone is sufficient for finding solutions.

3.6.2 Who with whom?

The number of different combinatorial problems that can be modeled and solved using integer domains is all-encompassing. Some applications seem to be quite astonishing to the beginner. Let us consider the following puzzle[5]:

Who went yesterday evening with whom when:
1. Andy enjoyed a concert.
2. Ben accompanied Olive.
3. Carl has not seen Eva.
4. Paula went to a cinema.
5. Eva was in a theater.
6. One boy and one girl went to an exhibition.

Dusty and Sabina belong also to the set of friends. Determine who went with whom and where if every boy spend the evening with some girl.

The solution is given by program `3_3_who_with_whom.ecl`[6]:

[5] Taken from [Bizam-75].
[6] This is an FS-type problem.

```
/*1*/     :-  lib(ic).

/*2*/    top:-
/*3*/          [Andy,Ben,Carl,Dusty]::[1..4],
/*4*/          [Olive, Eva,Paula,Sabina]::[1..4],
 %  concert=1, cinema=2, theater=3, exhibition=4
 %  It means: if eg. Ben=Olive=4, then
 %  Ben and Olive went to an exhibition

 %  Andy enjoyed a concert:
/*5*/          Andy#=1,

 %  Ben accompanied Olive:
/*6*/          Ben#=Olive,

 %  Carl has not seen Eva:
/*7*/          Carl#\=Eva,

 %  Paula went to a cinema:
/*8*/          Paula#=2,

 %  Eva was in a theater
/*9*/          Eva#=3,

 %  All persons are different:
/*10*/         Andy#\=Ben,
/*11*/         Andy#\=Carl,
/*12*/         Andy#\=Dusty,
/*13*/         Ben#\=Carl,
/*14*/         Ben#\=Dusty,
/*15*/         Carl#\=Dusty,

/*16*/         Olive#\=Eva,
/*17*/         Olive#\=Paula,
/*18*/         Olive#\=Sabina,
/*19*/         Eva#\=Paula,
/*20*/         Eva#\=Sabina,
/*21*/         Paula#\=Sabina,

/*22*/         write(Andy),write(" "),write(Ben),write(" "),
               write(Carl),write(" "),write(Dusty),nl,

/*23*/         write(Olive),write(" "),write(Eva),write(" "),
               write(Paula),write(" "),write(Sabina).
```

The program contains no `labeling(_)` built-in, used for initiating search. Its use would accelerate the inference. The solution generated is poorly under-

standable:

```
1 4 2 3
4 3 2 1
```

It means that:

```
Andy (first position on the boys list)
and Sabina (fourth position on the girls list)
enjoyed a concert (1).
Ben (second position on the boys list)
and Olive (first position on the girls list)
went to an exhibition (4).
Carl (third position on the boys list)
and Paula (third position on the girls list)
went to a cinema (2).
Dusty (fourth position on the boys list)
and Eva (second position on the girls list)
went to a theater (3).
```

The message readability will be improved in Section 4.4.3.

3.6.3 Students and languages

Problems where propagation alone is sufficient for obtaining a solution are sometimes astonishingly complex. This is the case for the following example taken from [Bizam-75]:

Five students of five nationalities spend their vacation on the Masurian Lakes. Its a Pole, a Hungarian, a Finn, a Swede and a German. Determine who speaks what language if:

1. Each student is fluent in one o more foreign languages, but only in those that are native for some of the remaining students.
2. There is no single language spoken by all of them.
3. Each student may speak with any other student using some language.
4. The common languages include native languages of all students.

5. On average each student speaks two foreign languages.
6. The Pole and the Hungarian speak three foreign languages.
7. While the Swede has been swimming, the remaining four students could speak a common language.
8. A common language could also be spoken while the Swede returned, but the Finn went rowing.
9. In order to speak Swedish, two student had to leave the group.
10. Polish and Finnish is spoken (as foreign language) by only two students.
11. The Pole and Finn may communicate using two languages, none of them being German.
12. The Hungarian and the Swede have only one common language.

This puzzle is solved by program `3_4_students_and_languages.ecl`[7]:

```
/*1*/   :- lib(ic).

/*2*/   top :-
/*3*/      Students=["Pole","Hungarian","Finn","Swede","German"],
/*4*/      Languages=["Polish","Hungarian","Finnish","Swedish","German"],

/*5*/      Pole=[PP,PH,PF,PS,PG],
/*6*/      Hungarian=[HP,HH,HF,HS,HG],
/*7*/      Finn=[FP,FH,FF,FS,FG],
/*8*/      Swede=[SP,SH,SF,SS,SG],
/*9*/      German=[GP,GH,GF,GS,GG],
/*10*/     L=[Pole,Hungarian,Finn,Swede,German],
/*11*/     Pole::0..1,
/*12*/     Hungarian::0..1,
/*13*/     Finn::0..1,
/*14*/     Swede::0..1,
/*15*/     German::0..1,
  % The meaning: if PF = 1, the Pole speaks Finnish;
  % if PF = 0, the Pole does not speak Finnish.

                %  constraint_0
                %  Each student speaks its native language::
/*16*/     PP#=1,
/*17*/     HH#=1,
```

[7] This is an FS-type problem.

```
/*18*/     FF#=1,
/*19*/     SS#=1,
/*20*/     GG#=1,

              %  constraint_1
              %  Each student speaks one or more foreign language,
              %  but only those that are native
              %  languages of the remaining students:
/*21*/     PH+PF+PS+PG#>0,
/*22*/     HP+HF+HS+HG#>0,
/*23*/     FP+FH+FS+FG#>0,
/*24*/     SP+SH+SF+SG#>0,
/*25*/     GP+GH+GF+GS#>0,

              %  constraint_2
              %  There is no language spoken by all students:
/*26*/     HP+FP+SP+GP#<4,
/*27*/     PH+FH+SH+GH#<4,
/*28*/     PF+HF+SF+GF#<4,
/*29*/     PS+HS+FS+GS#<4,
/*30*/     PG+HG+FG+SG#<4,

              %  constraint_3
              %  Each student may speak with any other
              %  student using some language:
/*31*/     constraint_3(Pole,Hungarian),
/*32*/     constraint_3(Pole,Finn),
/*33*/     constraint_3(Pole,Swede),
/*34*/     constraint_3(Pole,German),
/*35*/     constraint_3(Hungarian,Finn),
/*36*/     constraint_3(Hungarian,Swede),
/*37*/     constraint_3(Hungarian,German),
/*38*/     constraint_3(Finn,Swede),
/*39*/     constraint_3(Finn,German),
/*40*/     constraint_3(Swede,German),

              %  constraint_5
              %  On average each student speaks two foreign languages:
/*41*/     PH+PF+PS+PG+HP+HF+HS+HG+FP+FH+FS+FG+
                 SP+SH+SF+SG+GP+GH+GF+GS#=10,

              %  constraint_6
              %  The Pole and the Hungarian speak three foreign languages:
/*42*/     PH+PF+PS+PG#=3,
/*43*/     HP+HF+HS+HG#=3,

              %  constraint_7
              %  While the Swede has been swimming, the remaining
              %  four students could speak a common language:
```

```
/*44*/     constraint_7(HP,FP,GP,PH,FH,GH,PF,HF,GF,PG,HG,FG),

               %  constraint_8
               %  A common language could also be spoken while
               %  the Swede returned, but the Finn went rowing:
/*45*/     constraint_8(HP,SP,GP,PH,SH,GH,PS,HS,GS,PG,HG,SG),

               %  constraint_9
               %  In order to speak Swedish,two ,
               %  student had to leave the group:
/*46*/     PS+HS+FS+GS#=2,

               %  constraint_4
               %  The common languages include
               %  native languages of all students:
/*47*/     getval(p,1),
/*48*/     getval(h,1),
/*49*/     getval(f,1),
/*50*/     getval(s,1),
/*51*/     getval(g,1),

               %  constraint_10
               %  Polish and Finnish is spoken (as foreign language)
/*52*/     HP+FP+SP+GP#=1,
/*53*/     PF+HF+SF+GF#=1,

               %  constraint_11
               %  The Pole and Finn may communicate using,
               %  two languages, none of them being German:
/*54*/     constraint_11(PH,FH,FP,PF,PS,FS,PG,FG),

               %  constraint_12
               %  The Hungarian and the Swede have only one common language:
/*55*/     constraint_12(Hungarian,Swede),

/*56*/     solution(Students,L,Languages),!.

/*57*/   constraint_3([A1,A2,A3,A4,A5],[B1,B2,B3,B4,B5]):-
/*58*/     2#=A1+B1, setval(p,1);     %   attention: disjunction
/*59*/     2#=A2+B2, setval(h,1);
/*60*/     2#=A3+B3, setval(f,1);
/*61*/     2#=A4+B4, setval(s,1);
/*62*/     2#=A5+B5, setval(g,1).

/*63*/   constraint_7(HP,FP,GP,PH,FH,GH,PF,HF,GF,PG,HG,FG):-
/*64*/     HP#=1,FP#=1,GP#=1;    %   attention: disjunction
/*65*/     PH#=1,FH#=1,GH#=1;
/*66*/     PF#=1,HF#=1,GF#=1;
```

```
/*67*/     PG#=1,HG#=1,FG#=1.

/*68*/   constraint_8(HP,SP,GP,PH,SH,GH,PS,HS,GS,PG,HG,SG):-
/*69*/     HP#=1,SP#=1,GP#=1;
/*70*/     PH#=1,SH#=1,GH#=1;
/*71*/     PS#=1,HS#=1,GS#=1;
/*72*/     PG#=1,HG#=1,SG#=1.

/*73*/   constraint_11(PH,FH,FP,PF,PS,FS,PG,FG):-
/*74*/     constraint_11b(PG,FG),
/*75*/     constraint_11a(PH,FH,FP,PF,PS,FS).

/*76*/   constraint_11a(PH,FH,FP,PF,PS,FS):-
/*77*/     PH#=1,FH#=1,FP#=1;
/*78*/     PF#=1,FP#=1;
/*79*/     PS#=1,FS#=1,FP#=1;
/*80*/     PH#=1,PF#=1,FH#=1;
/*81*/     PH#=1,PS#=1,FH#=1,FS#=1;
/*82*/     PF#=1,PS#=1,FS#=1.

/*83*/   constraint_11b(PG,FG):-
/*84*/     PG#=0;FG#=0.

/*85*/   constraint_12([G1|_],[G2|_]):-
/*86*/     G1#=1,
/*87*/     G2#=1,
/*88*/     !.

/*89*/   constraint_12([G1|O1],[G2|O2]):-
/*90*/     constraint_12a(G1,G2),
/*91*/     constraint_12(O1,O2).

/*92*/   constraint_12a(G1,G2):-
/*93*/     G1#=0;        %   attention: disjunction
/*94*/     G2#=0.

/*95*/  solution([G1|O1],[G2|O2],L3):-
/*96*/    write(G1),writeln("  is spoken by:"),
/*97*/    solution1(G2,L3),
/*98*/    solution(O1,O2,L3).
/*99*/  solution([],[],_).

/*100*/  solution1([1|O1],[G2|O2]):-
/*101*/    write("   "),writeln(G2),
/*102*/    solution1(O1,O2).
/*103*/  solution1([],[]).

/*104*/  solution1([0|O1],[_|O2]):-
```

```
/*105*/    solution1(O1,O2).
```

The solution is:

```
Polish is spoken by:
 Pole
 Hungarian
 Swede
 German
Hungarian is spoken by:
 Pole
 Hungarian
 Finn
 German
Finnish is spoken by:
 Hungarian
 Finn
 Swede
Swedish is spoken by:
 Swede
 German
German is spoken by:
 Hungarian
 German
```

As seen, despite this problem complexity, it may be solved using only constraint propagation.

3.6.4 Righteous Oppositionists and Secret Collaborators

ECL^iPS^e has a library of symbolic constraints (`ic_symbolic`), useful for symbolic variables (defined by names). Using this library operations on set variables have to be prefixed by `&`. The following example demonstrates its uses.

After the fall of communism in Absurdoland, a chain of "Black and White" debating clubs mushroomed across the country. They were rather exclusive: its

membership was open only to former *Secret Collaborators* (of the resolved Communist Security Service) or former *Righteous Oppositionists* (hunted in the past by the Communist Security Service). Such a membership profile proved to be quite successful. It provided a fertile ground for contradictory discussions, loved by the general public, Main Stream TV media and journalists. It boosted also the consumption of all those beverages, which have a well-earned reputation of facilitating the understanding of complicated situations. The attractiveness of the discussions was further enhanced by the common knowledge that *Righteous Oppositionists* always tell the truth, whereas *Secret Collaborators* lie and tell the truth in alteration. The Main Stream tabloid "*News from the Sewer*" delegated to one of the clubs a Celebrated Journalist to write an in-depth report promoting the idea of reconciliation of those foes of the past. Unfortunately, the Celebrated Journalist had a problem: the club at the time of his arrival was populated by just three members, of whom `Member_1` and `Member_2` argued ferociously, evidently because they belonged to different groups of members. The journalist, not wishing to disturb the adversaries, simply asked `Member_3`, who did not take part in the argument, whether he was a former *Righteous Oppositionist*, or a former *Secret Collaborator*. Unfortunately, `Member_3` had already been drinking too much of the mentioned beverages; therefore he simply mumbled something quite unintelligible under his breath. The Celebrated Journalist asked therefore the remaining two members about what `Member_3` had said. `Member_1`, who perhaps thanks to some practice could understand the reply by `Member_3`, maintained that `Member_3` said he was a former *Righteous Oppositionist*. `Member_2` however first said that `Member_3` is a former *Secret Collaborator*, and next added that `Member_3` had been lying. Does the Celebrated Journalist has sufficient information to infer who is who[8]?

To gain some insight into the problem let's present its state space by a truth table as shown in Figure 3.13. There are three Boolean input variables (`M1`, `M21` and `M22`), denoting correspondingly the logical values of what `Member_1` said and what `Member_2` said the first and second time, with `0` meaning the corresponding member was lying and `1` meaning the corresponding member said the truth. Those three Boolean input variables can be combined in eight ways, as shown by the map.

The numbers inside the squares of the truth table correspond to logical values of the conjunction of all the problem constraints, `0` meaning the constraints failed, `1` meaning the constraints are satisfied:

[8]This is an FS-type problem.

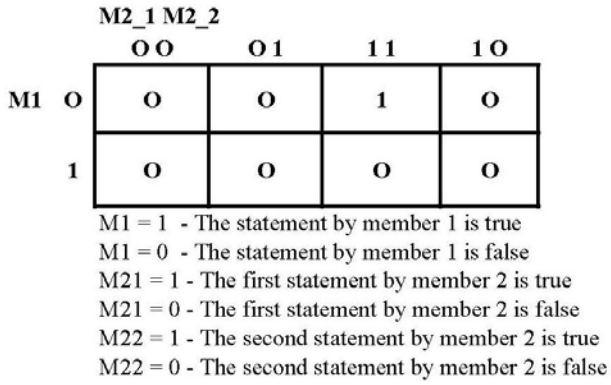


Figure 3.13: Truth table for the state space of the RO-SC story

- the first column is clearly false: no club member ever tells two lies in succession;
- the second column corresponds to a self-contradictory situation: if `Member_3` lied, then the first statement by `Member_2` cannot possibly by false;
- the same applies to the fourth column: if `Member_3` did not lie, then of course the first statement of `Member_2` cannot possibly be true;
- consider the bottom square of the third column: if the statement by `Member_1` is true, then both statements by `Member_2` cannot possibly be true;
- what remains is the top square of third column, which corresponds to a consistent state: if the statement by `Member_1` is false, then both statements of `Member_2` are true;
- it follows that `Member_1` and `Member_3` are former *Secret Collaborators*, and `Member_2` is a former *Righteous Oppositionist*, *Q.E.D.*

Assured that a reasonable and unique answer exists, let's use ECL^iPS^e to produce it. This is done by program `3_5_black_and_white.ecl`[9]:

[9]This follows roughly the program presented by J. Schimpf to the "Liars" problem, see [Schimpf-10a].

```
/*1*/   :- lib(ic).
/*2*/   :- lib(ic_symbolic).
/*3*/   :-local domain(club_member(righteous_oppositionist,secret_collaborator)).

/*4*/   top :-
/*5*/       solve(_).

/*6*/   solve([Member_1,Member_2,Member_3]):-
     % Declaring the symbolic domain:
/*7*/       [Member_1,Member_2,Member_3] &:: club_member,
     % Declaring binary variable domain:
/*8*/       [Member_3_possibly_said,Member_3_said, Member_1_possibly_said,
                Member_2_said_first, Member_2_said_next] :: 0..1,
/*9*/       Member_1 &\= Member_2,

     % % What Member_3 possibly said:
/*10*/      Member_3_possibly_said #= (Member_3 &=righteous_oppositionist),
/*11*/      single_utterance(Member_3, Member_3_possibly_said),

     % What Member_1 possibly said:
/*12*/      Member_1_possibly_said #= (Member_3_said #=Member_3_possibly_said),
/*13*/      single_utterance(Member_1, Member_1_possibly_said),

     % What Member_2 said first:
/*14*/      Member_2_said_first #= (Member_3 &=secret_collaborator),
/*15*/      single_utterance(Member_2,Member_2_said_first),

     % What Memeber_2 said next:
/*16*/      Member_2_said_next #=(Member_3_said #= 0),
/*17*/      single_utterance(Member_2,Member_2_said_next),
/*18*/      consecutive_utterances(Member_2, Member_2_said_first,Member_2_said_next),

/*19*/      ic_symbolic:indomain(Member_1),
/*20*/      ic_symbolic:indomain(Member_2),
/*21*/      ic_symbolic:indomain(Member_3),
/*22*/      writeln("Member_1":Member_1),
/*23*/      writeln("Member_2":Member_2),
/*24*/      writeln("Member_3":Member_3),
/*25*/      writeln("Member_2_said_first":Member_2_said_first),
/*26*/      writeln("Member_2_said_next":Member_2_said_next).

     % Righteous oppositionists always tell truth.
     % Secret collaborators may tell truth or falsehood:
/*27*/  single_utterance(Member, Truth) :-
/*28*/      (Member &= righteous_oppositionist) => Truth.
     % Check it using program \verb"test_TW_OE.ecl."

     % Secret collaborators lie and tell the
```

```
    % truth in strict alteration.
    % Righteous oppositionists always tell truth:
/*29*/  consecutive_utterances(Member, Truth1, Truth2) :-
/*30*/  (Member &= secret_collaborator) #= (Truth1 #\= Truth2).
    % Check it using program \verb"3_12_baw_check.ecl"
```

Following message is generated:

```
Member_1 : secret_collaborator
Member_2 : righteous_oppositionist
Member_3 : secret_collaborator
Member_2_said_first : 1
Member_2_said_next : 1
```

It should be remembered that the symbol `=>` denotes an implication as defined in logic, see Table 3.1. It differs from the Prolog implications, see Table 2.1.

ConX	ConY	ConX => ConY
True	True	True
False	True	True
False	False	True
True	False	False

Table 3.1: Definition of implication in logic as used in ECL^iPS^e

In line `/*28*/` *reification* is used, to be explained latter on in Section 5.6.4. In order to better understand program `3_5_black_and_white.ecl`, it is worthwhile to run program `3_6_baw_check.ecl`[10]:

```
/*1*/   :- lib(ic).
/*2*/   :- lib(ic_symbolic).
/*3*/   :-local domain(club_member(righteous_oppositionist,secret_collaborator)).
 /*4*/   top :-
% Consecutively one and only one of the lines /*5*/,...,/*15*/ is decommented.
% Depending upon the line decommented, the program generates an answer Yes or No.
/*5*/  %  single_utterance(righteous_oppositionist, 1). %  Yes
/*6*/  %  single_utterance(righteous_oppositionist, 0). %  No
/*7*/  %  single_utterance(secret_collaborator, 0). %  Yes
```

[10]This is an FS-type problem.

```
/*8*/  %  single_utterance(secret_collaborator, 1). %  Yes
/*9*/  %  consecutive_utterances(righteous_oppositionist, 1, 0). %  No
/*10*/ %  consecutive_utterances(righteous_oppositionist, 1, 1). %  Yes
/*11*/ %  consecutive_utterances(righteous_oppositionist, 0, 1). %  No
/*12*/ %  consecutive_utterances(secret_collaborator, 1, 0). %  Yes
/*13*/ %  consecutive_utterances(secret_collaborator, 0, 0). %  No
/*14*/ %  consecutive_utterances(secret_collaborator, 0, 1). %  Yes
/*15*/    consecutive_utterances(secret_collaborator, 1, 1). %  No

     % Righteous oppositionists always tell the truth.
     % A single utterance by a secret collaborators may be true or false
/*16*/  single_utterance(Club_Member, Truth) :-
/*17*/      (Club_Member &= righteous_oppositionist) => Truth.

     % Secret collaborators lie and tell the truth in alteration.
     % Righteous oppositionists always (in alteration as well) tell the truth:
/*18*/  consecutive_utterances(Club_Member, Truth1, Truth2) :-
/*19*/  (Club_Member &= secret_collaborator) #= (Truth1 #\= Truth2).
```

For the decommented line `/*15*/` the answer is: `No`.

The aim of programs in Section 3.6 was to show, that although constraint propagation is an incomplete inference method, in some cases it is sufficient for getting the solution. Because no backtracking was used, the approach relying only upon constraint propagation is sometimes denoted as *backtrack-free search*. Obviously, augmenting the discussed program with search (i.e. introducing `labeling/1`) creates no obstacle but usually accelerates the solving process.

The remaining examples in this chapter are such that constraint propagation alone is insufficient for getting the solution: constraint propagation has to be supported by search.

3.7 Propagation is most often not enough

The programs presented so far, which used only propagation, are exceptional. Normally search is needed to get a solution[11]. A series of example follows, for which - despite their seemingly simplicity - search is mandatory.

[11] Even for problems successfully solved with propagation only, search may be used to accelerate the solution.

3.7.1 Three equations

Consider program `3_7_three_equations.ecl`[12] for solving three linear equations in integers:

```
/*1*/   :- lib(ic).

/*2*/   top :-
/*3*/       [X,Y,Z]::0..6,

/*4*/       X + Y + Z #= 9,
/*5*/       write("Constraint X + Y + Z #= 9"),nl,
/*6*/       write("does not reduce domains:"),nl,
/*7*/       get_domain(X, LX),write("Domain of X ="),write(LX),nl,
/*8*/       get_domain(Y, LY),write("Domain of Y ="),write(LY),nl,
/*9*/       get_domain(Z, LZ),write("Domain of Z ="),write(LZ),nl,

/*10*/      2*X+4*Y+3*Z #= 28,
/*11*/      write("The additional constraint 2*X + 4*Y +3* Z #= 28"),
/*12*/      nl,write("neither reduces domains:"),nl,
/*13*/      get_domain(X, LLX),write("Domain of X ="),write(LLX),nl,
/*14*/      get_domain(Y, LLY),write("Domain of Y ="),write(LLY),nl,
/*14*/      get_domain(Z, LLZ),write("Domain of Z ="),write(LLZ),nl,

/*16*/      4*X+2*Y+Z #= 18,
/*17*/      write("At long last the constraint 4*X + 2*Y +Z #= 18"),
/*18*/      nl,write("reduces domains:"),nl,
/*19*/      get_domain(X, LLLX),write("Domain of X ="),write(LLLX),
/*20*/      nl,get_domain(Y, LLLY),write("Domain of Y ="),write(LLLY),
/*21*/      nl,get_domain(Z, LLLZ),write("Domain of Z ="),write(LLLZ),nl,
/*22*/      write("However, some values from the domains remain inconsistent."),nl,

/*23*/      labeling([X,Y,Z]),
/*24*/      write("Now,labeling is finishing the job of reducing domains:"),nl,
/*25*/      get_domain(X, KX),write("Domain of X ="),write(KX),nl,
/*26*/      get_domain(Y, KY),write("Domain of Y ="),write(KY),nl,
/*27*/      get_domain(Z, KZ),write("Domain of Z ="),write(KZ),nl,
/*28*/      write("and providing the unique solution:"),nl,
/*29*/      write("X = "),write(X),nl,
/*30*/      write("Y = "),write(Y),nl,
/*31*/      write("Z = "),write(Z),fail.

/*32*/   top:-
/*33*/      nl,write("No more solutions.").
```

[12] This is an FS-type problem.

The message is:

```
Constraint X + Y + Z ^= 9
does not reduce domains:
Domain of X =[0, 1, 2, 3, 4, 5, 6]
Domain of Y =[0, 1, 2, 3, 4, 5, 6]
Domain of Z =[0, 1, 2, 3, 4, 5, 6]

The additional constraint 2*X + 4*Y +3* Z #= 28
neither reduces domains:
Domain of X =[0, 1, 2, 3, 4, 5, 6]
Domain of Y =[0, 1, 2, 3, 4, 5, 6]
Domain of Z =[0, 1, 2, 3, 4, 5, 6]

At long last the constraint 4*X + 2*Y +Z #= 18
reduces domains:
Domain of X =[0, 1, 2, 3, 4]
Domain of Y =[1, 2, 3, 4, 5, 6]
Domain of Z =[0, 1, 2, 3, 4, 5, 6]
However, some values from the domains remain inconsistent.

Now, labeling is finishing the job of reducing domains:
Domain of X =[2]
Domain of Y =[3]
Domain of Z =[4]
and providing the unique solution:
X = 2
Y = 3
Z = 4
No more solutions.
```

3.7.2 Golfers

Using integer constraints simplifies also program `3_8_golfers.pl` from Section 2.4.1. This is illustrated by program `3_8_golfers.ecl`[13]:

```
/*1*/   :- lib(ic).

/*2*/   top :-
/*3*/       [Fred,Joe,Tom,Bob]::1..4,
            % Tom - variable denoting Tom's position in line.
```

[13] This is an FS-type problem.

```
/*4*/       [Red,Orange,Blue,Plaid]::1..4,
            % Blue - variable denoting the position of blue pants in line.

   % 2)The golfer to Fred's immediate right is wearing blue pants:
/*5*/       Blue#=Fred + 1,

   % (3)Joe is second in line:
/*6*/       Joe#=2,

   % (4)Bob is wearing plaid pants:
/*7*/       Bob#=Plaid,

   % 5)Tom isn't in position one or four, and he isn't
               % wearing the hideous orange pants:
/*8*/       Tom#\=1,
/*9*/       Tom#\=4,
/*10*/      Tom#\=Orange,

   % All golfers are different:
/*11*/      Fred#\=Joe,
/*12*/      Fred#\=Tom,
/*13*/      Fred#\=Bob,
/*14*/      Joe#\=Tom,
/*15*/      Joe#\=Bob,
/*16*/      Tom#\=Bob,

   % All colors are different:
/*17*/      Red#\=Orange,
/*18*/      Red#\=Blue,
/*19*/      Red#\=Plaid,
/*20*/      Orange#\=Blue,
/*21*/      Orange#\= Plaid,
/*22*/      Blue#\=Plaid,

/*13*/      labeling([Fred,Joe,Tom,Bob,Orange,
                            Blue,Red,Plaid]),

/*14*/      write("Fred,Joe,Tom,Bob"),nl,
/*15*/      write([Fred,Joe,Tom,Bob]),nl,
/*16*/      write("Red,Orange,Blue,Plaid"),nl,
/*17*/      write([Red,Orange,Blue,Plaid]),nl.
```

A following message is displayed:

```
Fred,Joe,Tom,Bob
[1, 2, 3, 4]
```

```
Red,Orange,Blue,Plaid
[3, 1, 2, 4]
```

It means that e.g. Joe is in position 2 in the golfers list and wears pants of a color corresponding to number 2 in the colors list, i.e. blue pants. The readability of the message will be taken care of in Section 4.4.4.

This time `labeling/1` is also needed to get the solution: constraint propagation is clearly insufficient.

3.7.3 Watchtowers

The necessity of using search is not related to the number of constraints. We already have seen example `3_4_students_and_languages.ecl` were in spite of a large number of constraints no search was needed for obtaining the solution. The following example is an "opposite" one: in spite of a small number of constraints, search has to be used to get the solution.

Consider a military base located on a square patch of land, surrounded by a wall with corners and middle sides strengthened by multilevel watchtowers. The guard in the corner watchtower may watch both adjacent wall sides. The guard in the middle side watchtower may watch only his side of the wall. How to allocate 12 guards in the watchtowers so that any side of the wall will be watched by 5 guards?

This is solved by program `3_9_watchtowers.ecl`[14]:

```
/*1*/   :- lib(ic).

/*2*/   top :-
/*3*/       Guards = [NW,N,NE,W,E,SW,S,SE],
                %NW - number of guards in watchtower NorthWest
                %E - number of guards in watchtower East

/*4*/       Guards :: 0..12,
/*5*/       sum(Guards) #= 12,
/*6*/       NW + N + NE #= 5,
/*7*/       NE + E + SE #= 5,
/*8*/       NW + W + SW #= 5,
/*9*/       SW + S + SE #= 5,
```

[14] This is an FS-type problem.

```
/*10*/      labeling(Guards),

/*11*/      printf("%3d%3d%3d\n", [NW,N,NE]),
/*12*/      printf("%3d   %5d\n", [W,    E]),
/*13*/      printf("%3d%3d%3d\n", [SW,S,SE]).
```

The solution is:

```
  0  0  5
  2     0
  3  2  0
```

In spite of the examples simplicity, `labeling/1` is needed to get a solution: constraint propagation alone is not sufficient.

3.7.4 Examination

A domain declaration may simplify the examination problem from Section 2.4.7 and accelerate its solution. This is shown by the `3_10_egzamination.ecl` program[15]:

```
/*1*/   :- lib(ic).
/*2*/   top :-
/*3*/      L=[M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,M11,M12,M13,M14,M15,
              M16,M17],
/*4*/      L :: 1..4,

/*5*/      M1 #\= M2,     /*6*/      M1 #\= M5,
/*7*/      M1 #\= M6,     /*8*/      M1 #\= M7,
/*9*/      M2 #\= M6,     /*10*/     M2 #\= M7,
/*11*/     M2 #\= M3,     /*12*/     M2 #\= M8,
/*13*/     M3 #\= M7,     /*14*/     M3 #\= M8,
/*15*/     M3 #\= M9,     /*16*/     M3 #\= M4,
/*17*/     M4 #\= M8,     /*18*/     M4 #\= M9,
/*19*/     M5 #\= M6,     /*20*/     M5 #\= M10,
/*21*/     M5 #\= M11,    /*22*/     M6 #\= M10,
/*23*/     M6 #\= M11,    /*24*/     M6 #\= M7,
/*25*/     M6 #\= M12,    /*26*/     M7 #\= M11,
/*27*/     M7 #\= M12,    /*28*/     M7 #\= M8,
/*29*/     M7 #\= M13,    /*30*/     M8 #\= M12,
/*31*/     M8 #\= M13,    /*32*/     M8 #\= M14,
/*33*/     M8 #\= M9,     /*34*/     M9 #\= M13,
/*35*/     M9 #\= M14,    /*36*/     M10 #\= M11,
```

[15] This is an FS-type problem.

```
/*37*/     M11 #\= M15,   /*38*/     M11 #\= M12,
/*39*/     M12 #\= M15,   /*40*/     M12 #\= M16,
/*41*/     M12 #\= M13,   /*42*/     M13 #\= M15,
/*43*/     M13 #\= M16,   /*44*/     M13 #\= M17,
/*45*/     M13 #\= M14,   /*46*/     M14 #\= M16,
/*47*/     M14 #\= M17,   /*48*/     M15 #\= M16,
/*49*/     M16 #\= M17,

/*50*/     labeling([M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,
                   M11,M12,M13,M14,M15,M16,M17]),

/*51*/     write(M1),write(", "),write(M2),
              write(", "),write(M3),write(", "),write(M4),nl,
/*52*/     write(M5),write(", "),write(M6),write(", "),write(M7),
              write(", "),write(M8),write(", "),write(M9),nl,
/*53*/     write(M10),write(", "),write(M11),write(", "),write(M12),
              write(", "),write(M13),write(", "),write(M14),nl,
/*54*/     write(M15),write(", "),write(M16),
              write(", "),write(M17), nl.
```

One of many possible solutions is given by:

```
   1, 2, 1, 2
2, 3, 4, 3, 4
4, 1, 2, 1, 2
     3, 4, 3
```

This time the solution was obtained immediately. This is a good example of the efficiency of search and propagation performed by $ECL^iPS^e - CPS$ as compared with search and unifications performed by $ECL^iPS^e - Prolog$.

3.7.5 Queens

Consider now a CLP-version of the Prolog program `2_14_queens_bs.pl`. The program `3_11_queens.ecl`[16] determines also safe placements for 8 queens, but seems to be more simple and readable than the former:

[16] This is an FS-type problem.

```
/*1*/   :- lib(ic).
/*2*/   top :-
/*3*/      queens(L),
/*4*/      write(L).

/*5*/   queens([X1,X2,X3,X4,X5,X6,X7,X8]):-
/*6*/      [X1,X2,X3,X4,X5,X6,X7,X8]::1..8,
/*7*/      safe([X1,X2,X3,X4,X5,X6,X7,X8]),
/*8*/      labeling([X1,X2,X3,X4,X5,X6,X7,X8]),
/*9*/      write([X1,X2,X3,X4,X5,X6,X7,X8]),nl,
/*10*/     fail.

/*11*/  queens(_):-
/*12*/     write("That's all!"),nl.

/*13*/  safe([]).
/*14*/  safe([H|T]):-
/*15*/     no_attack(H,T),
/*16*/     safe(T).

/*17*/  no_attack(X,Xs):-
/*18*/     no_attack(X,Xs,1).

/*19*/  no_attack(_,[],_).
/*20*/  no_attack(X,[Y|Ys],Nb):-
/*21*/     X #\= Y,
/*22*/     X #\= Y + Nb,
/*23*/     Y #\= X + Nb,
/*24*/     Nb1 is Nb+1,
/*25*/     no_attack(X,Ys,Nb1).
```

There are 92 placements, from which only the first and last three are presented:

```
[1, 5, 8, 6, 3, 7, 2, 4]
[1, 6, 8, 3, 7, 4, 2, 5]
[1, 7, 4, 6, 8, 2, 5, 3]
.......................
[8, 2, 5, 3, 1, 7, 4, 6]
[8, 3, 1, 6, 2, 5, 7, 4]
[8, 4, 1, 3, 6, 2, 7, 5]
```

This time `labeling/1` was also needed to get the solution; propagation alone was clearly insufficient.

3.7.6 Configuration

We should not forget about transforming the 3-element configuration program from Section 2.2.3 into a full-grown CLP program, given by `3_12_con_CLP.ecl`[17]:

```
/*1*/   :- lib(ic).
/*2*/   top :-
/*3*/       Components=[A_1,A_2,A_3,B_1,B_2,B_3,B_4,C_1,C_2],
/*4*/       Components :: 0..1,
/*5*/       Cost:: 1..2100,

 % Only one A-type element is needed:
/*6*/       A_1 + A_2 + A_3 #= 1,

 % Only one B-type element is needed:
/*7*/       B_1 + B_2 + B_3 + B_4 #= 1,

 % Only one C-type element is needed:
/*8*/       C_1 + C_2 #= 1,

 % Those are compatibility constraints:
/*9*/       C_1 + A_2 #=< 1,  % C_1 and A_2 not in the same configuration
/*10*/      B_2 + C_2 #=< 1,  % B_2 and C_2 not in the same configuration
/*11*/      C_2 + B_3 #=< 1,  % C_2 and B_3 not in the same configuration
/*12*/      B_4 + A_2 #=< 1,  % B_4 and A_2 not in the same configuration
/*13*/      B_3 + A_1 #=< 1,  % B_3 and A_1 not in the same configuration
/*14*/      A_3 + B_3 #=< 1,  % A_3 and B_3 not in the same configuration

/*15*/      Cost #= A_1 * 1900 + A_2 * 750 +
              A_3 * 900 + B_1 * 300 + B_2 * 500 + B_3 * 450 +
                  B_4 * 600 + C_1 * 700 + C_2 * 850,

/*16*/      labeling(Components),

/*17*/      writeln('Feasible configuration with cost':Cost),,
/*18*/      write_configuration([A_1,A_2,A_3,B_1,B_2,B_3,B_4,C_1,C_2],
                ["A_1","A_2","A_3","B_1","B_2","B_3","B_4","C_1","C_2"]),nl,nl,
/*19*/      fail.

/*20*/   top :-
/*21*/      write("Those are all feasible configurations.").

/*22*/   write_configuration([H1|T1],[H2|T2]):-
/*23*/      H1 is 1, write(H2),write("  "),
/*24*/      write_configuration(T1,T2).
```

[17] This is an FS-type problem.

```
/*25*/   write_configuration([H1|T1],[_|T2]):-
/*26*/      H1 is 0,
/*27*/      write_configuration(T1,T2).

/*28*/   write_configuration([],[]).
```

The solution is:

```
Feasible configuration with cost 2100:
A_3  B_2  C_1

Feasible configuration with cost 2050:
A_3  B_1  C_2

Feasible configuration with cost 1900:
A_3  B_1  C_1

Feasible configuration with cost 1900:
A_2  B_1  C_2

Those are all feasible configurations.
```

This time `labeling/1` was also needed.

3.8 Exercises

Equations and conditionals 1

Write a program to determine the smallest integer solution for the following equations and conditionals:
A + E = G
C + D = 10
E + F = 8
A + C = 6
If F <> 6 then H > F
If E <> 1 then H > B
If G <> 8 then F > B
If B <> 5 then G <> 5
If E <> 3 then C <> 4
where <> is a disequation.

Solution:
A=4, B=1, C=2, D=8, E=3, F=5, G=7, H=6

Equations and conditionals 2

Write a program to determine the smallest integer solution for the following equations and conditionals:
B + G = D
B + C = A
C + E + G = F
If D < A then C = 2
If D >= A then E = 2.

Solution:
A=5, B=4, C=1, D=7, E=2, F=6, G=3

A visit

The Smith family and their three children want to pay a visit but they do not all have the time to do so. Following are few hints who will go and who will not: If Mr Smith comes, his wife will come too. At least one of their two sons Matt and John will come. Either Mrs Smith or Tim will come, but not both. Either Tim and John will come, or neither will come. If Matt comes, then John and his father will also come. Write a program to determine who went and who did not.

Horse derby

At last horse derby, 10 fine horses completed the grueling 3 mile course. Predictably, as per every year, the results mysteriously went missing. However, various marshals remembered the following snippets of information: Sylvester lost to Zebra Wings. Zebra Wings beat Sylwester, Frogman's Flippers and Tweetie Pie. Fizzy Pop lost to Minty Mouse, Sylvester and CD Player. Frogman's Flippers beat Windy Miller, CD Player and Sylwester. Top Trumps lost to CD Player, Kool Kat and Tweetie Pie. CD Player beat Top Trumps and Fizzy Pop. Tweetie pie lost to Zebra Wings and Sylvester. Kool Kat lost to Tweetie Pie and Frogman's Flippers. Frogman's Flippers beat Fizzy Pop, Minty Mouse and CD Player. CD Player lost to Frogman's Flippers, Kool Kat and Tweetie Pie. Top Trumps beat Fizzy Pop and Windy Miller. Minty Mouse lost to Windy Miller and Sylwester. Windy Miller lost to Tweetie Pie and CD Player. Write a program to work out who finished where.

Spring fete

At the recent spring fete, four keen gardeners were displaying their fine roses. In total there were four colors and each rose appeared in two colors. Mr Green had a yellow rose. Mr Yellow did not have a red one. Mr Red had a blue rose but not a green one, whilst Mr Blue did not have a yellow one. One person with a red rose also had a green one. One person with a yellow rose also had a blue one. One of the persons with a green rose had no red. Neither of the persons with a yellow rose had a green one. No person has two roses of the same color and no two persons had the same two color roses and their names provide no clues. Write a program which settles who had which color roses.

Cake theft

During a recent police investigation, Chief Inspector Stone was interviewing five local villains to try and identify who stole Mrs Archer's cake from the mid-summers fair. Below is a summary of their statements:
1)Arnold: it wasn't Edward it was Brian
2)Brian: it wasn't Charlie it wasn't Edward
3)Charlie: it was Edward it wasn't Arnold
4)Derek: it was Charlie it was Brian
5)Edward:it was Derek it wasn't Arnold
It was well known that each suspect told exactly one lie. Write a program to determine who stole the cake.

Horse race

A gambler bet on a horse race, but the bookie wouldn't tell him the results of the race. The bookie gave clues as to how the five horses finished – which may have included some ties – and wouldn't pay the gambler off unless the gambler could determine how the five horses finished based on the following clues:
1. Penuche Fudge finished before Near Miss and after Whispered Promises.
2. Whispered Promises tied with Penuche Fudge if and only if Happy Go Lucky did not tie with Skipper's Gal.
3. Penuche Fudge finished as many places after Skipper's Gal as Skipper's Gal finished after Whispered Promises if and only if Whispered Promises finished before Near Miss.
The gambler thought for a moment, then answered correctly. Write a program to determine how did the five horses finish the race.

Grades

Five friends in the sixth form took the same combination of A-level subjects. Each obtained a different grade in each subject taken, and no two students had the same grade in the same subject. Write a program to determine grades obtained for each subject by each student, provided that:
- Andrew outscored Bridget in Physics, and Neil in Math.
- Wendy was the only girl to get a "C" grade, but she managed no "A" grades
- The pupil with an "E" in Math gained a "B" in Chemistry, but was not awarded a "C" in Physics.
- Paul's Physics grade was a "D" and his highest grade was a "C".
- The "B" in Math did not go to the same student as the "E" in Physics.
- Bridget's best result was in Chemistry, but her Math grade was lower than Paul's.

The Autumn Leaves Trail

Thousands of tourists drive the Autumn Leaves Trail each fall to enjoy the multicolored vista of changing seasons[18]. The Trail starts in Summerset and goes north 10.0 miles to Fallbrook. Five scenic spots highlight the drive, each providing parking along the narrow road with a spectacular view of a different Trail attraction; each scenic spot is at a different milepost designating its distance from Summerset in tenths of a mile. Given the road map data below, write a program to determine at what milepost along the Autumn Leaves Trail each viewpoint is located:

1. No two consecutive scenic spots are the same distance apart; the longest drive between any two consecutive locations (including end points) on the Trail is 3.6 miles, while the shortest is .4 miles.
2. The distance along the Autumn Leaves Trail from Summerset to Cucumber Creek equals the distance going north from Old Man Mountain to the White Oak Inn.
3. The Amish Covered Bridge, which isn't the last scenic spot along the route, is 1.0 miles south of Fallbrook.
4. The Cucumber Creek spot is twice as far from the Sugar Maple Farm stop as it is from the Old Man Mountain viewpoint.
5. The White Oak Inn and Cucumber Creek photographic opportunities lie more than 5.0 miles apart.
6. The first scenic spot on the Trail is at milepost 1.8 north of Summerset.

[18]This exercise is from http://aaa.allstarpuzzles.netdna-cdn.com/logic/00082.html

Gardens

Five friends have their gardens next to one another, where they grow three kinds of crops: fruits (apple, pear, nut, cherry), vegetables (carrot, parsley, gourd, onion) and flowers (aster, rose, tulip, lily)[19].

1. They grow 12 different varieties.
2. Everybody grows exactly 4 different varieties.
3. Each variety is at least in one garden.
4. Only one variety is in 4 gardens.
5. Only in one garden are all 3 kinds of crops.
6. Only in one garden are all 4 varieties of one kind of crops.
7. Pears are only in the two border gardens.
8. Paul's garden is in the middle with no lily.
9. Aster grower doesn't grow vegetables.
10. Rose grower doesn't grow parsley.
11. Nuts grower has also gourd and parsley.
12. In the first garden are apples and cherries.
13. Only in two gardens are cherries.
14. Sam has onions and cherries.
15. Luke grows exactly two kinds of fruit.
16. Tulips are only in two gardens.
17. Apples are in a single garden.
18. Only in one garden next to the Zick's is parsley.
19. Sam's garden is not on the border.
20. Hank grows neither vegetables nor asters.
21. Paul has exactly three kinds of vegetable.

Write a program to determine who has which garden and what is grown where.

Open House [20]

Five students in the local "gifted and talented" program (three girls named Brittany, Natalie, and Olive, and two boys named Emile and Moises) organized their school's open house this year. Each of these students is majoring in a different area of study (geography, language, math, philosophy, or sculpture). Some of these students enlisted one or more relatives to assist with the production of the open house (mother, father, or grandmother), though no one enlisted more than one of any kind of relative. Write a program to discover each student's full name (surnames are Brad-

[19] This exercise is from http://www.mathsisfun.com/puzzles

[20] This exercise is from http://brownbuffalo.sourceforge.net/

shaw, Henderson, Smith, Wu, and Zacher), area of study, and the relative or relatives, if any, of each child who assisted, provided that:
1. Smith (who isn't Moises or Olive) isn't the philosophy major.
2. Two of Wu's relatives assisted with the program.
3. Zacher enlisted fewer of his or her relatives to assist than at least one other student.
4. The sculpture major is the only one who enlisted no relatives to assist.
5. Brittany and Henderson each enlisted one parent; neither of them enlisted a grandmother, and neither of them is the math major.
6. Moises and the geography major either both enlisted their fathers' assistance, or neither of them did.
7. No two students of the same gender enlisted their mothers.
8. Bradshaw's father didn't assist.
9. Olive enlisted one more relative than the math major.
10. Natalie is the language major, and her father didn't assist.

Swimming race

Five competitors - A, B, C, D and E - enter a swimming race that awards gold, silver and bronze medals to the first three to complete it. Each of the following compound statements about the race is false, although one of two clauses in each may be true:
- A didn't win the gold, B didn't win the silver.
- D didn't win the silver and E didn't win the bronze.
- C won a medal, D didn't.
- A won a medal, C didn't.
- D and E both won medals.

Write a program to determine who won each of the medals.

Queue for plane tickets [21]

Five people are standing in a queue for plane tickets in Germany; each one has a name, an age, a favorite Internet website, a place they live, a hairstyle and a destination from the sets:
Their names are: Bob, Keeley, Rachael, Eilish and Amy, their ages: 14, 21, 46, 52 and 81, their favorite Internet websites: "Rush Limbaugh Show", "Conservapedia", "Chronicles: A Magazine of American Culture", "Jeff Rense Program" and "American Thinker", they live at a town, a city, a village, a farm and a youth hostel, their hairstyle is: afro, long, straight, curly and bald, their destinations are: France, Australia, England, Africa

[21] This exercise is from http://www.mathsisfun.com/puzzles

and Italy. Besides:
1. The person in the middle reads "Jeff Rense Program"
2. Bob is the first in the queue
3. The person who reads the "Rush Limbaugh Show" is next to the person who lives in a youth hostel
4. The person going to Africa is behind Rachael.
5. The person who lives in a village is 52.
6. The person who is going to Australia has straight hair.
7. The person traveling to Africa reads "Jeff Rense Program".
8. The 14 year old is at the end of the queue.
9. Amy reads "Chronicles: A Magazine of American Culture".
10. The person heading to Italy has long hair.
11. Keeley lives in a village.
12. The 46 year old is bald.
13. The fourth in the queue is going to England.
14. The people with curly and straight hair are standing next to each other.
15. The person who reads "Conservapedia" stands next to the person with an afro.
16. A person next to Rachael has an afro.
17. The 21 year old lives in a youth hostel.
18. The person who reads "Conservapedia" has long hair.
19. The 81 year old lives on a farm.
20. The person who is traveling to France lives in a town.
Write a program to determine names, ages, favorite Internet website, living places, hairstyles and destinations of all concerned.

Science Fair

Art and Bert were describing the result of the International Science Fair Extravaganza. There were three contestants, Louis, Rene, and Johannes. Art reported that Louis won the fair, while Rene came in second. Bert, on the other hand, reported that Johannes won the fair, while Louis came in second. In fact, neither Art nor Bert had given a correct report of the results of the science fair. Each of them had given one correct statement and one false statement. Write a program to determine what was the actual placing of the three contestants.

Chapter 4

CLP with global constraints for feasible solutions

4.1 Introductory remarks

The concepts introduced in this chapter and Chapter 6 are basic for modeling and solving complicated combinatorial problems. In order to create efficient platforms for modeling and solving CSP and COP, a set of fundamental concepts and predicates corresponding to these concepts is needed. For continuous dynamic systems, dealt with e.g. in mechanics and control engineering, the concepts needed had been developed and had matured over ages, starting with pioneering work by Newton and Leibnitz on differential equations. For combinatorial problems the concepts started to be developed with the advent of *Prolog* and *CLP*, and culminated in defining and programming a series of basic, extremely useful high-level abstracts implemented as *global constraints*, see [Baldiceanu-94] and [Baldiceanu-10]. *Global constraints* are constraints defining complex relations over a number of input lists of variables. They are supported by libraries `ic_global`, `lib(ic_cumulative)`, `lib(ic_edge_finder)`, `lib(ic_edge_finder3)`. `lib(branch_and_bound)` They are contrasted with already discussed elementary constraints with at most one input list, supported by `ic` and `branch_and_bound` libraries. The use of global predicates enhances program readability, declarativity and effectiveness while substantially decreasing the time needed to model the problem. The ECL^iPS^e CPS user may find elementary as well as global constraints in the *Alphabetical Predicate Index*

menu *ECLIPSe Documentation* from Figure 5.

The global predicates `alldifferent/1` and `element/3` are made available by invoking the needed library[1] by declaring:

```
:- lib(ic_global).
```

or

```
:- use_module(library(ic_global)).
```

4.2 The 'alldifferent/1' built-in

The built-in:

```
alldifferent(?List)
```

is fulfilled if all elements of the `List=[X1,...,Xn]` are pairwise different. This is one of the most useful and often used global constraints. Theoretically speaking it corresponds to the following set of `disequations`:

```
X1#\=X2,
X1#\=X3,
.......

X1#\=Xn,
X2#\=X3,
.......

X2#\=Xn,
.......

X(n-1)#\=Xn,
```

However, the search and propagation methods for `alldifferent([X1,...,Xn])` are much more efficient than those for the above definition.

[1] The ic library provides as well support for 'alldifferent/1' and 'element/3', but in a less effective way.

The `alldifferent/1` constraint is practically always used with `indomain/1` constraints, enforcing all values considered to be from the variable domains. Consider example `4_1_all_diff.ecl`[2], were it is required that `X,Y,Z` be a three-element variation of the four-set [1,2,3,4]:

```
/*1*/   :- lib(ic).

/*2*/   top :-
/*3*/       [X,Y,Z]::1..4,
/*4*/       alldifferent([X,Y,Z]),
/*5*/       indomain(X),
/*6*/       indomain(Y),
/*7*/       indomain(Z),
/*8*/       writeln("X":X),
/*9*/       writeln("Y":Y),
/*10*/      writeln("Z":Z),
/*11*/      fail.

/*12*/  top:-
/*13*/      write("That's it."),nl.
```

The message is:

```
X =1 Y =2 Z =3
X =1 Y =2 Z =4
X =1 Y =3 Z =2
X =1 Y =3 Z =4
X =1 Y =4 Z =2
X =1 Y =4 Z =3
X =2 Y =1 Z =3
X =2 Y =1 Z =4
X =2 Y =3 Z =1
X =2 Y =3 Z =4
X =2 Y =4 Z =1
X =2 Y =4 Z =3
X =3 Y =1 Z =2
X =3 Y =1 Z =4
X =3 Y =2 Z =1
X =3 Y =2 Z =4
X =3 Y =4 Z =1
```

[2]This is an FS-type problem.

```
X =3 Y =4 Z =2
X =4 Y =1 Z =2
X =4 Y =1 Z =3
X =4 Y =2 Z =1
X =4 Y =2 Z =3
X =4 Y =3 Z =1
X =4 Y =3 Z =2
That's it.
```

Thanks to `fail` in line `/*11*/`, all solutions for `alldifferent([X,Y,Z])` are determined. Obviously, if there are no solutions like for `4_2_all_diff.ecl`:

```
/*1*/   :- lib(ic).

/*2*/   top :-
/*3*/       [V,W,X,Y,Z]::1..4,
/*4*/       alldifferent([V,W,X,Y,Z]),
/*5*/       indomain(V),
/*6*/       indomain(W),
/*7*/       indomain(X),
/*8*/       indomain(Y),
/*9*/       indomain(Z),
/*10*/      writeln('V':V),
/*11*/      writeln('W':W),
/*12*/      writeln('X':X),
/*13*/      writeln('Y':Y),
/*14*/      writeln('Z':Z).,
```

the message is:

```
No.
```

4.3 The 'element/3' built-in

The built-in:

```
element(?Index, ++List, ?Value)
```

constraints `Value` to be at the position `Index` in the grounded integer list `List`.

This is also a very useful constraint, because it implements a relation between two domain variables, namely between a *subscripted* (indexed) variable from the `List` and the *subscript* (index) value for the variable from the `List`. I.e. for:

$$element(N, [c_1, c_2..., c_n], Y)$$

the constraint requires that:

$$Y = c_N.$$

Its importance is enhanced by the fact that either one or both `Index` and `Value` may be variables. This is illustrated by program `4_3_element.ecl`[3]:

```
/*1*/   :- lib(ic).

/*2*/   top:-
/*3*/       element(Index,[20,10,41,32],41),
/*4*/       writeln("Index ":Index),
/*5*/       element(2,[](20,10,41,32),Indexed_Value),
/*6*/       writeln("Indexed_Value ":Indexed_Value),
/*7*/       element(I,[20,10,41,32],I_V),
/*8*/       writeln("I ":I),
/*9*/       writeln("I_V":I_V).
```

The message is:

```
Index  : 3
Indexed_Value  : 10
I  : _955{1 .. 4}
I_V : _1087{[10, 20, 32, 41]}
```

The examples presented below are classified into the following two problem classes:

1. Feasible assignment problems, aiming at joining elements of some sets so as to fulfill constraints of belongness.
2. Feasible sequencing problems, aiming at ordering elements of some set so as to fulfill constraints of precedence.

The adjective *feasible* is used to distinguish the problems from *optimum* once discussed in Chapter 5.

[3]This is an FS-type problem.

4.4 Feasible assignment problems

Their essence is to find - for any element of some set - elements from some other sets so as to fulfill some constraints of belongness. *Tie constraints* define constraints among elements of various sets.

4.4.1 Send More Money

CLP is excellent for solving cryptarithmetic puzzles in the form of equations among unknown numbers whose digits are represented by letters. The following puzzle[4] belongs to the folklore of CLP:

There is a mathematical equation:

```
  S E N D
  M O R E
----------
M O N E Y
```

among unknown digits 0,1,2,3,4,5,6,7,8 and 9 represented by letters S,E,N,D,M,O,R,Y. The goal is to identify the value of each letter. The puzzle is solved by program `4_4_smm.ecl`[5]:

```
/*1*/  :- lib(ic).
/*2*/  top:-
/*3*/     sendmore(_).

/*4*/  sendmore(L) :-
/*5*/     L = [S,E,N,D,M,O,R,Y],
/*6*/     L :: [0..9],
/*7*/     alldifferent(L),
/*8*/     S #\= 0,
/*9*/     M #\= 0,
/*10*/    1000*S + 100*E + 10*N + D
             + 1000*M + 100*O + 10*R + E
             #= 10000*M + 1000*O + 100*N + 10*E + Y,
/*11*/    labeling(L),
```

[4] It is attributed to Henry Dudeney who published it in the July 1924 issue of *Strand Magazine*

[5] This is an FS-type problem.

```
/*12*/    write("  "),write(S),write(E),write(N),write(D),
/*13*/    write("    S E N D"),nl,
/*14*/    write(" "),write(M),write(0),write(R),write(E),
/*15*/    write("    M O R E"),nl,
/*16*/    write("  ----------------"),nl,
/*17*/    write(" "),write(M),write(0),write(N),write(E),write(Y),
/*18*/    write("  M O N E Y"),nl,nl.
```

The message is:

```
    9567      S E N D
    1085      M O R E
   ------  ----------
   10652    M O N E Y
```

4.4.2 FIFTEEN

A more advanced cryptarithmetic puzzle is known as `FIFTEEN`:
In the addition sum below digits have been replaced by letters and `@` symbols:

```
          @
    @@@FIVE
    @@FIVE@
  + @FIVE@@
-----------
    FIFTEEN
```

Different letters stand for different digits, the same letter stands for the same digit, an `@` symbol stands for any digit, which may be different in different `@` positions, and leading digits cannot be zero. If `FIVE` is divisible by `5` and `ELEVEN` is divisible by `11`, the program `4_5_FIFTEEN.ecl` determines what number is `FIFTEEN` and what digits are represented by all the symbols. However, the symbols `@` in different positions have to be named differently, e.g. like this

```
                    A1
   A4 A3 A2  F  I   V   E
   A7 A6  F  I  V   E  A5
+ A10  F  I  V  E  A9  A8
  ------------------------
    F  I  F  T  E   E   N
```

The program `4_5_FIFTEEN.ecl` is as follows:

```
/*1*/  :- lib(ic).
/*2*/  top:-
/*3*/     assert(counter(0)),
/*4*/     P = [F,I,V,E,T,N,L,A1,A2,A3,A4,A5,A6,A7,A8,A9,A10],
/*5*/     P :: [0..9],
/*6*/     alldifferent([F,I,V,T,N,L,E]),

/*7*/     F #\= 0,
/*8*/     E#\=0,
/*9*/     E #= 5,
/*10*/    A1#\=0,
/*11*/    A4#\=0,
/*12*/    A7#\=0,
/*13*/    A10#\=0,
/*14*/    my_modulo_5(F,I,V,E),
/*15*/    my_modulo_11(E,L,E,V,E,N),

/*16*/    A1 +
/*17*/    E + 10*V + 100*I + 1000*F + 10000*A2 + 100000*A3 + 1000000*A4 +
/*18*/    A5 + 10*E + 100*V + 1000*I + 10000*F + 100000*A6 + 1000000*A7 +
/*19*/    A8 + 10*A9 + 100*E + 1000*V + 10000*I + 100000*F + 1000000*A10 #=
/*19*/    1000000*F + 100000*I+ 10000*F + 1000*T + 100*E + 10*E + N,

/*20*/    labeling(P),
/*21*/    count,
/*22*/    counter(Solution_number),
/*23*/    write("Solution "), write(Solution_number),write(":"),nl,

/*24*/    write("  "),write("  "),write("  "),write(A1),
/*25*/    write("  "),write("  "),write("  "),write("  "),write("  "),
                  write("  "),write("  "),write("  "),write("  A1"),nl,
/*26*/    write(A4),write(A3),write(A2),write(F),write(I),write(V),write(  E),
/*27*/    write("  A4 A3 A2 F I  V  E"),nl,
/*28*/    write(A7),write(A6),write(F),write(I),write(V),write(E),write(A5),
/*29*/    write("  A7 A6  F I V  E A5"),nl,
/*30*/    write(A10),write(F),write(I),write(V),write(E),write(A9),write(A8),
/*31*/    write(" A10  F  I V E A9 A8"),nl,
/*32*/    write("------- --------------------"),nl,
/*33*/    write(F),write(I),write(F),write(T),write(E),write(E),write(N),
/*34*/    write("  F  I  F  T  E  E  N"),nl,nl,
/*35*/    fail.

/*36*/ top:-nl,nl,
/*37*/    write("That's everything!").
```

```
/*38*/ my_modulo_5(F,I,V,E):-
/*39*/    integers(X),
/*40*/    E+10*V+100*I+1000*F #= X*5.

/*41*/ my_modulo_11(E,L,E,V,E,N):-
/*42*/    integers(X),
/*43*/    N+10*E+100*V+1000*E+10000*L+100000*E #= X*11.

/*44*/ count:-
/*45*/    retract(counter(Old)),
/*46*/    New is Old + 1,
/*47*/    assert(counter(New)).
```

The problem has 18 solution, all with the same `FIFTEEN`. The first and the last one are as follows:

```
    Solution 1:
      8                   A1
1094085  A4 A3 A2  F  I  V  E
1540859  A7 A6  F  I  V  E A5
1408599 A10  F  I  V  E A9 A8
------- --------------------
4043551   F  I  F  T  E  E  N

.......................

    Solution 18:
      9                   A1
1594085  A4 A3 A2  F  I  V  E
1040859  A7 A6  F  I  V  E A5
1408598 A10  F  I  V  E A9 A8
------- --------------------
4043551   F  I  F  T  E  E  N
```

4.4.3 Who with whom again

Both global constraints discussed so far enable to simplify the program solving the *who with whom* puzzle from Section 3.6.2 while at the same time enabling the generation of a readable message. The modified program `4_6_who_with_whom_again.ecl`[6] is as follows:

[6]This is an FS-type problem.

```
/*1*/   :- lib(ic).
/*2*/   top :-
/*3*/         [Andy, Ben, Carl, Dusty]::[1..4],
/*4*/         [Olive, Eva, Paula, Sabina]::[1..4],
  % concert=1, cinema=2, theater=3, exhibition=4
  % It means:   if e. g.   Ben=Olive=4, then
  % Ben and Olive went to an exhibition

  % Andy enjoyed a  concert:
/*5*/         Andy#=1,

  % Ben accompanied Olive:
/*6*/         Ben#=Olive,

  % Carl has not seen Eva:
/*7*/         Carl#\=Eva,

  % Paula went to a cinema:
/*8*/         Paula#=2,

  % Eva went to a theater:
/*9*/         Eva#=3,

/*10*/        alldifferent([Andy,Ben,Carl,Dusty]),
/*11*/        alldifferent([Olive,Eva,Paula,Sabina]),

/*12*/        write(Andy),write(" "),write(Ben),write(" "),
/*13*/        write(Carl),write(" "),write(Dusty),nl,
/*14*/        write(Olive),write(" "),write(Eva),write(" "),
/*15*/        write(Paula),write(" "),write(Sabina),nl,nl,
  % End of solution part.

  % Beginning of message part:
  % Determining the numbers for boys on the boy list:
/*16*/         element(Number_of_First_Boy,[Andy,Ben,Carl,Dusty],1),
/*17*/         element(Number_of_Second_Boy,[Andy,Ben,Carl,Dusty],2),
/*18*/         element(Number_of_Third_Boy,[Andy,Ben,Carl,Dusty],3),
/*19*/         element(Number_of_Fourth_Boy,[Andy,Ben,Carl,Dusty],4),

  % Determining the numbers for girls on the girl list:
/*20*/         element(Number_of_First_Girl,[Olive, Eva,Paula,Sabina],1),
/*21*/         element(Number_of_Second_Girl,[Olive, Eva,Paula,Sabina],2),
/*22*/         element(Number_of_Third_Girl,[Olive, Eva,Paula,Sabina],3),
/*23*/         element(Number_of_Fourth_Girl,[Olive, Eva,Paula,Sabina],4),

  % Translating numbers for boys to names:
/*24*/         name_of_boy(Number_of_First_Boy,Name_of_1_boy),
/*25*/         name_of_boy(Number_of_Second_Boy,Name_of_2_boy),
/*26*/         name_of_boy(Number_of_Third_Boy,Name_of_3_boy),
```

```
/*27*/         name_of_boy(Number_of_Fourth_Boy,Name_of_4_boy),

  % Translating numbers for girls to names:
/*28*/         name_of_girl(Number_of_First_Girl,Name_of_1_girl),
/*29*/         name_of_girl(Number_of_Second_Girl,Name_of_2_girl),
/*30*/         name_of_girl(Number_of_Third_Girl,Name_of_3_girl),
/*31*/         name_of_girl(Number_of_Fourth_Girl,Name_of_4_girl),

/*32*/         write(Name_of_1_boy),write(" and "),
/*33*/         write(Name_of_1_girl), write(" enjoyed a concert."),nl,
/*34*/         write(Name_of_2_boy),write(" and "),
/*35*/         write(Name_of_2_girl), write(" went to a cinema."),nl,
/*36*/         write(Name_of_3_boy),write(" and "),
/*37*/         write(Name_of_3_girl), write(" went to a theater."),nl,
/*38*/         write(Name_of_4_boy),write(" and "),
/*39*/         write(Name_of_4_girl), write(" went to an exhibition."),nl.

/*40*/ name_of_boy(1,"Andy").
/*41*/ name_of_boy(2,"Ben").
/*42*/ name_of_boy(3,"Carl").
/*43*/ name_of_boy(4,"Dusty").
/*44*/ name_of_girl(1,"Olive").
/*45*/ name_of_girl(2,"Eva").
/*46*/ name_of_girl(3,"Paula").
/*47*/ name_of_girl(4,"Sabina").
```

The program generates the message:

```
1 4 2 3
4 3 2 1

Andy and Sabina enjoyed a concert.
Carl and Paula went to a cinema.
Dusty and Eva went to a theater.
Ben and Olive went to an exhibition.
```

4.4.4 Golfers again

Both global constraints discussed so far may also be used to simplify the program solving the *golfers* puzzle from Section 3.7.2 while at the same time enabling the

generation of a readable message. The new program `4_7_golfers_again.ecl`[7] is as follows:

```
/*1*/   :- lib(ic).
/*2*/   top :-
/*3*/     [Fred,Joe,Tom,Bob]::1..4,
        % Tom - variable denoting Tom's position in line.
/*4*/     alldifferent([Fred,Joe,Tom,Bob]),

/*5*/     [Red,Orange,Blue,Plaid]::1..4,
        % Blue - variable denoting the position of blue pants in line.

        % (1) Someone is wearing red pants:
/*5*/     alldifferent([Red,Orange,Blue,Plaid]),

        % (2) The golfer to Fred's immediate right is wearing blue pants:
/*6*/     Blue#=Fred+1,

        % (3) Joe is second in line:
/*7*/     Joe#=2,

        % (4) Bob is wearing plaid pants:
/*8*/     Bob#=Plaid,

        % (5) Tom isn't in position one or four,
        % and he isn't  wearing the hideous orange pants:
/*9*/     Tom#\=1,
/*10*/    Tom#\=4,
/*11*/    Tom#\=Orange,

/*12*/    labeling([Fred,Joe,Tom,Bob,Orange,Blue,Red,Plaid]),

/*13*/    write("Fred,Joe,Tom,Bob"),nl,
/*14*/    write([Fred,Joe,Tom,Bob]),nl,
/*15*/    write("Red,Orange,Blue,Plaid"),nl,
/*16*/    write([Red,Orange,Blue,Plaid]),nl,
        % End of problem solving part

        % Beginning of message generating part:
        % The point at issue: finding pairs (Name_of_golfer, color_of_pants)
        % Number of golfer at position n, n=1,..4:
/*17*/    element(Golfer_with_number_1,[Fred,Joe,Tom,Bob],1),
/*18*/    element(Golfer_with_number_2,[Fred,Joe,Tom,Bob],2),
/*19*/    element(Golfer_with_number_3,[Fred,Joe,Tom,Bob],3),
/*20*/    element(Golfer_with_number_4,[Fred,Joe,Tom,Bob],4),
```

[7] This is an FS-type problem.

```
       % Translating golfer numbers into golfer names
/*21*/    name_of_golfer(Golfer_with_number_1,Name_of_golfer_1),
/*22*/    name_of_golfer(Golfer_with_number_2,Name_of_golfer_2),
/*23*/    name_of_golfer(Golfer_with_number_3,Name_of_golfer_3),
/*24*/    name_of_golfer(Golfer_with_number_4,Name_of_golfer_4),

       % Number of color at position n, n=1,..4:
/*25*/    element(color_with_number_1,[Red,Orange,Blue,Plaid],1),
/*26*/    element(color_with_number_2,[Red,Orange,Blue,Plaid],2),
/*27*/    element(color_with_number_3,[Red,Orange,Blue,Plaid],3),
/*28*/    element(color_with_number_4,[Red,Orange,Blue,Plaid],4),

       % Translating color numbers into color names:
/*29*/    color(color_with_number_1,color_1),
/*30*/    color(color_with_number_2,color_2),
/*31*/    color(color_with_number_3,color_3),
/*32*/    color(color_with_number_4,color_4),

       % Joining  elements of pairs (Name_of_golfer, color_of_pants):
/*33*/write(Name_of_golfer_1),write(" wears "),write(color_1),write(" pants."),nl,
/*34*/write(Name_of_golfer_2),write(" wears "),write(color_2),write(" pants."),nl,
/*35*/write(Name_of_golfer_3),write(" wears "),write(color_3),write(" pants."),nl,
/*36*/write(Name_of_golfer_4),write(" wears "),write(color_4),write(" pants."),nl.

/*37*/    name_of_golfer(1,"Fred").
/*38*/    name_of_golfer(2,"Joe").
/*39*/    name_of_golfer(3,"Tom").
/*40*/    name_of_golfer(4,"Bob").

/*41*/    color(1,"red").
/*42*/    color(2,"orange").
/*43*/    color(3,"blue").
/*44*/    color(4,"plaid").
```

he following message is generated:

```
Fred,Joe,Tom,Bob
[1, 2, 3, 4]
Red,Orange,Blue,Plaid
[3, 1, 2, 4]

Fred wears orange pants.
Joe wears blue pants.
Tom wears red pants.
Bob wears plaid pants.
```

From this program and from the previous one it can be seen that ECL^iPS^e CLP is decidedly more powerful for problem solving than for generating messages displaying solutions. For both the `4_6_who_with_whom_again.ecl` and the `4_7_golfers_again.ecl` program, the message generating part was more voluminous and verbose than the problem solving part.

4.4.5 Three cubes again

The *three cubes* program from Section 2.4.2 could also be simplified with the help of the two global constraints discussed so far. The constraints are also useful for generating a readable message. The new program `4_8_three_cubes_again.ecl`[8] is as follows:

```
/*1*/   :-lib(ic).
/*2*/   top:-
/*3*/         color=[Black,Grey, White],
/*4*/         Size=[Small, Large, Medium],
/*5*/         [Black,Grey, White] :: [1..3],
/*6*/         [Small, Large, Medium] :: [1..3],
/*7*/         alldifferent([Black,Grey, White]),
/*8*/         alldifferent([Small, Large, Medium]),
/*9*/         Cubes = [ cube(1,_,_), cube(2,_,_), cube(3,_,_)],
      % cube(Cube_number,Cube_size,Cube_color)
/*10*/        Constraints = [ cube(Black,_,black),
                   cube(Grey,Size_of_grey_cube,grey),
                      cube(White,_,white),

      %(2) The small cube has number 2:
                      cube(2,Small,_),
      % Nothing is known about the medium cube:
                      cube(Medium,medium,color_of_medium_cube),
      % Nothing is known about the large cube:
                      cube(Large,large,color_of_large_cube),
                      cube(3,Size_3,_) ],

      %(1) The large cube is brighter than the medium cube:
/*11*/        brighter(color_of_large_cube,color_of_medium_cube),

      %(3) The number of the black cube is greater than the one on the white cube
/*12*/        Black#>White,

      %(4) The size of cube with number 3
      % is smaller than the size of the grey cube:
```

[8] This is an FS-type problem.

```
/*13*/        smaller_size(Size_3,Size_of_grey_cube),

  % The elements  of  "Constraints" list must be grounded
  %e xactly like corresponding  elements from "Cubes" list:
/*14*/        grounding(Constraints, Cubes),
/*15*/        writeln(Color), writeln(Size),
  %  End of solution part.

 /*The solution for this part is as follows:
      [3, 1, 2]
      [2, 1, 3]
for variables:
      [Black, Grey, White]
      [Small, Large, Medium]*/

  % Beginning of message generating part. The problem: find triples
  % (Number,Color,Size) with same value of elements.

  % Number of color on position n, n=1,2,3:
/*16*/        element(Number_of_color_1,[Black,Grey, White],1),
/*17*/        element(Number_of_color_2,[Black,Grey, White],2),
/*18*/        element(Number_of_color_3,[Black,Grey, White],3),

  % Translating number of color into name of color:
/*19*/        color(Number_of_color_1,Name_of_color_1),
/*20*/        color(Number_of_color_2,Name_of_color_2),
/*21*/        color(Number_of_color_3,Name_of_color_3),

  % Number of size on position n, n=1,2,3:
/*22*/        element(Number_of_size_1,[Small,Large,Medium],1),
/*23*/        element(Number_of_size_2,[Small,Large,Medium],2),
/*24*/        element(Number_of_size_3,[Small,Large,Medium],3),

  % Translating number of size into name of size:
/*25*/        size(Number_of_size_1,Name_of_size_1),
/*26*/        size(Number_of_size_2,Name_of_size_2),
/*27*/        size(Number_of_size_3,Name_of_size_3),

  % Joining  elements of triples (Name_of_color, Cube_number, Name_of_size):
/*28*/        write("The "),write(Name_of_color_1),write(" cube with number 1"),
              write(" is "),write(Name_of_size_1),nl,
/*29*/        write("The "),write(Name_of_color_2),write(" cube with number 2"),
              write(" is "),write(Name_of_size_2),nl,
/*30*/        write("The "),write(Name_of_color_3),write(" cube with number 3"),
              write(" is "),write(Name_of_size_3),nl.
```

```
/*31*/   color(1,black).
/*32*/   color(2,grey).
/*33*/   color(3,white).

/*34*/   size(1,small).
/*36*/   size(2,large).
/*36*/   size(3,medium).

/*37*/   smaller_size(small,large).
/*38*/   smaller_size(small,medium).
/*39*/   smaller_size(medium,large).

/*40*/   brighter(white,grey).
/*41*/   brighter(white,black).
/*42*/   brighter(grey,black).

/*43*/   grounding([],_).
/*44*/   grounding([H|T],List):-
/*45*/        member(H,List),
/*46*/        grounding(T,List).
```

The complete solution is:

```
[3, 1, 2]
[2, 1, 3]
The grey cube with number 1 is large
The white cube with number 2 is small
The black cube with number 3 is medium
```

4.4.6 Queens again

The *alldifferent/1* build-in may be used for a rather original solution to the 8 queens problem. The corresponding program `4_9_queens_again.ecl`[9] is as follows:

```
/*1*/   :-lib(ic).

/*2*/   top:-
/*3*/     queens(_).
```

[9]This is an FS-type problem.

```
/*4*/   queens([X1,X2,X3,X4,X5,X6,X7,X8]):-
/*5*/     [X1,X2,X3,X4,X5,X6,X7,X8]::1..8,
/*6*/     [X11,X22,X33,X44,X55,X66,X77,X88]::1..16,
/*7*/     [X18,X27,X36,X45,X54,X63,X72,X81]::1..16,
/*8*/     alldifferent([X1,X2,X3,X4,X5,X6,X7,X8]),
/*9*/     X11 #= X1+1,
/*10*/    X22 #= X2+2,
/*11*/    X33 #= X3+3,
/*12*/    X44 #= X4+4,
/*13*/    X55 #= X5+5,
/*14*/    X66 #= X6+6,
/*15*/    X77 #= X7+7,
/*16*/    X88 #= X8+8,
/*17*/    alldifferent([X11,X22,X33,X44,X55,X66,X77,X88]),

/*18*/    X18 #= X1+8,
/*19*/    X27 #= X2+7,
/*20*/    X36 #= X3+6,
/*21*/    X45 #= X4+5,
/*22*/    X54 #= X5+4,
/*23*/    X63 #= X6+3,
/*24*/    X72 #= X7+2,
/*25*/    X81 #= X8+1,
/*26*/    alldifferent([X18,X27,X36,X45,X54,X63,X72,X81]),

/*27*/    labeling([X1,X2,X3,X4,X5,X6,X7,X8]),
/*28*/    write([X1,X2,X3,X4,X5,X6,X7,X8]),nl,fail.

/*29*/    queens(_):-
/*30*/    write("That's it!").
```

The solution generated is the same as for program `3_11_queens.ecl`, see Section 3.7.5.

4.4.7 Seven machines - seven tasks

Allocating resources between tasks is a typical combinatorial application, successfully solved by CLP languages. This is illustrated by the following example:

Any one of seven machines may perform any of seven different tasks, but at different cost, as shown by Table 4.1. The tasks should be allocated between machines so as to keep the overall cost below the threshold equal to `185`.
The solution is given by program `4_10_7_machines_7_tasks.ecl`[10]:

[10]This is an FS-type problem.

Machine	Task						
	1	2	3	4	5	6	7
1	15	23	43	27	76	43	91
2	45	76	32	39	72	37	48
3	56	45	87	75	34	76	29
4	13	45	34	51	52	21	76
5	45	49	18	48	58	98	23
6	23	25	29	39	52	41	12
7	76	98	86	41	34	76	77

Table 4.1: Task costs for machines

```
/*1*/   :- lib(ic).
/*2*/   top :-
/*3*/     [O1,O2,O3,O4,O5,O6,O7]::1..7,
/*4*/     [K1,K2,K3,K4,K5,K6,K7]::0..100,
/*5*/     alldifferent([O1,O2,O3,O4,O5,O6,O7]),
/*6*/     element(O1,[15,23,43,27,76,43,91],K1),
/*7*/     element(O2,[45,76,32,39,72,37,48],K2),
/*8*/     element(O3,[56,45,87,75,34,76,29],K3),
/*9*/     element(O4,[13,45,34,51,52,21,76],K4),
/*10*/    element(O5,[45,49,18,48,58,98,23],K5),
/*11*/    element(O6,[23,25,29,39,52,41,12],K6),
/*12*/    element(O7,[76,98,86,41,34,76,77],K7),

/*13*/    K1+K2+K3+K4+K5+K6+K7 #< 185,
/*14*/    labeling([K1,K2,K3,K4,K5,K6,K7]),
/*15*/    display_results([O1,K1,O2,K2,O3,K3,O4,K4,
                O5,K5,O6,K6,O7,K7],1),
/*16*/    K is K1+K2+K3+K4+K5+K6+K7,
/*17*/    write("Cost = "),write(K),
/*18*/    L=[O1,K1,O2,K2,O3,K3,O4,K4,O5,K5,O6,K6,O7,K7],
/*19*/    write(L).

/*20*/  display_results([],_):-
/*21*/    !.
/*22*/  display_results([A,B|R],N):-
/*23*/    write("Machine "),write(N),write(" is performing task "),write(A),
                    write(" costing "),write(B),write("."),nl,
/*24*/    M is N+1,
/*25*/    display_results(R,M).
```

Now we asked to be shown all solutions using the option `more` form ECL^iPS^e, Main Menu. This results in:

```
Machine 1 is performing task 1 costing 15.
Machine 2 is performing task 4 costing 39.
Machine 3 is performing task 7 costing 29.
Machine 4 is performing task 6 costing 21.
Machine 5 is performing task 3 costing 18.
Machine 6 is performing task 2 costing 25.
Machine 7 is performing task 5 costing 34.
Overall cost =  81
[O1,K1,O2,K2,O3,K3,O4,K4,O5,K5,O6,K6,O7,K7]  =
[1, 15, 4, 39, 7, 29, 6, 21, 3, 18, 2, 25, 5, 34]

Machine 1 is performing task 1 costing 15.
Machine 2 is performing task 4 costing 39.
Machine 3 is performing task 2 costing 45.
Machine 4 is performing task 6 costing 21.
Machine 5 is performing task 3 costing 18.
Machine 6 is performing task 7 costing 12.
Machine 7 is performing task 5 costing 34.
Overall cost = 184
[O1,K1,O2,K2,O3,K3,O4,K4,O5,K5,O6,K6,O7,K7]  =
[1, 15, 4, 39, 2, 45, 6, 21, 3, 18, 7, 12, 5, 34]

Machine 1 is performing task 2 costing 23.
Machine 2 is performing task 6 costing 37.
Machine 3 is performing task 5 costing 34.
Machine 4 is performing task 1 costing 13.
Machine 5 is performing task 3 costing 18.
Machine 6 is performing task 7 costing 12.
Machine 7 is performing task 4 costing 41.
Overall cost =  178
[O1,K1,O2,K2,O3,K3,O4,K4,O5,K5,O6,K6,O7,K7]  =
[2, 23, 6, 37, 5, 34, 1, 13, 3, 18, 7, 12, 4, 41]

Machine 1 is performing task 4 costing 27.
Machine 2 is performing task 6 costing 37.
Machine 3 is performing task 7 costing 29.
Machine 4 is performing task 1 costing 13.
Machine 5 is performing task 3 costing 18.
Machine 6 is performing task 2 costing 25.
Machine 7 is performing task 5 costing 34.
Overall cost = 183
```

```
[O1,K1,O2,K2,O3,K3,O4,K4,O5,K5,O6,K6,O7,K7]  =
[4, 27, 6, 37, 7, 29, 1, 13, 3, 18, 2, 25, 5, 34]
```

4.4.8 Three machines - three from five tasks

A more complicated allocation problem is the following:

Any one of three machines may be used to perform any of five tasks, but at different cost, as shown in Table 4.2.

Machine	Task				
	1	2	3	4	5
1	1	11	5	7	13
2	4	6	2	8	10
3	6	3	9	12	15

Table 4.2: Task costs for machines

This time three selected tasks should be allocated to three machines available so as to keep the overall cost below the threshold of `10`. The solution is given by program `4_11_3_machines_3_from_5_tasks.ecl`[11]:

```
/*1*/   :- lib(ic).
/*2*/   top :-
/*3*/     [O1,O2,O3] :: 1..5,
  % The list of task numbers contains three of five task numbers.
  % E.g. O2 is the task number for the task performed by machine 2.

/*4*/     [K1,K2,K3] :: 1..10,
  % The list of task costs contains three of five task costs.
  % E.g. K2 is the task cost for the O2 task.

/*5*/     alldifferent([O1,O2,O3]),

  % [1,11,5,7,13] - list of task costs for machine 1:
/*6*/     element(O1,[1,11,5,7,13],K1),

  % [4,6,2,8,10] - list of task costs for machine 2:
/*7*/     element(O2,[4,6,2,8,10],K2),
```

[11] This is an FS-type problem.

```
  % [6,3,9,12,15] - list of task costs for machine 23:
/*8*/     element(O3,[6,3,9,12,15],K3),

/*9*/     K1+K2+K3 #=< 10,
/*10*/    labeling([K1,K2,K3]),
/*11*/    display_results([O1,K1,O2,K2,O3,K3],1).

/*12*/  display_results([A,B|R],N):-
/*13*/    write("Machine "),write(N),write(" is performing task "),write(A),
                        write(" costing "),write(B),write("."),nl,
/*14*/    M is N+1,
/*15*/    display_results(R,M).
/*16*/  display_results([],_).
```

The message is:

```
Machine 1 is performing task 1 costing 1.
Machine 2 is performing task 3 costing 2.
Machine 3 is performing task 2 costing 3.
```

4.4.9 Three machines - five tasks

An additional complication is introduced by the following example:

Consider once more the task cost table 4.2, but this time assume that all five tasks have to be completed by the only three machines available. To do this we transform this problem to the already solved problem were the number of machines was equal to the number of tasks. So each machine gets a double and a fictitious task `6` with `0` cost is introduced, as shown in 4.3:

Machine	Task					
	1	2	3	4	5	6
M1	1	11	5	7	13	0
M12	1	11	5	7	13	0
M2	4	6	2	8	10	0
M22	4	6	2	8	10	0
M3	6	3	9	12	15	0
M32	6	3	9	12	15	0

Table 4.3: Task costs for machines and their doubles

The solution is given by program `4_12_3_machines_5_tasks.ecl`[12]:

```
/*1*/   :- lib(ic).
/*2*/   top :-
/*3*/     [M1,M2,M3,M12,M22,M32] :: 1..6,
 % E.g. M12 = 4 means that machine 1 is performing task 4.
/*4*/     [K1,K2,K3,K12,K22,K32] :: 0..16,
 % E.g. K12 = 7 means that machine 1 is performing task 4 with cost 7.
/*5*/     alldifferent([M1,M2,M3,M12,M22,M32]),

/*6*/     element(M1,[1,11,5,7,13,0],K1),
 % E.g. M12 = 4 means that machine 1 is performing task 4 with cost 7:
/*7*/     element(M12,[1,11,5,7,13,0],K12),
/*8*/     element(M2,[4,6,2,8,10,0],K2),
/*9*/     element(M22,[4,6,2,8,10,0],K22),
/*10*/    element(M3,[6,3,9,12,15,0],K3),
/*11*/    element(M32,[6,3,9,12,15,0],K32),

/*12*/    K1+K2+K3+K12+K22+K32 #=< 25,
/*13*/    labeling([K1,K2,K3,K12,K22,K32]),
/*14*/    Cost is K1+K2+K3+K12+K22+K32,
/*15*/    write("Overall cost = "),write(Cost),nl,
/*16*/    write("[M1,K1,M2,K2,M3,K3,M12,K12,M22,K22,M32,K32]"),nl,
/*17*/    L=[M1,K1,M2,K2,M3,K3,M12,K12,M22,K22,M32,K32],
/*18*/    write(L),nl,
/*19*/    display_results(L,1),
/*20*/    !,nl.

/*21*/  display_results([],_).
/*22*/  display_results([A,B|R],N):-
/*23*/    not(B = 0),
/*24*/    N =< 3,
/*25*/    write("Machine "),write(N),write(" is performing task "),write(A),
                write(" costing "),write(B),write("."),nl,
/*26*/    M is N+1,
/*27*/    display_results(R,M).
/*28*/  display_results([A,B|R],N):-
/*29*/    not(B = 0),
/*30*/    N > 3,
/*31*/    M is N - 3,
/*32*/    write("Machine "),write(M),write(" is performing task "),write(A),
                write(" costing "),write(B),write("."),nl,
/*33*/    Q is N+1,
/*34*/    display_results(R,Q).

/*35*/  display_results([_,_|R],N):-
```

[12] This is an FS-type problem.

```
/*36*/    M is N+1,
/*37*/    display_results(R,M).
```

Because of the constraints in lines /*3*/,...,/*11*/, the fictitious task 6 will never be performed.

The message generated is:

```
Overall cost = 23
[M1,K1,M2,K2,M3,K3,M12,K12,M22,K22,M32,K32]
[1, 1, 3, 2, 6, 0, 4, 7, 5, 10, 2, 3]
Machine 1 is performing task 1 costing 1.
Machine 2 is performing task 3 costing 2.
Machine 1 is performing task 4 costing 7.
Machine 2 is performing task 5 costing 10.
Machine 3 is performing task 2 costing 3.
```

4.5 Feasible timetabling

4.5.1 Five rooms

Another puzzle badly in need of the alldifferent/1 built-in is the five rooms puzzle, which may be considered as a rather simple and naive time-tabling problem. This puzzle is a modification of the well-known *Zebra* puzzle[13], which also forms part of the Prolog and CLP folklore:

To five rooms should be attributed five colors, five days, five subjects, five subject marks, and five teaching technologies. The:

```
  room colors (red,green,blue,white,yellow),
  days of the week (Monday,Tuesday,Wednesday,Thursday,Friday),
  subjects (physics,mathematics,informatics,economics,English),
  subject marks (dull,difficult,interesting,most_interesting,
                 nothing_special),
  teaching technologies (computer,internet,video,chalk_blackboard,
                  projector)
```

are subject to following constraints:

```
  (1)  The physics class is in the red room.
```

[13]It is attributed to Lewis Carrol (1832-1898), the author of *Alice's Adventures in Wonderland* and *Through the Looking Glass*

```
  (2)  The English class needs a video set.
  (3)  Mathematics is run in the first room from the left side.
  (4)  The class in the yellow room is dull.
  (5)  The class in the room next to the computer room is
                interesting.
  (6)  The mathematics class is in the room next to the blue room.
  (7)  The class considered as nothing special is run using chalk
                and blackboard.
  (8)  The class on Thursday is most interesting.
  (9)  Informatics is on Tuesday.
  (10) Economics is difficult.
  (11) The class next to the Internet class is dull.
  (12) In the green room classes are on Friday.
  (13) The green room is on the right side of the white room.
  (14) In the middle room classes are on Wednesdays.

First - a complete assignment has to be determined:
1)What classes, in what rooms, on what days, with what marks and
  with what technologies are run throughout the week?

Next - a partial assignment has to be determined:
2a)What class is run on Monday?
2b)What class and in what room is run using the projector?
```

The puzzle is solved using program `4_13_five_rooms.ecl`[14]:

```
/*1*/   :-lib(ic).

/*2*/   top:-
/*3*/     Days = [Monday,Tuesday,Wednesday,Thursday,Friday],
/*4*/     Colors = [Red,Green,Blue,White,Yellow],
/*5*/     Subjects = [Physics,Mathematics,Informatics,Economics,English],
/*6*/     Marks = [Dull,Difficult,Interesting,Most_Interesting,Nothing_Special],
/*7*/     Technology = [Computer,Internet,Video,ChalkBlackboard,Projector],

/*8*/     Days :: 1..5,
/*9*/     colors :: 1..5,
/*10*/    Subjects :: 1..5,
/*11*/    Marks :: 1..5,
/*12*/    Technology :: 1..5,
```

[14]This is an FS-type problem.

```
/*13*/    alldifferent(Days),
/*14*/    alldifferent(colors),
/*15*/    alldifferent(Subjects),
/*16*/    alldifferent(Marks),
/*17*/    alldifferent(Technology),

%(1)  The physics class is in the red room:
/*18*/    Physics#=Red,
%(2)  The English class needs a video set:
/*19*/    English#=Video,
%(3)  Mathematics is run in the first room from the left side:
/*20*/    Mathematics is 1,
%(4)  The class in the yellow room is dull:
/*21*/    Dull#=Yellow,
%(5)  The class in the room next to the computer room is interesting:
/*22*/    next_to(Interesting,Computer,1),
%(6)  The mathematics class is in the room next to the blue room:
/*23*/    next_to(Mathematics,Blue,1),
%(7)  The class considered as nothing special is run using chalk and blackboard:
/*24*/    Nothing_Special#=ChalkBlackboard,
%(8)  The class on Thursday is most interesting:
/*25*/    Most_Interesting#=Thursday,
%(9)  Informatics is on Tuesday:
/*26*/    Informatics#=Tuesday,
%(10) Economics is difficult:
/*27*/    Economics#=Difficult,
%(11) The class next to the Internet class is dull:
/*28*/    next_to(Dull,Internet,1),
%(12) In the green room classes are on Friday:
/*29*/    Green#=Friday,
%(13) The green room is on the right side of the white room:
/*30*/    Green#=White+1,
%(14) In the middle room classes are on Wednesdays:
/*31*/    Wednesady#=3,
/*32*/    flatten([Days, colors, Subjects,Marks,Technology], List),
/*33*/    labeling(List),nl,nl,
/*34*/write("Complete assignment:"),nl,

/*35*/    write("Days = "),write(Days),nl,
/*36*/    write("Colors = "), write(colors),nl,
/*37*/    write("Subjects = "),write(Subjects),nl,
/*38*/    write("Marks = "),write(Marks),nl,
/*39*/    write("Technology = "),write(Technology),nl,nl,

/*40*/write("Partial assignment:"),nl,
/*41*/    SubjectsNames = [Physics-"Physics",Mathematics-"Mathematics",
              Informatics-"Informatics", Economics-"Economics",English-"English"],
/*42*/    memberchk(Monday-MondayDays, SubjectsNames),
```

Days = [Monday, Tuesday, Wednesday, Thursday, Friday],
Colours = [Red, Green, Blue, White, Yellow],
Subjects = [Physics, Mathematics, Informatics, Economics, English],
Marks = [Dull, Difficult, Interesting, Most_Interesting, Nothing_Special],
Technology = [Computer, Internet, Video, Chalk_Blackboard, Projector]

Solution:
Days = [1, 2, 3, 4, 5]
Colours = [3, 5, 2, 4, 1]
Subjects = [3, 1, 2, 5, 4]
Marks = [1, 5, 2, 4, 3]
Technology = [1, 2, 4, 3, 5]

Monday	Tuesday	Wednesday	Thursday	Friday
Yellow	Blue	Red	White	Green
Mathematics	Informatics	Physics	English	Economics
Dull	Interesting	Nothing_Special	Most_Interesting	Difficult
Computer	Internet	Chalk_Blackboard	Video	Projector

Figure 4.1: Five rooms timetable

```
/*43*/    memberchk(Projector-ProjectorTechnology,SubjectsNames),
/*44*/    printf("%w is taught on Monday.", [MondayDays]),nl,
/*45*/    printf("%w is taught using the projector.",[ProjectorTechnology]).

/*46*/   next_to(X,Y,Z):-
/*47*/     X+Z#=Y.
/*48*/   next_to(X,Y,Z):-
/*49*/     X#=Y+Z.
```

The calculated timetable including graphics and lists are shown by Figure 4.1.

4.5.2 Ten rooms

Let us extend the size of the five rooms problem from Section 4.5.1 by considering ten rooms, to which ten colors, ten time slots, ten classes, ten marks and ten technologies had to be attributed. The:

```
room colors (red, green, blue, white, yellow, pink, violet,
             orange, brown, grey),
days a_m and p_m (Monday_a_m,Tuesday_a_m,Wednesady_a_m,
                  Thursday_a_m,Friday_a_m,Monday_p_m,
```

```
                  Tuesday_p_m,Wednesady_p_m,Thursday_p_m,
                  Friday_p_m),
  subjects (physics,mathematics,informatics,economics,English,
            chemistry,German,history,music,electronics),
  subject marks (dull,difficult,interesting,most_interesting,
                 nothing_special,exhausting,funny,popular,
                 singing,absorbing),
  technology (computer,internet,video,chalk_blackboard,projector,
            reagents,dictionaries,maps,piano,oscilloscope))
```

are subject to following constraints:

```
  (1)  The physics class is in the red room.
  (2)  The English class needs a video set.
  (3)  Mathematics is run in the first room from the left side.
  (4)  The class in the yellow room is dull.
  (5)  The class in the room next to the computer room is
               interesting.
  (6)  The mathematics class is in the room next to the blue room.
  (7)  The class considered as nothing special is run using chalk
               and blackboard.
  (8)  The class on Thursday is most interesting.
  (9)  Informatics is on Tuesday.
  (10) Economics is difficult.
  (11) The class next to the Internet class is dull.
  (12) In the green room classes are on Friday.
  (13) The green room is on the right side of the white room.
  (14) In the middle room classes are on Wednesdays.
  (15) In the pink room classes are on Monday p_m
  (16) For the chemistry class reagents are used
  (17) Classes on Monday p_m are exhausting
  (18) The Projector is next to the room where reagent are used:
  (19) On Monday p_m is a class in the pink room
  (20) In the violet room are dictionaries
  (21) The violet room is next to the pink room
  (22) German is taught next to the room where chemistry is taught
  (23) On Tuesday p_m the class is funny
  (24) The room with dictionaries is on the left side of the
               orange room
  (25) The class run on the right side of the German class is
```

```
              popular
  (26) On Wednesday p_m a class is run in the orange room
  (27) For the history class maps are needed
  (28) Piano is in room number 9
  (29) There is much singing in the music class
  (30) The piano is in the brown room
  (31) A class in the brown room is run Thursday p_m
  (32) Electronics is taught in the room next to the room were
              music is taught
  (33) For teaching electronics an oscilloscope is needed
  (34) The class that makes use of the oscilloscope is absorbing
  (35) On Friday p_m the class is in the grey room.

First - a complete assignment has to be determined:
1)What classes, in what rooms, on what days, with what marks and
  with what technologies are run throughout the week?

Next - a partial assignment has to be determined:
2a)What class is run on Monday?
2b)What class and in what room is run using the projector?
```

The puzzle is solved using program `4_14_ten_rooms.ecl`[15]:

```
/*1*/   :-lib(ic).
/*2*/   top:-
/*3*/     Days = [Monday_a_m,Tuesday_a_m,Wednesady_a_m,Thursday_a_m,Friday_a_m,
            Monday_p_m,Tuesday_p_m,Wednesady_p_m,Thursday_p_m,Friday_p_m],
/*4*/     Colors = [Red,Green,Blue,White,Yellow,Pink,Violet,Orange,Brown,Grey],
/*5*/     Subjects = [Physics,Mathematics,Informatics,Economics,English,
            Chemistry,German,History,Music,Electronics],
/*6*/     Marks = [Dull,Difficult,Interesting,Most_Interesting,Nothing_Special,
            Exhausting,Funny,Popular,Singing,Absorbing],
/*7*/     Technology = [Computer,Internet,Video,ChalkBlackboard,Projector,
            Reagents,Dictionaries,Maps,Piano,Oscilloscope],

/*8*/     Days :: 1..10,
/*9*/     Colors :: 1..10,
/*10*/    Subjects :: 1..10,
/*11*/    Marks :: 1..10,
/*12*/    Technology :: 1..10,
/*13*/    alldifferent(Days),
```

[15]This is an FS-type problem.

```
/*14*/    alldifferent(colors),
/*15*/    alldifferent(Subjects),
/*16*/    alldifferent(Marks),
/*17*/    alldifferent(Technology),

%(1)  The physics class is in the red room:
/*18*/    Physics#=Red,

%(2)  The English class needs a video set:
/*19*/    English#=Video,

%(3)  Mathematics is run in the first room from the left side:
/*20*/    Mathematics is 1,

%(4)  The class in the yellow room is dull:
/*21*/    Dull#=Yellow,

%(5)  The class in the room next to the computer room is interesting:
/*22*/    adjacent(Interesting,Computer,1),

%(6)  The mathematics class is in the room next to the blue room:
/*23*/    adjacent(Mathematics,Blue,1),

%(7)  The class considered as nothing special is run using chalk and blackboard:
/*24*/    Nothing_Special#=ChalkBlackboard,

%(8)  The class on Thursday a_m is most interesting:
/*25*/    Most_Interesting#=Thursday_a_m,

%(9)  Informatics is on Tuesday a_m:
/*26*/    Informatics#=Tuesday_a_m,

%(10) Economics is difficult:
/*27*/    Economics#=Difficult,

%(11) The class is dull next to the Internet class:
/*28*/    adjacent(Dull,Internet,1),

%(12) In the green room classes are on Friday a_m:
/*29*/    Green#=Friday_a_m,

%(13) The green room is on the right side of the white room:
/*30*/    Green#=White+1,

%(14) In the middle room classes are on Wednesdays a_m:
/*31*/    Wednesady_a_m#=3,

%(15) In the pink room classes are on Monday p_m:
/*32*/    Pink#=Monday_p_m,
```

```
%(16) For the chemistry class reagents are used:
/*33*/    Chemistry#=Reagents,

%(17) Classes on Monday p_m are exhausting:
/*34*/    Monday_p_m#=Exhausting,

%(18) The Projector is next to the room where reagent are used:
/*35*/    adjacent(Projector,Reagents,1),

%(19) On Monday p_m is a class in the pink room:
/*36*/    Monday_p_m#=Pink,

%(20) In the violet room are dictionaries:
/*37*/    Violet#=Dictionaries,

%(21) The violet room is next to the pink room:
/*38*/    adjacent(Violet,Pink,1),

%(22) German is taught next to the room where chemistry is taught:
/*39*/    adjacent(German,Chemistry,1),

%(23) On Tuesday p_m the class is funny:
/*40*/    Tuesday_p_m#=Funny,

%(24) The room with dictionaries is on the left side of the orange room:
/*41*/    Orange#=Dictionaries+1,

%(25) The class run on the right side of the German class is popular:
/*42*/    Popular#=German+1,

%(26) On Wednesday p_m a class is run in the orange room
/*43*/    Wednesady_p_m#=Orange,

%(27) For the history class maps are needed:
/*44*/    History#=Maps,

%(28) Piano is in room number 9:
/*45*/    Piano is 9,

%(29) There is much singing in the music class:
/*46*/    Music#=Singing,

%(30) The piano is in the brown room:
/*47*/    Piano#=Brown,

%(31) A class in the brown room is run Thursday p_m:
/*48*/    Brown#=Thursday_p_m,
```

```
%(32) Electronics is taught in the room next to the room were music is taught:
/*49*/    adjacent(Electronics,Music,1),

%(33) For teaching electronics an oscilloscope is needed:
/*50*/    Elektronika#=Oscilloscope,

%(34)  The class that makes use of the oscilloscope is absorbing:
/*51*/    Oscilloscope#=Absorbing,

%(35) On Friday p_m the class is in the grey room:
/*52*/    Friday_p_m#=Grey,

/*53*/    flatten([Days, colors, Subjects,Marks, Technology], List),
/*54*/    labeling(List),

          % Complete assignment:
/*55*/    write("Complete assignment:"),nl,
/*56*/    write("Days =        "),write(Days),nl,
/*57*/    write("colors =     "), write(colors),nl,
/*58*/    write("Subjects = "),write(Subjects),nl,
/*59*/    write("Marks =      "),write(Marks),nl,
/*60*/    write("Technology =   "),write(Technology),nl,nl,

          % Partial assignment:
/*61*/    write("Partial assignment:"),nl,
/*62*/    SubjectsNames = [Physics-"Physics", Mathematics-"Mathematics",
                           Informatics-"Informatics", Economics-"Economics",
                           English-"English",Chemistry-"Chemistry",
                           German-"German",History-"History",Music-"Music",
                           Electronics-"Electronics"]
/*63*/    memberchk(Monday_a_m-MondayDays,SubjectsNames),
/*64*/    memberchk(Projector-ProjectorTechnology,SubjectsNames),
/*65*/    printf("%w is taught on Monday.", [MondayDays]),nl,
/*66*/    printf("%w is taught using the projector.",[ProjectorTechnology]).

/*67*/  adjacent(X,Y,Z):-
/*68*/    X+Z#=Y.
/*69*/  adjacent(X,Y,Z):-
/*70*/    X#=Y+Z.
```

The timetables including graphics and lists are shown in Figures 4.2 and 4.3.

Days = [Monday_am, Tuesday_am, Wednesday_am, Thursday_am, Friday_am,
Monday_pm, Tuesday_pm, Wednesday_pm, Thursday_pm, Friday_pm],
Colours = [Red, Green, Blue, White, Yellow, Pink, Violet, Orange, Brown, Grey],
Subjects = [Physics, Mathematics, Informatics, Economics, English,
Chemistry, German, History, Music, Electronics],
Marks = [Dull, Difficult, Interesting, Most_Interesting, Nothing_Special,
Exhausting, Funny, Popular, Singing, Absorbing],
Technology = [Computer, Internet, Video, Chalk_Blackboard, Projector,
Reagents, Dictionaries, Maps, Piano, Oscilloscope]

Solution 1:
Days = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
Colours = [3, 5, 2, 4, 1, 6, 7, 8, 9, 10]
Subjects = [3, 1, 2, 5, 4, 6, 7, 8, 9, 10]
Marks = [1, 5, 2, 4, 3, 6, 7, 8, 9, 10]
Technology = [1, 2, 4, 3, 5, 6, 7, 8, 9, 10]

Monday_am	Tuesday_am	Wednesday_am	Thursday_am	Friday_am
Yellow	Blue	Red	White	Green
Mathematics	Informatics	Physics	English	Economics
Dull	Interesting	Nothing_Special	Most_Interesting	Difficult
Computer	Internet	Chalk_Blackboard	Video	Projector

Monday_pm	Tuesday_pm	Wednesday_pm	Thursday_pm	Friday_pm
Pink	Violet	Orange	Brown	Grey
Chemistry	German	History	Music	Elektronics
Exhausting	Funny	Popular	Singing	Absorbing
Reagents	Dictionaries	Maps	Piano	Oscilloscope

Solution 2:
Days = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
Colours = [3, 5, 2, 4, 1, 6, 7, 8, 9, 10]
Subjects = [3, 1, 2, 5, 8, 6, 7, 4, 9, 10]
Marks = [1, 5, 2, 4, 3, 6, 7, 8, 9, 10]
Technology = [1, 2, 8, 3, 5, 6, 7, 4, 9, 10]

Monday_am	Tuesday_am	Wednesday_am	Thursday_am	Friday_am
Yellow	Blue	Red	White	Green
Mathematics	Informatics	Physics	History	Economics
Dull	Interesting	Nothing_Special	Most_Interesting	Difficult
Computer	Internet	Chalk_Blackboard	Maps	Projector

Monday_pm	Tuesday_pm	Wednesday_pm	Thursday_pm	Friday_pm
Pink	Violet	Orange	Brown	Grey
Chemistry	German	English	Music	Elektronics
Exhausting	Funny	Popular	Singing	Absorbing
Reagents	Dictionaries	Video	Piano	Oscilloscope

Figure 4.2: Ten rooms timetable - solution 1 and 2

Days = [Monday_am, Tuesday_am, Wednesday_am, Thursday_am, Friday_am,
Monday_pm, Tuesday_pm, Wednesday_pm, Thursday_pm, Friday_pm],
Colours = [Red, Green, Blue, White, Yellow, Pink, Violet, Orange, Brown, Grey],
Subjects = [Physics, Mathematics, Informatics, Economics, English,
Chemistry, German, History, Music, Electronics],
Marks = [Dull, Difficult, Interesting, Most_Interesting, Nothing_Special,
Exhausting, Funny, Popular, Singing, Absorbing],
Technology = [Computer, Internet, Video, Chalk_Blackboard, Projector,
Reagents, Dictionaries, Maps, Piano, Oscilloscope]

Solution 3:
Days = [1, 2, 3, 7, 5, 6, 4, 8, 9, 10]
Colours = [3, 5, 2, 4, 1, 6, 7, 8, 9, 10]
Subjects = [3, 1, 2, 5, 4, 6, 7, 8, 9, 10]
Marks = [1, 5, 2, 7, 3, 6, 4, 8, 9, 10]
Technology = [1, 2, 4, 3, 5, 6, 7, 8, 9, 10]

Monday_am	Tuesday_am	Wednesday_am	Thursday_am	Friday_am
Yellow	Blue	Red	Violet	Green
Mathematics	Informatics	Physics	German	Economics
Dull	Interesting	Nothing_Special	Most_Interesting	Difficult
Computer	Internet	Chalk_Blackboard	Dictionaries	Projector

Monday_pm	Tuesday_pm	Wednesday_pm	Thursday_pm	Friday_pm
Pink	White	Orange	Brown	Grey
Chemistry	English	History	Music	Elektronics
Exhausting	Funny	Popular	Singing	Absorbing
Reagents	Video	Maps	Piano	Oscilloscope

Solution 4:
Days = [1, 2, 3, 7, 5, 6, 4, 8, 9, 10]
Colours = [3, 5, 2, 4, 1, 6, 7, 8, 9, 10]
Subjects = [3, 1, 2, 5, 8, 6, 7, 4, 9, 10]
Marks = [1, 5, 2, 7, 3, 6, 4, 8, 9, 10]
Technology = [1, 2, 8, 3, 5, 6, 7, 4, 9, 10]

Monday_am	Tuesday_am	Wednesday_am	Thursday_am	Friday_am
Yellow	Blue	Red	Violet	Green
Mathematics	Informatics	Physics	German	Economics
Dull	Interesting	Nothing_Special	Most_Interesting	Difficult
Computer	Internet	Chalk_Blackboard	Dictionaries	Projector

Monday_pm	Tuesday_pm	Wednesday_pm	Thursday_pm	Friday_pm
Pink	White	Orange	Brown	Grey
Chemistry	History	English	Music	Elektronics
Exhausting	Funny	Popular	Singing	Absorbing
Reagents	Maps	Video	Piano	Oscilloscope

Figure 4.3: Ten rooms timetable - solution 3 and 4

4.5.3 All Things to All People

The `element/3` built-in is almost always used with the `alldifferent/1` built-in. This is illustrated by the following example:

The Absurdoland's party *All Things to All People* is a popular political force to be reckoned with. At its Headquarters each Friday a meeting takes place with the agenda devoted solely to next week dispatching of party activists to local communities to meet with local activists, voters and supporters, and persuade people to vote for the party candidates in the forthcoming elections. Last Friday the discussion concentrated upon visiting three important local communities, Lower Hole, Upper Hole and Middle Hole, by three trusted and experienced party activists, Mr Blather, Mr Jabber and Ms Fable. The visits were supposed to take place only on Monday, Tuesday or Wednesday, by one party activist each day, because they were also badly needed at the Headquarters. The problem to be settled is who should go where. Each of the party activists has special wishes and hindrances to be taken into account:
1)Activist Blather decided never to travel again to Lower Hole, because at his last stay there he was invited for lunch to a shabby roadside eatery pot, where the local activists and supporters presented him with a complete set of Chinese ball-pens; well, he did not boast about this to his party colleagues.
2)Activist Jabber has no objections for going to Lower Hole or to Upper Hole, but not on Tuesdays, because his sponsoring benefactor, the Famous Businessman, whom he used to meet at some randomly selected grave at the Lower- or Upper Hole cemetery, traditionally devotes each Tuesday to one of his girlfriends. Mr Jabber does not intend to give up those meetings because at each of them he is presented by the Famous Businessman with a plastic bag filled with cash and some memos about things he should take care of.
3)To Upper Hole Mr Jabber does not want to go on Monday as well, because on Mondays all Escort Service Agencies in Upper Hole have a day off.
4)To Lower Hole nobody wants travel on Mondays because then all bars and restaurants try to sell their Sunday left-overs.
5)Mr Blather should not be dispatched to Upper Hole because at his last stay there he had considerable problems in explaining this item of the Party Political Program that promises state guarantees for loans taken by any unemployed who wishes - for a planned future business activity - to buy a new SUV of the well-known make "Luxus".
6)Mr Blather may be dispatched to Lower Hole, but not on Monday, because Mondays are traditional extensions of his customary weekends.

7)Ms Fable should not be dispatched to Middle Hole, because last time there she refused to support the request of the Middle Hole Party Chairman to be distinguished by the widely aspired "Pour le Fraude" golden medal that she herself has not got yet.

Is it possible to find a dispatch solution that gives justice to all the presented wishes and hindrances?

This problem is solved by program `4_15_delegations.ecl`[16]:

```
/*1*/   :-lib(ic).
/*2*/   top:-
/*3*/       Towns=[Destination_of_Blather, Destination_of_Jabber,
            Destination_of_Fable],
/*4*/       Towns::1..3,
  % Visited towns:   1 - Lower Hole, 2 - Upper Hole, 3 - Middle Hole.
  % If e.g  Destination_of_Blather=3, then Blather is dispatched to Middle Hole.
/*5*/       alldifferent(Towns),

/*6*/       Days=[Monday,Tuesday,_],
/*7*/       Days::1..3,
  % Visited Towns:  1 - Lower Hole, 2 - Upper Hole, 3 - Middle Hole.
  % If e.g. Tuesday=3, then on Tuesday someone is dispatched to Middle Hole.
/*8*/       alldifferent(Days),

  % 1) Mr  Blather is not going to Lower Hole:
/*9*/       Destination_of_Blather #\= 1,

  % 2) Mr Jabber has no objections for going to Lower Hole,
  %   or to Upper Hole, but not on Tuesdays:
/*10*/      constraint_2(Destination_of_Jabber,Tuesday),

  % 3) Mr Jabber does not wish to travel to Upper Holes on Mondays:
/*11*/     constraint_3(Destination_of_Jabber,Monday),

  % 4) To Lower Hole nobody wishes to travel on Mondays:
/*12*/     Monday #\= 1,

  % 5) Mr Blather should not be dispatched to Upper Hole:
/*13*/     Destination_of_Blather #\= 2,

  % 6) Mr Blather may be dispatched to Lower Hole, but not on Mondays:
/*14*/     constraint_6(Destination_of_Blather,Monday),
```

[16]This is an FS-type problem.

```
  % 7) Ms Fable should not be dispatched to Middle Hole:
/*15*/     Destination_of_Fable #\= 3,

/*16*/     write("Towns = "),writeln(Towns),
/*17*/ write("Days = "),writeln(Days),nl,
  % End of problem solving part

  % Beginning of solution writing part:
  % The idea is to determine 3-tuples (Name,Destination,Day)
  % having the same number:

  % Number of destination on position 1:
/*18*/     element(1,Towns,Number_of_destination_on_position_1),

  % Number of day on position Number_of_destination_on_position_1:
/*19*/     element(Number_of_day_1,Days,Number_of_destination_on_position_1),

  % Number of destination on position 2:
/*20*/     element(2,Towns,Number_of_destination_on_position_2),

  % Number of day on position Number_of_destination_on_position_2:
/*21*/     element(Number_of_day_2,Days,Number_of_destination_on_position_2),

  % Number of destination on position 3:
/*22*/     element(3,Towns,Number_of_destination_on_position_3),

  % Number of day on position Number_of_destination_on_position_3:
/*23*/     element(Number_of_day_3,Days,Number_of_destination_on_position_3),

  % Translating destination number into destination name:
/*24*/     destination(Number_of_destination_on_position_1,Name_of_destination_1),
/*25*/     destination(Number_of_destination_on_position_2,Name_of_destination_2),
/*26*/     destination(Number_of_destination_on_position_3,Name_of_destination_3),

  %  Translating day number into day name:
/*27*/     day(Number_of_day_1,Name_of_day_1),
/*28*/     day(Number_of_day_2,Name_of_day_2),
/*29*/     day(Number_of_day_3,Name_of_day_3),

  % Merging elements of the 3-tuples:
/*30*/     write("Mr Blather will be dispatched to "),
             write(Name_of_destination_1),write(Name_of_day_1),nl,
/*31*/     write("Mr Jabber will be dispatched to "),
             write(Name_of_destination_2),write(Name_of_day_2),nl,
/*32*/     write("Ms Fable will be dispatched to "),
             write(Name_of_destination_3),write(Name_of_day_3),nl.

/*33*/   constraint_2(Destination_of_Jabber,Tuesday):-
/*34*/     Destination_of_Jabber #= 1,
```

```
/*35*/     Tuesday #\= 1;
/*36*/     Destination_of_Jabber #= 2,
/*37*/     Tuesday #\= 2.
/*38*/    constraint_2(_,_).

/*39*/   constraint_3(Destination_of_Jabber,Monday):-
/*40*/     Destination_of_Jabber #= 2,
/*41*/     Monday #\= 2.
/*42*/   constraint_3(_,_).

/*43*/    constraint_6(Destination_of_Blather,Monday):-
/*44*/        Destination_of_Blather #= 3,
/*45*/        Monday #\= 3.
/*46*/   constraint_6(_,_).

  % Translating destination number into destination name:
/*47*/   destination(1,"Lower Hole").
/*48*/   destination(2,"Upper Hole").
/*49*/   destination(3,"Middle Hole").


  % Translating day number into day name:
/*50*/   day(1," on Monday.").
/*51*/   day(2," on Tuesday.").
/*52*/   day(3," on Wednesday.").
```

The message generated is:

```
Towns = [3, 1, 2]
Days = [2, 3, 1]

Mr Blather will be dispatched to Middle Hole on Tuesday.
Mr Jabber will be dispatched to Lower Hole on Wednesday.
Ms Fable will be dispatched to Upper Hole on Monday.
```

4.6 Data handling

The solution of complicated and large problems may require some additional knowledge about data structures and their handling.

4.6.1 Structures and arrays

Structures, abbreviated by *struct*, are handy for presenting and processing data from nested relational data bases. Their use is declared by `local struct()` templates, like e.g.:

```
:- local struct(person(name, address, age)).
:- local struct(employee(p:person, salary)).
```

where the structure `person` is nested in structure `employee` using field `p`. The following example illustrates the use of the structures. It is given by commands in *command mode*, see Section 0.3, for which a response is generated:

```
This is a command:
 [eclipse 1]:
 :- local struct(person(name, address, age)).
 :- local struct(employee(p:person, salary)).
 Employee = employee with [name: "Jan Kowalski"", age: 26,
             salary: 4000, address: "Gliwice, Kormoranow 5"],
 arg(name of employee, Employee, Name),
 arg(age of employee, Employee, Age),
 arg(salary of employee, Employee, Salary).

This is the response:
 Employee = employee(person("Jan Kowalski",
             "Gliwice, Kormoranow 5", 26), 4000)
 Name = "Jan Kowalski"
 Age = 26
 Salary = 4000
```

Important data structures are multidimensional *arrays*. They are singled out by the prefix [] and handled by following built-ins:

1)a one-dimensional array with 4 elements may be constructed by calling `dim(Array,[4])`. This results in a one-dimensional array with four free elements:

```
This is a command:
 [eclipse 2]: dim(Array,[4]).
```

```
This is the response:
Array = [](_169, _170, _171, _172)
```

2)a 2-dimensional `3 x 2` array may be constructed as follows:

```
This is a command:
[eclipse 3]: dim(Array,[3,2]).

This is the response:
Array = []([](_181, _182), [](_178, _179), [](_175, _176))
```

3)The dimensions of a 2-dimensional array may be calculated as follows:

```
This is a command:
[eclipse 4]: Array = []([](a,b,c),[](d,e,f)),dim(Array,D).

This is the response:
D = [2, 3]
```

4)The built-in `dim/2` may also serve to determine the elements of an array:

```
This is a command:
[eclipse 5]: Array = [](a, b, c, d), dim(Array,D).

This is the response:
Array = [](a, b, c, d)
D = [4]
```

5)A 1-dimensional array may have lists as its elements. The number of elements of such array is equal to the number of lists, e.g.:

```
This is a command:
[eclipse 6]: Array=[]([5 ,7 ,1 ,20 ],[14 ,8 ,100,300],
        [2 ,20 ,50 ,12 ] ),
dim(Array,[M]).

This is the response:
Array = []([5, 7, 1, 20], [14, 8, 100, 300],
```

```
[2, 20, 50, 12])
M = 3
```

6)The number of elements of a list may be determined using the `length(?List, ?Length)` built-in, e.g.:

```
This is a command:
[eclipse 7]: length([a,b,c,d], Length_of_list).

This is the response:
Length_of_list = 4
```

The `length/2` built-in may also be used for list construction, e.g.:

```
This is a command:
[eclipse 8]: length(List, 4).

This is the response:
List = [_166, _168, _170, _172]
```

7)The `I`-th row of an array with `M` columns can be determined by calling `Row_I is Array(I,1..M]`, where `Row_I` is a list, e.g.:

```
This is a command:
[eclipse 9]: Array = []([](a,b,c),[](d,e,f)),
             Second_Row is Array[2,1..3].

This is the response:
 Array = []([](a, b, c), [](d, e, f))
 Second_Row = [d, e, f]
```

8)The `J`th column of an array with `N` rows can be determined by calling `Column_J is Array(1..N,J]`, where `Column_J` is a list, e.g.:

```
This is a command:
[eclipse 10]: Array = [](a,b,c),[](d,e,f)),
              Third_Column is Array[1..2,3].
```

```
This is the response:
Array = [](](a, b, c), [](d, e, f))
Third_Column = [c, f]
```

4.6.2 How to get hold of matrix elements?

If some operations have to be done on successive rows of a matrix, the presented approach of getting hold of the rows cannot be used. Instead the built-in:

```
arg(Row_number,ArrayMatrix,ArrayMatrixRow)
```

is to be used. It determines (as an array) the matrix row of given number. This is illustrated by program `4_16_extracting_elements.ecl`[17]:

```
/*1*/   :- lib(ic).
/*2*/   array_matrix(ArrayMatrix):-
/*3*/         ArrayMatrix=[](
/*4*/         [](1,2,3,4,5),
/*5*/         [](6,7,8,9,10),
/*6*/         [](11,12,13,14,15)
/*7*/         ).

/*8*/   top:-
/*9*/         array_matrix(ArrayMatrix),
/*10*/        arg(2,ArrayMatrix,ArrayMatrixRow),
/*11*/        writeln("ArrayMatrixRow":ArrayMatrixRow),
/*12*/        ArrayMatrixRow=..[[]|ListMatrixRow],
/*13*/        writeln("ListMatrixRow":ListMatrixRow),
/*14*/        element(3,ListMatrixRow,Element_2_3),
/*15*/        writeln("Element_2_3":Element_2_3).
```

The message is:

```
ArrayMatrixRow : [](6, 7, 8, 9, 10)
ListMatrixRow : [6, 7, 8, 9, 10]
Element_2_3 : 8
```

While being interested only in a specific element of the matrix, a simpler approach is recommended: the element is available by calling the predicate

[17] This is an FS-type problem.

`ArrayMatrix[Element_coordinates]`, like this:

```
ArrayMatrix=[](
                          [](1,2,3,4,5),
                          [](6,7,8,9,10),
                          [](11,12,13,14,15)
                          ),
X is ArrayMatrix[2,3].
```

The message is:

```
ArrayMatrix = [](1, 2, 3, 4, 5), [](6, 7, 8, 9, 10), [](11, 12, 13, 14, 15))
X = 8
```

4.6.3 Recursions and iterations - bye, bye declarativity!

For ECL^iPS^e *Prolog* - as for any other Prolog - data is processed chiefly using recursions. Prolog people just love recursions. E.g. consecutive elements of a list may be obtained by the simple private predicate `write_list/1` defined by the recursion from program `4_17_write_list.ecl`[18]:

```
/*1*/   top :-
/*2*/         write_list([1,2,3]).

/*3*/   write_list([X|Xs]):-
/*4*/         writeln(X),
/*5*/         write_list(Xs).
/*6*/   write_list([]).
```

The essence of recursion amounts to defining the `write_list/1` by itself. The program generates a message:

```
1
2
3
```

Recursive programming is functioning - as has been demonstrated many times - also in ECL^iPS^e CPS. The ECL^iPS^e designers decided however to supplement recursions by *iterations*, the essence of which amounts to calling the

[18] This is an FS-type problem.

same predicate, in a loop, for changing data. Such iterations are not used in Prolog programs; their presence in ECL^iPS^e CPS seems to be a concession to programmers accustomed to procedural programming. As a result some of Prolog programs declarativity as well as readability has been lost.

The basic iterative built-in is `do/2` used as:

```
+iteration_definition(X) do +goal(X)
```

for calling `goal(X)` according to `iteration_definition(X)`.

Following `iteration_definitions` may be used:

1) `foreach(X,List) do goal(X)` is iterating `goal(X)` for all `X` from the list `List`. `X` is a local variable for `goal(X)`. E.g.:

```
This is a command:
[eclipse 1]: (foreach(X, [1,2,3]) do writeln(X)).

This is the response:
1
2
3
X = X
```

The response is the same as that obtained by the private `write_list/1` predicate. However, `foreach(X,List)` may also be used for constructing lists:

```
This is a command:
[eclipse 2]: (foreach(X, [1,2,3]), foreach(Y,List) do Y is X+5).

This is the response:
X = X
Y = Y
List = [6, 7, 8]
```

The possibility to construct data structures is common to the majority of iteration definitions. It lessens somehow the *burden of procedurality* from those definitions. It has been used in program `4_18_scalar_product.ecl` (see 4.6.5),

calculating the scalar products of two vectors presented as lists. This scalar product has been in turn used in the `5_14_knapsack_1.ecl` program, see 5.6.3.

2) `foreacharg(X,Predicate) do goal(X)` is iterating `goal(X)` for all `X` given by arguments (free or grounded) of the predicate `Predicate`. E.g.:

```
 This is a command:
[eclipse 3]: (foreacharg(X, s(p,q,R,5)) do writeln(X)).

This is the response:
p
q
R
5
X = X
R = R
```

The built-in `foreacharg(X,Predicate)` cannot be used for constructing predicates because of the ambiguity of this concept.

3) `foreacharg(X,Predicate,I) do goal(X)` is iterating `goal(X)` for all `X` given by arguments (free or grounded) of the predicate `Predicate`, while delivering the numbers `I` of positions `X` in the predicate. E.g.:

```
This is a command:
[eclipse 3]: (foreacharg(X, s(p,q,R,5),I) do writeln(X),writeln(I)).

This is the response:
p
1
q
2
R
3
5
4
X = X
R = R
I = I
```

4) `param(Variable_1,Variable_2,...)` is used for introducing variables into loops of the `do` iterations. In other words, `Variable_1,Variable_2,...` are declared as global, in contrast to other loop variables, which by default are always local. This is illustrated by determining all pairs of list elements:

```
This is a command:
[eclipse 4]: List = [1,2,3],
( foreach(X, List), param(List) do
   ( foreach(Y,List), param(X) do
   write(X),write(" "),write(Y),nl
   )
).

This is the response:
1  1
1  2
1  3
2  1
2  2
2  3
3  1
3  2
3  3
List = [1, 2, 3]
X = X
Y = Y
```

Another example of using `param()` in a `for()` loop is given by the more concise than `4_9_queens_again.ecl` version of the queen placement problem in `4_19_queens_one_more_time.ecl`, see Section 4.6.4.
5)`count(I,Min,Max) do goal(I)` is iterating `goal(I)` for all integers `I` from the range `[Min...Max]`. `I` is (obviously) a local variable for `goal(I)`. This is illustrated by constructing a list of integers:

```
This is a command:
[eclipse 5]: (count(I,1,4), foreach(I,List) do true).

This is the response:
I = I
```

```
List = [1, 2, 3 ,4]
```

6) `for(I,MinExpr,MaxExpr)do goal(I)` is iterating `goal(I)` for all integer variables `I` from the range `[MinExpr...MaxExpr]`. `I` is (obviously) a local variable for `goal(I)`, and `MinExpr` as well as `MaxExpr` may be arithmetic expressions. This construct may be used only for controlling iterations, i.e. `MaxExpr` must be grounded. This is illustrated by constructing a list of integers:

```
This is a command:
[eclipse 6]: (for(I,1,5), foreach(I,List) do true).

This is the response:
 I = I
 List = [1, 2, 3, 4, 5]
```

7) `for(I,MinExpr,MaxExpr,Delta)do goal(I)` is iterating `goal(I)` for integer variables `I` from the range `[MinExpr...MaxExpr]` incremented with `Delta`. `I` is (obviously) a local variable for `goal(I)`, and `MinExpr` as well as `MaxExpr` may be arithmetic expressions. This construct may be used only for controlling iterations, i.e. `MaxExpr` must be grounded. This is illustrated by constructing a list of integers:

```
This is a command:
[eclipse 7]: (for(I,1,5,2), foreach(I,List) do true).

This is the response:
 I = I
 List = [1, 3, 5]
```

8) `multifor(List,ListMin,ListMax)do goal(List)` is a generalization for `for/3` presented in `6)` when iterations have to be performed for a number of variables. `multifor` is iterating `goal(List)` for all integer variables from the `List` for ranges given by lists `ListMin` and `ListMax`. `List` is (obviously) a local variable for `goal(List)`, and `MinExpr` i `MaxExpr` may contain the same number of arithmetic expressions. This construct may be used only for controlling iterations, i.e. `MaxExpr` must be grounded. The example is:

```
This is a command:
[eclipse 8]: (multifor([I,J],[1,2],[2,4]) do writeln([I,J]),
                   K is I+J, writeln([K])).
```

```
This is the response:
 [1, 2]
 [3]
 [1, 3]
 [4]
 [1, 4]
 [5]
 [2, 2]
 [4]
 [2, 3]
 [5]
 [2, 4]
 [6]
 I = I
 J = J
 K = K
```

An interesting application for `multifor/3` is given by *sudoku* puzzles, see program `4_20_sudoku.ecl` in Section 4.7.1.

9) `multifor(List,ListMin,ListMax,ListDelta) do goal(List)` is a generalization for `multifor(List,ListMin,ListMax)do goal(List)` presented in `8)` for integer variables incremented with `ListDelta`. The example is:

```
This is a command:
[eclipse 9]: (multifor([I,J],[1,2],[2,5],[1,2]) do writeln([I,J]),
                    K is I+J, writeln([K])).

This is the response:
 [1, 2]
 [3]
 [1, 4]
 [5]
 [2, 2]
 [4]
 [2, 4]
 [6]
 I = I
```

```
J = J
K = K
```

10) `fromto(First,In,Out,Last)do goal(In,Out)` is the most general iterator. It iterates `goal(In,Out)` by starting with `In = First`, thus computing a first value for `Out`. This value is swapped at the second iteration for `In`, and so on: at each iteration the value of OUT computed from previous `In` is swapped for the next `In`, until `Out = Last` and the iteration stops. `In` and `Out` are local variables for `goal`. The `fromto/4` performance is illustrated by computing the sum of a list of integers:

```
This is a command:
[eclipse 10]: (foreach(X,[10,20,30]),
        fromto(0,In,Out,Sum) do Out is InX).+

This is the response:
 X = X
 In = In
 Out = Out
 Sum = 60
```

`fromto/4` may also be used for reversing lists:

```
This is a command:
[eclipse 11]: (foreach(X,[10,20,30]),
        fromto([],In,[X|In],Reversed_list) do true).

This is the response:
 X = X
 In = In
 Reversed_list = [30, 20, 10]
```

For sophisticated applications of `fromto/4` the `First` argument is grounded only at the end of iterations. This occurs for various variable filtering schemes, e.g.:

```
This is a command:
[eclipse 12]: (foreach(X,[5,3,8,1,4,6]),
    fromto(List,In,Out,[]) do
        X>3 -> In=[X|Out] ; Out=In).
```

```
This is the response:
 X = X
 List = [5, 8, 4, 6]]
 In = In
 Out = Out
```

The `4_21_queens_for_the_last_time.ecl` program (see Section 4.7.2) illustrates another situation, for which `First` is not grounded till the end of iterations.

4.6.4 Queens one more time

Iterations allow to express the queens placement problem from Section 4.4.6 in a more compact way, as shown in program `4_19_queens_one_more_time.ecl`[19]:

```
/*1*/   :- lib(ic).
/*2*/   top:-
/*3*/       queens(_,_).

/*4*/   queens(N, Chessboard) :-
/*5*/       size_of_chessboard(N),
/*6*/       dim(Chessboard, [N]),
/*7*/       Chessboard[1..N] :: 1..N,

/*8*/       (for(I,1,N), param(Chessboard,N) do
/*10*/      (for(J,I+1,N), param(Chessboard,I) do
/*11*/      Chessboard[I] #\= Chessboard[J],
/*12*/      Chessboard[I] #\= Chessboard[J]+J-I,
/*13*/      Chessboard[I] #\= Chessboard[J]+I-J
/*14*/      )
/*15*/      ),

/*16*/      labeling(Chessboard),
/*17*/      writeln(Chessboard).

/*18*/      size_of_chessboard(4).
```

The message corresponds to the already obtained solution:

```
     [](2, 4, 1, 3)
     [](3, 1, 4, 2),
```

[19] This is an FS-type problem.

this time using arrays.

4.6.5 Scalar product

The scalar product (or dot product) of two vectors presented as lists:
`[a1,a2,...,an]` `[b1,b2,...,bn]`
is given by:
`a1*b1 + a2*b2 + ... an*bn.`
This can be computed by program `4_18_scalar_product.ecl`[20]:

```
/*1*/   :- lib(ic).
/*2*/   top:-
/*3*/       scalar_product([1,2,3,4],[10,20,30,40],_).

/*4*/   scalar_product(List_1,List_2,Scalar_product):-
/*5*/       (foreach(V1, List_1),
/*6*/       foreach(V2, List_2),
/*7*/       foreach(Product,Product_list)
/*8*/       do
/*9*/       Product is V1 * V2
/*10*/      ),
/*11*/      Scalar_product #= sum(Product_list),nl,
/*12*/      write("Scalar product = "),writeln(Scalar_product),nl.
```

The message is:

```
Scalar product = 300
```

4.7 More feasible assignment problems

4.7.1 Sudoku

Sudoku is a combinatorial number-placement puzzle. The goal is to fill the cells of a 9×9 gridded table with digits from `1` to `9` so that each column, each row, and each of the nine 3×3 gridded sub-tables that compose the grid (called

[20] This is an FS-type problem.

"boxes") contains all of the digits from `1` to `9`. Initially the table is partially completed in a way that assures a unique solution.

The program `4_20_sudoku.ecl`[21] is solving sudoku puzzles using the built-in `multifor/3`. It is a slightly modified version of the program available at the website [Schimpf-10]:

```
/*1*/   :- lib(ic).
/*2*/   top:-
/*3*/       write("Declare puzzle number (1,2 or 3):"),nl,
/*4*/       read_token(Number, integer),
/*5*/       solve(Number).

/*6*/   solve(Number):-
/*7*/       problem(Number, Board),
/*8*/       write_board(Board),
/*9*/       sudoku(Board),
/*10*/      write_board(Board).

/*11*/  sudoku(Board):-
/*12*/      Board[1..9,1..9] :: 1..9,
/*13*/      (for(I,1,9), param(Board)
/*14*/      do
/*15*/            Row is Board[I,1..9],
/*16*/            alldifferent(Row),
/*17*/            Col is Board[1..9,I],
/*18*/            alldifferent(Col)
/*19*/      ),
/*20*/      (multifor([I,J],1,9,3), param(Board)
/*21*/      do
/*22*/            (multifor([K,L],0,2),
/*23*/            param(Board,I,J),
/*24*/            foreach(X,Square)
/*25*/            do
/*26*/                X is Board[I+K,J+L]
/*26*/            ),
/*27*/      alldifferent(Square)
/*28*/      ),
/*29*/      term_variables(Board, Variables),
/*30*/      labeling(Variables).

/*31*/  write_board(Board):-
/*32*/      (for(I,1,9), param(Board)
/*33*/      do
/*34*/            (for(J,1,9), param(Board,I)
/*34*/            do
```

[21] This is an FS-type problem.

```
/*35*/               X is Board[I,J],
/*36*/               (var(X) -> write("  _") ; printf("%2d", [X]))
/*37*/           ), nl
/*38*/     ), nl.

   problem(1, [](
    [](_, _, 2, _, 6, _, _, _, 3),
    [](_, _, _, 5, 9, _, _, 7, _),
    [](_, 6, _, _, _, 4, _, _, _),
    [](_, _, _, _, _, 1, _, 3, 7),
    [](_, 9, 7, _, _, _, 8, 1, _),
    [](4, 1, _, 7, _, _, _, _, _),
    [](_, _, _, 8, _, _, _, 9, _),
    [](_, 2, _, _, 7, 5, _, _, _),
    [](6, _, _, _, 4, _, 3, _, _))).

   problem(2, [](
    [](_, _, 5, _, 7, 4, _, 6, _),
    [](9, _, _, _, _, 3, _, _, _),
    [](_, _, 1, _, _, _, _, 3, 2),
    [](_, _, _, 9, _, _, _, _, 5),
    [](_, _, _, _, _, _, _, _, _),
    [](4, _, _, _, _, 6, _, _, _),
    [](8, 6, _, _, _, _, 7, _, _),
    [](_, _, _, 1, _, _, _, _, 8),
    [](_, 2, _, 8, 6, _, 5, _, _))).

   problem(3, [](
    [](_, _, _, _, _, 1, 2, _, _),
    [](_, _, _, _, _, _, 9, 6, _),
    [](_, _, _, 7, 4, 6, _, _, 8),
    [](_, 9, 3, _, 2, _, _, 1, _),
    [](_, 8, _, _, _, _, _, 9, _),
    [](_, 6, _, _, 5, _, 8, 2, _),
    [](1, _, _, 5, 6, 7, _, _, _),
    [](_, 3, 4, _, _, _, _, _, _),
    [](_, _, 6, 3, _, _, _, _, _))).
```

The solution of problem 3 gives:

```
  _  _  _  _  _  1  2  _  _
  _  _  _  _  _  _  9  6  _
  _  _  _  7  4  6  _  _  8
  _  9  3  _  2  _  _  1  _
  _  8  _  _  _  _  _  9  _
```

```
_ 6 _ _ 5 _ 8 2 _
1 _ _ 5 6 7 _ _ _
_ 3 4 _ _ _ _ _ _
_ _ 6 3 _ _ _ _ _

6 5 8 9 3 1 2 4 7
3 4 7 2 8 5 9 6 1
9 1 2 7 4 6 5 3 8
4 9 3 6 2 8 7 1 5
2 8 5 1 7 3 4 9 6
7 6 1 4 5 9 8 2 3
1 2 9 5 6 7 3 8 4
5 3 4 8 1 2 6 7 9
8 7 6 3 9 4 1 5 2
```

4.7.2 Queens for the last time

The program `4_21_queens_for_the_last_time.ecl`[22] illustrates the application of `fromto/4` to the queen placement problem:

```
/*1*/   :- lib(ic).
/*2*/   top:-
/*3*/       four_queens(4,_).

/*4*/   four_queens(N, Chessboard):-
/*5*/       length(Chessboard, N),
/*6*/       Chessboard:: 1..N,

/*7*/       (fromto(Chessboard,[Position_1|Next_positions],
                                Next_positions,[])
/*8*/       do
/*9*/            (foreach(Position_2, Next_positions),
/*10*/            param(Position_1),
/*11*/            count(Distance,1,_)
/*12*/            do
/*13*/                Position_2 #\= Position_1,
/*14*/                Position_2 - Position_1 #\= Distance,
/*15*/                Position_1 - Position_2 #\= Distance
/*16*/            )
/*17*/      ),
```

[22] This is an FS-type problem.

```
/*18*/      labeling(Chessboard),
/*19*/      write("Chessboard = "),writeln(Chessboard).
```

The message is:

```
      Chessboard = [2, 4, 1, 3]
      Chessboard = [3, 1, 4, 2]
```

4.7.3 Implicit domain declaration - lectures again

Consider again the lecture example from Section 2.4.10. It may be solved by an CLP program `4_22_lectures.ecl` with implicit domain declaration:

```
/*1*/  :- lib(ic).
/*2*/  top :-
/*3*/    Lectures =
              [
              lecture(1, _, _, _, _, _, _),
              lecture(2, _, _, _, _, _, _),
              lecture(3, _, _, _, _, _, _)
              ],

  % The implicit domain declaration from  line /*3*/ holds
  %  for all lecture() predicates from the Constraints[] list below:

  % From the definition of  'Lectures'  follows that  integers
  %  Andrew, Barbara and Christopher should be grounded to  values 1, 2, 3:

/*4*/    Constraints =
              [
              lecture(Andrew, "Andrew", _, _, _, _, _),
              lecture(Barbara, "Barbara",_,_, _, _, _),
              lecture(Christopher, "Christopher",_, _, _, _, _),

  % From the definition of  'Lectures'  follows that  integers
  %  Knowledge_Engineering, Econometric_Models and Artificial_Intelligence
  %  should be grounded to  values 1, 2, 3:
    lecture(Knowledge_Engineering, _, "Knowledge Engineering", _, _, _, _),
    lecture(Econometric_Models, _, "Econometric Models", _, _, _, _),
    lecture(Artificial_Intelligence, _, "Artificial Intelligence", _, _, _, _),

  % From the definition of  'Lectures'  follows that  integers
  %  Tuesday, Wednesday and Thursday should be grounded to  values 1, 2, 3:
              lecture(Tuesday, _, _, "Tuesday", _, _, _),
```

```
            lecture(Wednesday, _, _, "Wednesday", _, _, _),
            lecture(Thursday, _, _, "Thursday", _, _, _),

  % From the definition of  'Lectures'  follows that  integers
  %  H2_00, H3_45 and H5_30 should be grounded to  values 1, 2, 3:
            lecture(H2_00, _, _, _, "2:00 p.m.", _, _),
            lecture(H3_45, _, _, _, "3:45 p.m.", _, _),
            lecture(H5_30, _, _, _, "5:30 p.m.", _, _),

  % From the definition of  'Lectures'  follows that  integers
  %  R104, RD3 and RK2 should be grounded to  values 1, 2, 3:
            lecture(R104, _, _, _, _, "104", _),
            lecture(RD3, _, _, _, _, "D3", _),
            lecture(RK2, _, _, _, _, "K2", _),

  % From the definition of  'Lectures'  follows that  integers
  %  Paul, Jones and Smith should be grounded to  values 1, 2, 3:
            lecture(Paul, _, _, _, _, _, "Paul"),
            lecture(Jones, _, _, _, _, _, "Jones"),
            lecture(Smith, _, _, _, _, _, "Smith")  ],

% 1) Andrew will attend the lecture by Professor Paul:
/*5*/    Andrew #= Paul,

% 2) Tuesdays lecture does not start at 2:00 p.m:
/*6*/    Tuesday #\= H2_00,

% 3) The lecture on "Knowledge Engineering" does not start at 5:30 p.m:
/*7*/    Knowledge_Engineering #\= H5_30,

% 4) Thursdays lecture start at 3:45 p.m:
/*8*/    Thursday #= H3_45,

% 5) Christopher will attend the lecture on "Econometric Models":
/*9*/    Christopher #= Econometric_Models,

% 6) Barbara would like to attend the Tuesday lecture:
/*10*/   Barbara #= Tuesday,

% 7) The lecture on "Artificial Intelligence" is delivered in Room D3:
/*11*/   Artificial_Intelligence #= RD3,

%8) Wednesdays lectures are not delivered in Room 104:
/*12*/   Wednesday #\= R104,

% 9) Professor Smith is not delivering the lecture "Econometric Models":
/*13*/   Smith #\= Econometric_Models,
```

```
% 10) Professor Jones is not delivering his lecture in Room K2:
/*14*/   Jones #\= RK2,

/*14*/   grounding(Constraints, Lectures),
/*15*/   (foreach(Lecture, Lectures) do writeln(Lecture)),!.

 %   All elements of the Constraints[] list must be grounded to some
 %   values of the elements of the Lectures[] list:

/*16*/  grounding([],_).
/*17*/  grounding([H|T],Lectures) :-
/*18*/     member(H,Lectures),
/*19*/     grounding(T,Lectures).
```

The message displays the solution:

```
lecture(1, Andrew, Knowledge Engineering, Wednesday, 2:00 p.m., K2, Paul)
lecture(2, Barbara, Artificial Intelligence, Tuesday, 5:30 p.m., D3, Smith)
lecture(3, Christopher, Econometric Models, Thursday, 3:45 p.m., 104, Jones)
```

4.7.4 Stable marriages

The stable marriages problem is best described by a quote from [Wirth-75]:

> *"Assume that two disjoint sets A and B of equal cardinality n are given. Find a set of n pairs (a; b) such that a ε A and b ε B satisfy some constraints. Many different criteria for such pairs exist; one of them is the rule called 'stable marriage rule.' Assume that A is a set of men and B is a set of women. Each man and each woman has stated distinct preferences for their partners. If the n couples are chosen such that there exists a man and a woman who are not married, but who would prefer each other to their actual marriage partners, then the assignment is said to be unstable. If no such pair exists, it is called stable. This situation characterizes many related problems in which assignments have to be made according to preferences, such as, for example, the choice of a school by pupils, the choice of recruits by different branches of the armed services, etc. The example of marriages is particularly intuitive; note, however, that the stated list of preferences is invariant and does not change after a particular assignment has been made. This rule simplifies the*

problem, but it also represents a distortion of reality."
Niklaus Wirth, *"Algorithms + Data Structures = Programs."*

A marriage between a man m (from a set of men) and a woman w (from a set of women) is thus considered stable if and only if for any outsider (o):

1. whenever man m ranks another female outsider o(from a set of women) higher than his current wife w, the female outsider$o$ prefers her husband to m, and

2. whenever women w ranks another male outsider o(from a set of men) higher than her current husband m, the male outsider o prefers his wife to w.

This is illustrated by Figure 4.4.

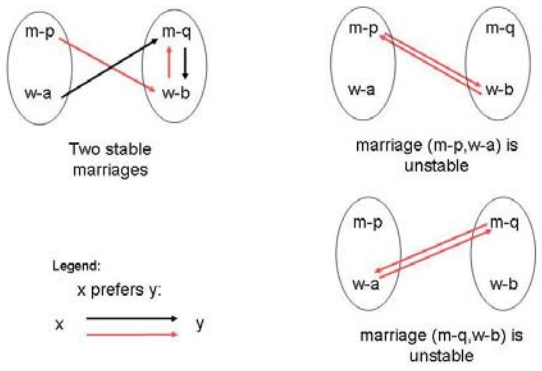


Figure 4.4: Examples of stable and unstable marriages

As can be seen, the marriage (`m-p, w-a`) is unstable because `m-p` prefers `w-b` more than his wife `w-a`, and `w-b` prefers `m-p` more than her husband `m-q`.

A set of marriages is stable if it does not contain unstable pairs.

Let us consider the following example with three women (`woman_1, woman_2` and `woman_3`) and three men (`man_1, man_2`

Women	High pref...Low pref		
woman_1	man_2	man_1	man_3
woman_2	man_3	man_2	man_1
woman_3	man_1	man_3	man_2

Table 4.4: Women are ranking men

Men	High pref...Low pref		
man_1	woman_2	woman_1	woman_3
man_2	woman_3	woman_2	woman_1
man_3	woman_1	woman_3	woman_2

Table 4.5: Men are ranking women

and `man_3r`), with rankings shown in Tables 4.4 and 4.5:

The ordering in rows of Tables 4.4 and 4.5 denotes the ranking of persons involved: e.g. the first choice of `woman_1` is `man_2` (`woman_1` likes `man_2` most), her second choice is `man_1`, and her last choice is `man_3`.

When given a married pair, let's say (`man_r-woman_a`) and `man_q-woman_b`), if `woman_a` prefers `man_q` more than her current husband `man_r`, and `man_r` prefers `woman_b` more than his current wife `woman_a` (i.e. their summary ranking numbers may be lowered by pairing `man_q-woman_a`) and `man_r-woman_b`), then the pair `man_r-woman_a` is called a *dissatisfied pair*. A set of marriages is said to be stable if there are no dissatisfied pairs.

Intuitively, there are three stable solutions to this problem:

1. Men get their first choice and ladies their third:
   `man_1-woman_2, man_2-woman_3, man_3-woman_1`,
   all pairs have summary ranking 4.

2. Women get their first choice and men their third:
   `man_2-woman_1, man_3-woman_2, man_1-woman_3`,
   all pairs have summary ranking 4.

3. All participants get their second choice:
   `man_1-woman_1, man_2-woman_2, man_3-woman_3`,
   all pairs have summary ranking 4.

All three are stable because instability requires both participants to be happier (i.e. having a lower summary choice) with an alternative match. The data shown in Tables 4.4 and 4.5 has to be (in a CLP program) put in a different way, like this:

```
   problem(1,
% 1=man_1, 2=man_2,   3=man_3:
 /*woman_1:*/   []([](2, 1, 3),   % rankByWomen:
 /*woman_2:*/      [](3, 2, 1),   % women are ranking men:  woman_1 likes
 /*woman_3:*/      [](1, 3, 2)),  % most man_2, next-man_1, and least-man_3

% 1=woman_1, 2=woman_2, 3=woman_3
 /*man_1:*/           []([](2, 1, 3),    % rankByMen:
 /*man_2:*/              [](3, 2, 1),    % men are ranking women: man_1 likes most
 /*man_3:*/              [](1, 3, 2))).  % woman_2, next-woman_1, and least-woman_3
```

The problem is solved by the rather sophisticated program `4_23_stable_marriage.ecl` due to Kjellerstrand ([Kjellerstrand-13]). It invokes a library called *Propria* that implements a generalized propagation technique. If it's not loaded there is an instantiation fault while reading data. The program is:

```
/*1*/  :-lib(ic).
/*2*/  :-lib(ic_global).
/*3*/  :-lib(ic_search).
/*4*/  :-lib(propia).
/*5*/  top :-
/*6*/          all_solutions(0),
/*7*/          all_solutions(1),
/*8*/          all_solutions(2),
/*9*/          all_solutions(3),
/*10*/            all_solutions(4),
/*11*/            all_solutions(5).

/*12*/ all_solutions(Problem) :-
/*13*/             printf("\nProblem %d:\n", [Problem]),
/*14*/             findall([Husband,Wife], stable_marriage(Problem,Husband,Wife),L),
    % On corresponding positions of lists
   % 'Husband' and 'Wife' are stable marriages:

/*15*/           (foreach([H,W], L) do
/*16*/             write("Husband: "),write(H),nl,
```

```
/*17*/            write("Wife   : "),write(W),nl,nl
/*18*/            ).

/*19*/ stable_marriage(Problem,Husband,Wife) :-
/*20*/            problem(Problem, RankByWomen,RankByMen),

/*21*/            dim(RankByWomen,[NumWomen,NumMen]),
/*22*/            dim(RankByMen,[NumMen,NumWomen]),

/*23*/            dim(Wife,[NumMen]),
/*24*/            Wife #:: 1..NumWomen,

/*25*/            dim(Husband,[NumWomen]),
/*26*/            Husband #:: 1..NumMen,

/*27*/            ic_global:alldifferent(Wife),
/*28*/            ic_global:alldifferent(Husband),

   % Rankings are tested on all possible pairings for men and for women:
   % if the fact that any man M who ranks an outsider woman O higher
   % than his wife implies that the outsider woman O prefers her husband to M:
/*29*/            ( for(M,1,NumMen) * for(O,1,NumWomen),
/*30*/            param(RankByMen,RankByWomen,Wife,Husband) do
/*31*/            (RankByMen[M,O] #< RankByMen[M, Wife[M]]) =>
/*32*/            (RankByWomen[O,Husband[O]] #< RankByWomen[O,M])
/*33*/            ),

   % and if the fact that any woman W who ranks an outsider male O higher
   % than her husband implies  that the outsider male O prefers his wife to W:
/*34*/            ( for(W,1,NumWomen) * for(O,1,NumMen),
/*35*/            param(RankByMen,RankByWomen,Wife,Husband) do
/*36*/            (RankByWomen[W,O] #< RankByWomen[W,Husband[W]]) =>
/*37*/            (RankByMen[O,Wife[O]] #< RankByMen[O,W])
/*38*/            ),
    % then the marriages are stable.

    % Husbands are paired with wifes for the same lists positions:
/*39*/            ( for(W,1,NumWomen), param(Husband, Wife) do
/*40*/            Wife[Husband[W]] #= W
/*41*/            ),

   % Wifes are paired with husbands for the same lists positions:
/*42*/            ( for(M,1,NumMen), param(Husband, Wife) do
/*43*/            Husband[Wife[M]] #= M
/*44*/            ),

   % flatten the list of lists [Wife,Husband] for labeling purposes:
/*45*/            term_variables([Wife,Husband],Vars),
```

```
/*46*/            labeling(Vars).

   problem(0,
   []([](1, 2),   % rankByWomen
      [](1, 2)),

   []([](2, 1),    % rankByMen
      [](2, 1))).
```

From [Wikipedia-13]:

```
   problem(1,
   []([](2, 1, 3),%  rankByWomen
      [](3, 2, 1),
      [](1, 3, 2)),

   []([](2, 1, 3), %  rankByMen
      [](3, 2, 1),
      [](1, 3, 2))).
```

From [van Hentenryck-99]:

```
   problem(2,
   []([](1, 2, 4, 3, 5),  % rankByWomen
       [](3, 5, 1, 2, 4),
       [](5, 4, 2, 1, 3),
       [](1, 3, 5, 4, 2),
       [](4, 2, 3, 5, 1)),

   []([](5, 1, 2, 4, 3),  % rankByMen
       [](4, 1, 3, 2, 5),
       [](5, 3, 2, 4, 1),
       [](1, 5, 4, 3, 2),
       [](4, 3, 2, 1, 5))).
```

From [Kjellerstrand-13]:

```
   problem(3,
   []([](7, 3, 8, 9, 6, 4, 2, 1, 5),  % rankByWomen
       [](5, 4, 8, 3, 1, 2, 6, 7, 9),
       [](4, 8, 3, 9, 7, 5, 6, 1, 2),
       [](9, 7, 4, 2, 5, 8, 3, 1, 6),
       [](2, 6, 4, 9, 8, 7, 5, 1, 3),
       [](2, 7, 8, 6, 5, 3, 4, 1, 9),
       [](1, 6, 2, 3, 8, 5, 4, 9, 7),
       [](5, 6, 9, 1, 2, 8, 4, 3, 7),
       [](6, 1, 4, 7, 5, 8, 3, 9, 2)),

   []([](3, 1, 5, 2, 8, 7, 6, 9, 4),  % rankByMen
       [](9, 4, 8, 1, 7, 6, 3, 2, 5),
       [](3, 1, 8, 9, 5, 4, 2, 6, 7),
       [](8, 7, 5, 3, 2, 6, 4, 9, 1),
```

```
      [](6, 9, 2, 5, 1, 4, 7, 3, 8),
      [](2, 4, 5, 1, 6, 8, 3, 9, 7),
      [](9, 3, 8, 2, 7, 5, 4, 6, 1),
      [](6, 3, 2, 1, 8, 4, 5, 9, 7),
      [](8, 2, 6, 4, 9, 1, 3, 7, 5))).
```

From [Hunt-13]:

```
problem(4,
[]([](1,2,3,4),% rankWomen
    [](4,3,2,1),
    [](1,2,3,4),
    [](3,4,1,2)),

[]([](1,2,3,4),% rankByMen
    [](2,1,3,4),
    [](1,4,3,2),
    [](4,3,1,2))).
```

From [Ahriz-13]:

```
problem(5,
[]([](1,5,4,6,2,3),
    [](4,1,5,2,6,3),
    [](6,4,2,1,5,3),
    [](1,5,2,4,3,6),
    [](4,2,1,5,6,3),
    [](2,6,3,5,1,4)),

[]([](1,4,2,5,6,3),
    [](3,4,6,1,5,2),
    [](1,6,4,2,3,5),
    [](6,5,3,4,2,1),
    [](3,1,2,4,5,6),
    [](2,3,1,6,5,4))).
```

The solutions are:

```
Problem 0:
Husband: [](2, 1)
Wife   : [](2, 1)

Problem 1:
Husband: [](2, 3, 1)
Wife   : [](3, 1, 2)

Husband: [](3, 1, 2)
Wife   : [](2, 3, 1)
```

```
Husband: [](1, 2, 3)
Wife   : [](1, 2, 3)

Problem 2:
Husband: [](4, 1, 2, 5, 3)
Wife   : [](2, 3, 5, 1, 4)

Husband: [](2, 1, 4, 5, 3)
Wife   : [](2, 1, 5, 3, 4)

Husband: [](2, 3, 4, 1, 5)
Wife   : [](4, 1, 2, 3, 5)

Problem 3:
Husband: [](7, 5, 9, 8, 3, 6, 1, 4, 2)
Wife   : [](7, 9, 5, 8, 2, 6, 1, 4, 3)

Husband: [](6, 5, 9, 8, 3, 7, 1, 4, 2)
Wife   : [](7, 9, 5, 8, 2, 1, 6, 4, 3)

Husband: [](6, 4, 9, 8, 3, 7, 1, 5, 2)
Wife   : [](7, 9, 5, 2, 8, 1, 6, 4, 3)

Husband: [](6, 1, 4, 8, 5, 9, 3, 2, 7)
Wife   : [](2, 8, 7, 3, 5, 1, 9, 4, 6)

Husband: [](6, 4, 1, 8, 5, 7, 3, 2, 9)
Wife   : [](3, 8, 7, 2, 5, 1, 6, 4, 9)

Husband: [](6, 1, 4, 8, 5, 7, 3, 2, 9)
Wife   : [](2, 8, 7, 3, 5, 1, 6, 4, 9)

Problem 4:
Husband: [](1, 4, 2, 3)
Wife   : [](1, 3, 4, 2)

Husband: [](1, 2, 4, 3)
Wife   : [](1, 2, 4, 3)

Problem 5:
Husband: [](1, 2, 6, 3, 5, 4)
Wife   : [](1, 2, 4, 6, 5, 3)

Husband: [](1, 2, 6, 3, 4, 5)
Wife   : [](1, 2, 4, 5, 6, 3)

Husband: [](1, 2, 4, 3, 6, 5)
Wife   : [](1, 2, 4, 3, 6, 5)
```

4.8 Feasible sequencing

Feasible sequencing aims at determining the order of elements from some set so as to fulfill *neighbourhood constraints*, i.e. constraints determining the position of each element with respect to the elements.

4.8.1 Car assembly line sequencing

This example is both an opportunity to present two important global built-in predicates and to show their application. The predicates are:

1. The `sequence_total/7` predicate, defined as:

```
sequence_total(+Min, +Max, +Low, +High, +K, +Vars, ++Values)
```

where number of values taken from the list of different integers `Values` is between a non-negative `Low` and positive `High` integer for all sequences of `K` integers from the list of integers `Vars`, and the total occurrence of each integer in `Vars` is between `Min` and `Max`

The "strangeness" of this predicate is due to the fact that it was custom-tailored for modelling some situation on car assembly lines.

2. The `occurrences/3` predicate, defined as:

```
occurrences(++Value, +List, ?N)
```

that is fulfilled if the value `Value` occurs `N` times in `List`.

Sequencing is the process of determining the precise order of some items, e.g. car bodies on a car assembly line to meet a given production order. ECL^iPS^e is - because of some special global constraint - well-suited for solving such problems. Consider the following example. In a car assembly line, car bodies are moving on conveyors through different work stations, each specialized for a particular job, such as installing the engine, installing the power seats, installing wheels etc. For each car entering a work station, a crew of assemblers from that

station moves with the car while performing their jobs. The speed of the assembly line is such as to allow the crews to finish their jobs while the car bodies are in their stations. E.g. if the installation of power seats takes 16 minutes and a new car body enters the assembly line every 4 minutes, then (assuming that each car needs a power seats), the station for power seats installation needs a capacity to handle $16/4 = 4$ car bodies, i.e. it has to be staffed by 4 power seats handling crews. However, because not each car requires a power seat, in order to save instrumentation and labour, the capacity of the power seats station may be smaller, e.g. the station may have only 3 crews to handle power seats. That means the station can cope with no more than 3 cars requiring power seats out of any sequence of 4 cars. In shorthand - the power seats station has a *capacity constraint* 3/4. Now its up to the assembly line scheduler to assure that the entire sequence of car bodies feed into the assembly line has no 4-bodies subsequences with more than 3 bodies requiring power seats. Consider the capacity requirements for four car models to be produced with five options as shown in Table 4.6:

Option	Capacity constraints	Models produced			
		1	2	3	4
Sunroof	3/5	-	×	×	-
CD changer	4/5	×	-	-	×
Automatic transmission	4/5	×	×	-	×
Power seats	3/4	×	×	×	-
Parking assistant	1/2	×	-	×	-
Number of cars required		30	30	30	30

Table 4.6: Capacity constraints for car assembly line: x - option required, - - option not required

The notion of capacity constraint is illustrated for the case of power seat workstation in Figure 4.5.

A solution (one of a large multitude of possible solutions) for the sequence of 120 car bodies feed into the assembly line so that the capacity constraints of all work stations are satisfied is determined by program `4_24_car_assembling.ecl` using two powerful global constraints: `occurrences/3` and `sequence_total/7`:

```
/*1*/   :- lib(ic).
/*2*/   :- lib(ic_global).
```

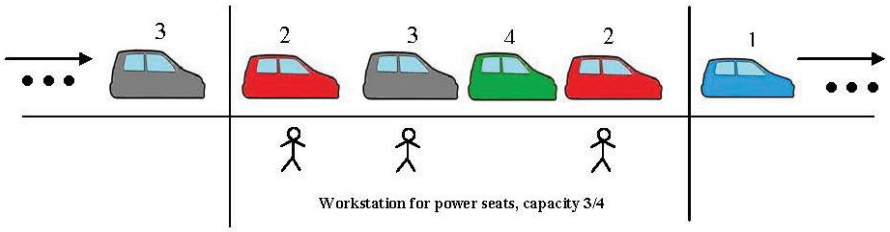


Figure 4.5: The meaning of workstation capacity constraints

```
/*3*/  top:-
/*4*/    length(L,120),
/*5*/    L::1..4,
 %    1 - Model 1, 2 - Model 2,  3 - Model 3, 4 - Model 4

 % Constrain numbers of produced models using global constraint 'occurrences/3':
 %          occurrences(++Value, +Vars, ?N)
 %  The integer 'Value' occurs 'N' times in integer list 'Vars'

         % The value 1 occurs  30 times in L:
/*6*/    occurrences(1, L, 30),

         % The value 2 occurs  30 times in L:
/*7*/    occurrences(2, L, 30),

         % The value 3 occurs  30 times in L:
/*8*/    occurrences(3, L, 30),

         % The value 4 occurs  30 times in L:
/*9*/    occurrences(4, L, 30),

 %  Constrain capacity of workstations using global constraint 'sequence_total/7':
 %          sequence_total(+Min, +Max, +Low, +High, +K, +Vars, ++Values)
 %  The number of integers taken from integer list 'Values' is between 'Low' and
 % 'High'  for all sequences of 'K' integers in  integer list 'Vars',
 %  and the total occurrence of each integer in 'Vars' is between 'Min' and 'Max'

 % Sunroofs - at least none and at most 3 of any consecutive 5 integers in L
 % are from list [2,3]; at least 60 and at most 60 integers in L are
 % from list [2,3]:
/*10*/   sequence_total(60, 60, 0, 3, 5, L, [2,3]),
```

```
 %   CD changer - at least none and at most 4 of any consecutive 5 integers in L
 %   are from list [1,3,4]; at least 90 and at most 90 integers in L are
 %   from list [1,3,4]:
/*11*/   sequence_total(90, 90, 0, 4,  5, L, [1,3,4]),

 %   Automatic transmission - at least none and at most 4 of any consecutive
 % 5 integers in L are from list [1,2,4]; at least 90 and at most 90 integers
 % in L are from list [1,2,4]:
/*12*/   sequence_total(90, 90,0, 4,  5, L, [1,2,4]),

 %   Power seats - at least none and at most 3 of any consecutive 4 integers
 % in L are from list [1,2,3]; at least 90 and at most 90 integers in L are
 %     from list [1,2,3]:
/*13*/   sequence_total( 90, 90, 0, 3,4, L, [1,2,3]),

 %   Parking assistant - at least none and at most 1 of any consecutive 2 integers
 %  in L are from list [1,3]; at least 60 and at most 60 integers in L are from
 %  list [1,3]:
/*14*/   sequence_total( 60, 60, 0, 1, 2, L, [1,3]),

/*15*/   labeling(L),
/*16*/   write_list(L).

/*17*/ write_list([H|T]):-
/*18*/    write(H),write(", "),
/*19*/    write_list(T).
/*20*/    write_list([_]).
```

The solution is:

```
   L=[
 1, 2, 1, 4, 3, 2, 1, 4, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1,
 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1,
 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1,
 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1,
 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1,
 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 3]
```

To check that it satisfies the power seats capacity constraint, one has to look at every subsequence of 4 cars, i.e.:

```
 1, 2, 1, 4,  It's O.K
 2, 1, 4, 3,  It's O.K
 1, 4, 3, 2,  It's O.K
```

and so on, for each capacity constraint. Quite a job! So it's worthwhile to

present the solution as the sequencing diagrams from Figure 4.6, where capacity constraints correspond to colour patterns.

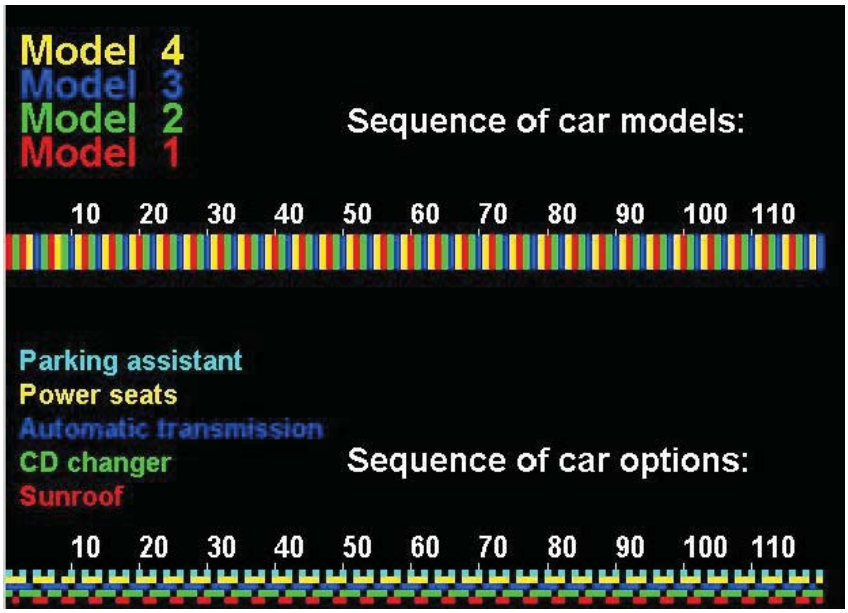


Figure 4.6: Car assembly line sequencing

4.8.2 Bob's Shish Kebab

This example is yet another opportunity to present the important built-in:

```
occurrences(++Value, +List, ?N)
```

that is fulfilled if the value `Value` occurs `N` times in `List`, and to show its usage for solving the *Bob's Shish Kebab* example that enjoys the reputation of being a tough one (a 5 star puzzle), see [Edmund-98]. It is befitting to end the series of rather simple FS-type problems, solved using global built-ins, with a more taxing problem. The *Bob's Shish Kebab* problem is as follows:

Bob and Patty invited their friends Javier and Marie over for a cookout. On the menu were grilled marinated beef cubes and four kinds of vegetables –

mushrooms, onions, peppers, and tomatoes – that were put onto skewers. The skewer that each person made had three beef cubes and one piece of three kinds of vegetables – each person disliked a different vegetable and omitted it from his or her skewer. The six pieces can be numbered 1 to 6 from the handle to the point of the skewer. Can you tell what item each person had in each position, provided that:

1. No kebab had two beef cubes right next to each other.
2. No one's beef cubes were in the same three positions as anyone else's.
3. One shish kebab's first three items (numbers 1, 2, and 3 respectively) were beef, pepper, and mushroom; this wasn't Javier's.
4. One skewer had beef cubes in positions 1, 3, and 5, and a tomato wedge in position 6.
5. Bob, who loves onions and included a chunk on his skewer, had other vegetables in both positions 4 and 5.
6. On the four kebabs, the items in position 5 were beef, mushroom, onion, and tomato.
7. Each onion chunk was immediately between two beef cubes.
8. No pepper was immediately between two beef cubes.
9. Marie can't stand mushroom and left them off her skewer.
10. At least two kebabs had the same vegetable in the same position at least once.

It contains a lot of negative conditions (i.e. conditions stating that something is not true) that may cause difficulties. A systematic way to handle them (its essence is to use predicates defining negated conditions) is presented by program `4_25_kebab.ecl`[23]:

```
/*1*/  :- lib(ic).
/*2*/  :-lib(ic_global).
 /*3*/  top:-

  %   Bi- element on position i of Bob's skewer
  %   Pi- item on position i of Pati's skewer
  %   Ji- element on position i of Javier's skewer
  %   Mi- element on position i of Marie's skewer
  %   1-mushroom, 2-pepper, 3-onion, 4-tomato, 5-beef

/*4*/       Bob=[B1,B2,B3,B4,B5,B6,B7],
/*5*/       Bob::1..5,
```

[23]This is an FS-type problem.

```
/*6*/       Patty=[P1,P2,P3,P4,P5,P6,P7],
/*7*/       Patty::1..5,
/*8*/       Javier=[J1,J2,J3,J4,J5,J6,J7],
/*9*/       Javier::1..5,
/*10*/      Marie=[M1,M2,M3,M4,M5,M6,M7],
/*11*/      Marie::1..5,

  %   Example: J3 = 5 means Javier had beef at the third position.

  %   Positions 7 on each skewers denote vegetables disliked
  %   by the corresponding person: each person disliked a
  %   different vegetable and omitted it from his or her skewer:
/*12*/      [B7,P7,J7,M7]::1..4,
/*13*/      ic_global: alldifferent([B7,P7,J7,M7]),

  %    Constraint 1 - No kebab had two beef cubes right next to each other:
/*14*/      constraint_1([Bob,Patty,Javier,Marie]),

  %    Constraint 2 - No one's beef cubes were in the same three positions
  %    as anyone else's:
/*15*/      constraint_2([Bob,Patty,Javier,Marie]),

  %    Constraint 3 - One shish kebab's first three items (numbers 1, 2,
  %    and 3 respectively) were beef, pepper, and mushroom; this wasn't Javier's.
/*16*/      constraint_3(Bob,Patty,Marie),

  %    Constraint 4 - One skewer had beef cubes in positions 1, 3, and 5, and
  %    a tomato wedge in position 6.
/*17*/      constraint_4(Bob,Patty,Javier,Marie),

  %    Constraint 5 -  Bob, who loves onions and included a chunk
  %    on his skewer,had other vegetables in both positions 4 and 5:
/*18*/      constraint_5([_,_,_,B4,B5,_,_]),

  %    Constraint 6 - On the four kebabs, the items in position 5 were beef,
  %    mushroom, onion, and tomato:
/*19*/      constraint_6(Bob,Patty,Javier,Marie),

  %    Constraint 7 - Each onion chunk was immediately between two beef cubes
/*20*/      constraint_7([Bob,Patty,Javier,Marie]),

  %    Constraint 8 - No pepper was immediately between two beef cubes:
/*21*/      constraint_8([Bob,Patty,Javier,Marie]),

  %    Constraint 9 - Marie can't stand mushroom and left them off her skewer:
/*22*/      constraint_9(Marie),

  %    Constraint 10 - At least two kebabs had the same vegetable in the
  %    same position at least once:
```

```
/*23*/      constraint_10([Bob,Patty,Javier,Marie]),

  %     Each skewer has 3 beef cubes:
/*24*/      occurrences(5, [B1,B2,B3,B4,B5,B6], 3),
/*25*/      occurrences(5, [P1,P2,P3,P4,P5,P6], 3),
/*26*/      occurrences(5, [J1,J2,J3,J4,J5,J6], 3),
/*27*/      occurrences(5, [M1,M2,M3,M4,M5,M6], 3),

  %     Each skewer has one piece of three kinds of vegetables,
  %    the fourth vegetable rejected:
/*28*/      occurrences(1, [B1,B2,B3,B4,B5,B6,B7], 1),
/*29*/      occurrences(1, [P1,P2,P3,P4,P5,P6,P7], 1),
/*30*/      occurrences(1, [J1,J2,J3,J4,J5,J6,J7], 1),
/*31*/      occurrences(1, [M1,M2,M3,M4,M5,M6,M7], 1),

/*32*/      occurrences(2, [B1,B2,B3,B4,B5,B6,B7], 1),
/*33*/      occurrences(2, [P1,P2,P3,P4,P5,P6,P7], 1),
/*34*/      occurrences(2, [J1,J2,J3,J4,J5,J6,J7], 1),
/*35*/      occurrences(2, [M1,M2,M3,M4,M5,M6,M7], 1),

/*36*/      occurrences(3, [B1,B2,B3,B4,B5,B6,B7], 1),
/*37*/      occurrences(3, [P1,P2,P3,P4,P5,P6,P7], 1),
/*38*/      occurrences(3, [J1,J2,J3,J4,J5,J6,J7], 1),
/*39*/      occurrences(3, [M1,M2,M3,M4,M5,M6,M7], 1),

/*40*/      occurrences(4, [B1,B2,B3,B4,B5,B6,B7], 1),
/*41*/      occurrences(4, [P1,P2,P3,P4,P5,P6,P7], 1),
/*42*/      occurrences(4, [J1,J2,J3,J4,J5,J6,J7], 1),
/*43*/      occurrences(4, [M1,M2,M3,M4,M5,M6,M7], 1),

/*44*/      labeling([B1,B2,B3,B4,B5,B6,B7,P1,P2,P3,P4,P5,P6,P7,
                J1,J2,J3,J4,J5,J6,J7,M1,M2,M3,M4,M5,M6,M7]),

/*45*/      write("Bob's skewer:    "),translate(B1),write("  "),translate(B2),
                write("  "), translate(B3),write("  "), translate(B4),
                write("  "),translate(B5),write("  "), translate(B6),nl,

/*46*/      write("Patty's skewer:  "),translate(P1),write("  "),translate(P2),
                write("  "),translate(P3),write("  "), translate(P4),
                write("  "),translate(P5),write("  "), translate(P6),nl,

/*47*/      write("Javier's skewer: "),translate(J1),write("  "),translate(J2),
                write("  "),translate(J3),write("  "),translate(J4),
                write("  "),translate(J5),write("  "),translate(J6),nl,

/*48*/      write("Marie's skewer:  "), translate(M1),write("  "),translate(M2),
                write("  "),translate(M3),write("  "), translate(M4),
                write("  "), translate(M5),write("  "), translate(M6),nl,nl.
```

```
/*49*/  translate(1):-write("mushroom").
/*50*/  translate(2):-write("pepper  ").
/*51*/  translate(3):-write("onion   ").
/*52*/  translate(4):-write("tomato  ").
/*53*/  translate(5):-write("beef    ").

  %    Constraint 1 - No kebab had two beef cubes right next to each other:[24]
/*54*/  constraint_1([H|T]):-
/*55*/      check_1(H),
/*56*/      constraint_1(T).
/*57*/  constraint_1([]).

/*58*/  check_1([A,B,C,D,E,F,_]):-
/*59*/      ~two_beef_cubes_next_to_each_other(A,B),
/*60*/      ~two_beef_cubes_next_to_each_other(B,C),
/*61*/      ~two_beef_cubes_next_to_each_other(C,D),
/*62*/      ~two_beef_cubes_next_to_each_other(D,E),
/*63*/      ~two_beef_cubes_next_to_each_other(E,F).

/*64*/  two_beef_cubes_next_to_each_other(X,Y):-
/*65*/      X#=5,Y#=5.

  %    Constraint 2 - No one's beef cubes were in the same three positions
  %    as anyone else's:
/*66*/  constraint_2([Bob,Patty,Javier,Marie]):-
/*67*/      check_2(Bob,Patty),check_2(Bob,Javier),check_2(Bob,Marie),
/*68*/      check_2(Patty,Javier),check_2(Patty,Marie),check_2(Javier,Marie).

/*69*/  check_2([B1,B2,B3,B4,B5,B6,_],[P1,P2,P3,P4,P5,P6,_]):-
/*70*/      ~in_the_same_positions(B1,B3,B5,P1,P3,P5),
/*71*/      ~in_the_same_positions(B1,B3,B6,P1,P3,P6),
/*72*/      ~in_the_same_positions(B1,B4,B6,P1,P4,P6),
/*73*/      ~in_the_same_positions(B2,B4,B6,P2,P4,P6).

/*74*/  in_the_same_positions(X1,X2,X3,Y1,Y2,Y3):-
/*75*/      X1#=5,
/*76*/      X2#=5,
/*77*/      X3#=5,
/*78*/      Y1#=X1,
/*79*/      Y2#=X2,
/*80*/      Y3#=X3.

  %    Constraint 3 - One shish kebab's first three items (numbers 1, 2,
  %    and 3 respectively) were beef, pepper, and mushroom; this wasn't Javier's:
/*81*/  constraint_3([B1,B2,B3,_,_,_,_],[P1,P2,P3,_,_,_,_],[M1,M2,M3,_,_,_,_]):-
/*82*/      (
```

[24] The Reader will excuse the Author for repeating the constraint statements, but this non-redundancy makes for easier grasping the programs essence.

```
/*83*/      (B1#=5, B2#=2, B3#=1);
/*84*/      (P1#=5, P2#=2, P3#=1);
/*85*/      (M1#=5, M2#=2, M3#=1)
/*86*/      ).

  %    Constraint 4 - One skewer had beef cubes in positions
  %    1, 3, and 5, and a tomato wedge in position 6.
/*87*/  constraint_4(Bob,Patty,Javier,Marie):-
/*88*/      (had_beef_cubes_in_positions_1_3_5_6(Bob);
/*89*/      had_beef_cubes_in_positions_1_3_5_6(Patty);
/*90*/      had_beef_cubes_in_positions_1_3_5_6(Javier);
/*91*/      had_beef_cubes_in_positions_1_3_5_6(Marie)).

/*92*/  had_beef_cubes_in_positions_1_3_5_6([X1,_,X3,_,X5,X6,_]):-
/*93*/      X1#=5, X3#=5, X5#=5, X6#=4.

  %    Constraint 5 - Bob, who loves onions and included a chunk
  %    on his skewer, had other vegetables in both positions 4 and 5:
/*94*/  constraint_5([_,_,_,B4,B5,_,_]):-
/*95*/      B4#\=5, B5#\=5, B4#\=B5, B4#\=3, B5#\=3.

  %    Constraint 6 - On the four kebabs, the items in position 5 were
  %    beef, mushroom, onion, and tomato:
/*96*/  constraint_6([_,_,_,_,B5,_,_],[_,_,_,_,P5,_,_],[_,_,_,_,J5,_,_],
                       [_,_,_,_,M5,_,_]):-
/*97*/      (
/*98*/      (B5#=5; P5#=5; J5#=5; M5#=5),
/*99*/      (B5#=1; P5#=1; J5#=1; M5#=1),
/*100*/     (B5#=3; P5#=3; J5#=3; M5#=3),
/*101*/     (B5#=4; P5#=4; J5#=4; M5#=4)
/*102*/     ).

  %    Constraint 7 - Each onion chunk was immediately between two beef cubes:
/*103*/ constraint_7([Bob,Patty,Javier,Marie]):-
/*104*/     check_7(Bob),check_7(Patty),
/*105*/     check_7(Javier),check_7(Marie).

/*106*/ check_7([A,B,C,_,_,_,_]):-
/*107*/     A#=5,B#=3,C#=5.
/*108*/ check_7([_,B,C,D,_,_,_]):-
/*109*/     B#=5,C#=3,D#=5.
/*110*/ check_7([_,_,C,D,E,_,_]):-
/*111*/     C#=5,D#=3,E#=5.
/*112*/ check_7([_,_,_,D,E,F,_]):-
/*113*/     D#=5,E#=3,F#=5.
/*114*/ check_7([_,_,_,_,_,_,G]):-
/*115*/     G#=3.

  %    Constraint 8 - No pepper was immediately between two beef cubes:
```

```
/*116*/ constraint_8([Bob,Patty,Javier,Marie]):-
/*117*/     check_8(Bob),check_8(Patty),
/*118*/     check_8(Javier),check_8(Marie).

/*119*/ check_8([A,B,C,D,E,F,_]):-
/*120*/     ~pepper_was_between_two_beef_cubes(A,B,C),
/*121*/     ~pepper_was_between_two_beef_cubes(B,C,D),
/*122*/     ~pepper_was_between_two_beef_cubes(C,D,E),
/*123*/     ~pepper_was_between_two_beef_cubes(D,E,F).

/*124*/ pepper_was_between_two_beef_cubes(X,Y,Z):-
/*125*/     X#=5,Y#=2,Z#=5.

  %    Constraint 9 - Marie can't stand mushroom and left them off her skewer:
/*126*/ constraint_9([M1,M2,M3,M4,M5,M6,M7]):-
/*127*/     M7#=1,
/*128*/     M1#\=1, M2#\=1, M3#\=1,
/*129*/     M4#\=1, M5#\=1, M6#\=1.

  %    Constraint 10 - At least two kebabs had the same
  %    vegetable in the same position at least once:
/*130*/  constraint_10([Bob,Patty,Javier,Marie]):-
/*131*/     (
/*132*/     check_10(Bob,Patty);
/*133*/     check_10(Bob,Javier);
/*134*/     check_10(Bob,Marie);
/*135*/     check_10(Patty,Javier);
/*136*/     check_10(Patty,Marie);
/*137*/     check_10(Javier,Marie)
/*138*/     ).

/*139*/ check_10([X1,X2,X3,X4,X5,X6,_],[Y1,Y2,Y3,Y4,Y5,Y6,_]):-
/*140*/     (
/*141*/     (X1#\=5,X1#=Y1);
/*142*/     (X2#\=5,X2#=Y2);
/*143*/     (X3#\=5,X3#=Y3);
/*144*/     (X4#\=5,X4#=Y4);
/*145*/     (X5#\=5,X5#=Y5);
/*146*/     (X6#\=5,X6#=Y6)
/*147*/     ).
```

The message displays a unique solution:

Bob's skewer:	beef	onion	beef	pepper	mushroom	beef
Patty's skewer:	beef	pepper	mushroom	beef	tomato	beef
Javier' skewer:	beef	onion	beef	mushroom	beef	tomato
Marie's skewer:	pepper	beef	tomato	beef	onion	beef

4.8.3 Dinner calamity

Sometimes variables have a "cyclic" nature, like days in a week, months in a year, places around a circle. Some care is needed to handle them, as illustrated by the following example.

Mr and Mrs Davis invited their friends, Mr and Mrs Astor, Mr and Mrs Blake, Mr and Mrs Crane for a dinner served on an elegant retro styled hexagonal table. However, their nice conversation unexpectedly turned sour because some fundamental political differences have emerged. As the result of a heating discussion:
1) Mrs Astor was insulted by Mr Blake, who sat next to her.
2) Mr Blake was insulted by Mrs Crane, who sat opposite him.
3) Mrs Blake was insulted by Mrs Astor, who set opposite her.
4) The hostess (Mrs Davis) was insulted by the only person to sit between two men.
Knowing additionally that:
5)The hostess was the only person to sit between each of a married couple, and
6)Mrs Davis sat opposite to Mr Davis,
we have to determine who was sitting where and who insulted the hostess. The problem is solved by program `4_26_dinner_calamity.ecl`[25]:

```
/*1*/   :- lib(ic).
/*2*/   top :-
/*3*/       Places = [MrsAstor, MrAstor, MrBlake, MrsBlake,
                        MrsCrane, MrCrane, MrsDavis, MrDavis],
/*4*/       Places :: 1..8,
 % The places are numbered as shown in Figure \ref{Fig.4.11}.

 % Meaning of variables: if e.g. Mr Astor = 7, then Mr Astor is sitting on place 7
 % The occupant of one place may be fixed:
/*5*/       MrsAstor = 1,

 % Any person is occupying only one place:
/*6*/       alldifferent(Places),

 % 1) Mrs Astor was insulted by Mr Blake,
 %    who sat next to her on her left:
```

[25]This is an FS-type problem.

```
/*7*/       MrBlake = 2,

 % 2) Mr Blake was insulted by Mrs Crane, who sat opposite him:
/*8*/       opposite(2,MrsCrane),

 % 3) Mrs Blake sat opposite to Mrs Astor
/*9*/       opposite(MrsBlake,1),

 % 4) The hostess  was insulted by the only person to sit between two men.
/*10*/      (member(Insulter,[MrsBlake,MrsCrane,MrCrane]),
                 in_between(MrAstor,Insulter,2);
/*11*/       member(Insulter,[MrsBlake,MrsCrane]),
                 in_between(MrAstor,Insulter,MrCrane);
/*12*/       member(Insulter,[MrsBlake,MrsCrane,MrCrane, MrsDavis]),
                 in_between(MrAstor,Insulter,MrDavis);
/*13*/       member(Insulter,[MrAstor, MrsBlake, MrsCrane,MrDavis]),
                 in_between(2,Insulter,MrCrane);
/*14*/       member(Insulter,[MrAstor, MrsBlake,  MrsCrane, MrCrane]),
                 in_between(2,Insulter,MrDavis);
/*15*/       member(Insulter,[MrAstor, MrsBlake,  MrsCrane]),
                 in_between(MrCrane,Insulter,MrDavis)),

 % 5) The hostess (Mrs Davis) was the only person
 % to sit between each of a married couple
/*16*/      (in_between(MrsBlake, MrsDavis, 2);
/*17*/       in_between(1, MrsDavis, MrAstor);
/*18*/       in_between(MrsCrane, MrsDavis, MrCrane)),

 % 6) Mrs Davis sat opposite to Mr Davis:
/*196*/      opposite(MrsDavis,MrDavis),

/*20*/      labeling([MrsAstor, MrAstor, MrBlake, MrsBlake,
                    MrsCrane, MrCrane, MrsDavis, MrDavis]),

/*21*/      writeln([MrsAstor, MrAstor, MrBlake, MrsBlake,
                     MrsCrane, MrCrane, MrsDavis, MrDavis]),
/*22*/      List_names=["Mrs Astor","Mr Astor","Mr Blake","Mrs Blake",
                    "Mrs Crane","Mr Crane","Mrs Davis","Mr Davis"],
/*23*/      List_positions=[MrsAstor, MrAstor, MrBlake, MrsBlake,
                     MrsCrane, MrCrane, MrsDavis, MrDavis],
/*24*/      get_insulter(List_names,List_positions,Insulter,Insulter_name),

/*25*/      write("Mrs Astor was sitting at place "), write("1"),writeln("."),
/*26*/      write("Mr Astor was sitting at place "), write(MrAstor),writeln("."),
/*27*/      write("Mrs Blake was sitting at place "), write(MrsBlake),writeln("."),
/*28*/      write("Mr Blake was sitting at place "), write("2"),writeln("."),
/*29*/      write("Mrs Crane was sitting at place "), write(MrsCrane),writeln("."),
/*30*/      write("Mr Crane was sitting at place "), write(MrCrane),writeln("."),
/*31*/      write("Mrs Davis was sitting at place "), write(MrsDavis),writeln("."),
```

```
/*32*/      write("Mr Davis was sitting at place "), write(MrDavis),writeln("."),
/*33*/      writeln("The hostess (Mrs Davis) was insulted by the "),
/*34*/      write("person sitting at place "),write(Insulter), write(", who was "),
/*35*/      write(Insulter_name),writeln("."),nl.

/*36*/   next_to(A,B):-
/*37*/      B #= A + 1;
/*38*/      A #= B + 1.

/*39*/   next_to(8,1).
/*40*/   next_to(1,8).

/*41*/   in_between(A,X,B):-
/*42*/      X #= A + 1,
/*43*/      X #= B - 1;
/*44*/      X #= A - 1,
/*45*/      X #= B + 1.

/*46*/   in_between(7,8,1).
/*47*/   in_between(1,8,7).
/*48*/   in_between(8,1,2).
/*49*/   in_between(2,1,8).

/*50*/   opposite(A,B):-
/*51*/      B #= A + 4;
/*52*/      A #= B + 4.

/*53*/   get_insulter([_|T_names],[H_position|T_position],X,Insulter):-
/*54*/      not(X = H_position),
/*55*/      get_insulter(T_names,T_position,X,Insulter).
/*56*/   get_insulter([H_names|_],[H_position|_],X,Insulter):-
/*57*/      X = H_position,
/*58*/      Insulter = H_names.
```

The solution is:

```
[1, 7, 2, 5, 6, 3, 8, 4]
Mrs Astor was sitting at place 1.
Mr Astor was sitting at place 7.
Mrs Blake was sitting at place 5.
Mr Blake was sitting at place 2.
Mrs Crane was sitting at place 6.
Mr Crane was sitting at place 3.
Mrs Davis was sitting at place 8.
Mr Davis was sitting at place 4.
```

```
The hostess (Mrs Davis) was insulted by the person
sitting at place 3 who was Mr Crane.
```

It is depicted on Figure 4.7.

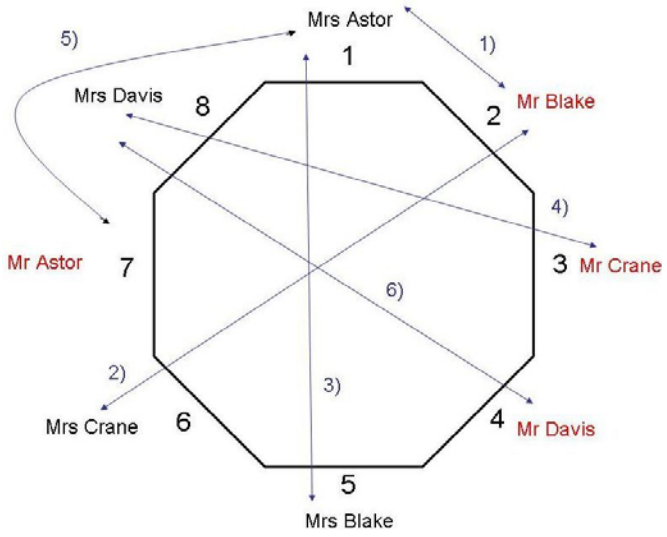


Figure 4.7: Dinner calamity solution

4.9 Exercises

Stones of Heaven [26]

Wan Li, a dealer in Chinese antiques and artifacts, had an excellent month recently when he made sales to four customers from around the world – Finland, Italy, Japan and United States – who were willing and able to pay very good prices. The four items were rare jade figurines (a belt buckle, dragon, grasshopper and horse), each carved from a different color of jade (dark green, light green, red and white). Each piece dates from a different Chinese dynasty (Ching, Ming, Sung and Tang). Write a program to

[26] This exercise is from http://brownbuffalo.sourceforge.net/

match each figurine with its color and dynasty, and give the home country of each buyer, if: 1. The rare white dragon (which the American did'nt buy) did'nt come from the Sung dynasty. 2. The exquisite belt buckle (which was'nt any shade of green) was created in 618 A.D. for an emperor of the Tang dynasty. 3. Three of the figurines were: the one bought by the Finn (which was'nt the dragon), the one from the Ching dynasty (which did'nt go to the buyer from Japan) and the light green object (which was'nt the horse). 4. The American decided against both the grasshopper and the piece from the Sung dynasty, neither of which she felt would match her home decor. Determine: Item – Color – Dynasty – Country of buyer.

Lectures [27]

Last week at school was made varied by a series of lectures, one each day (Monday through Friday), in the auditorium. None of the lectures was particularly interesting (on choosing a college, physical hygiene, modern art, nutrition, and study habits), but the students figured that anything that got them out of fourth period was okay. The lecturers were two women named Alice and Bernadette, and three men named Charles, Duane, and Eddie; last names were Felicidad, Garber, Haller, Itakura, and Jeffreys. Write a program to find each day's lecturer and subject, provided: 1. Alice lectured on Monday. 2. Charles's lecture on physical hygiene wasn't given on Friday. 3. Dietician Jeffreys gave the lecture on nutrition. 4. A man gave the lecture on modern art. 5. Ms. Itakura and the lecturer on proper study habits spoke on consecutive days, in one order or the other. 6. Haller gave a lecture sometime after Eddie did. 7. Duane Felicidad gave his lecture sometime before the modern art lecture.

City council meeting

At the last meeting of the local city council, each member (Mr. Akerman, Ms. Baird, Mr. Chatham, Ms. Duval, and Mr. Etting) had to vote on five motions, number 1 to 5 in the clues below. Write a program to determine how each one voted on each motion, provided that:

1. Each motion got a different number of yes votes.
2. In all, the five motions got three more yes votes than no votes.
3. No two council members voted the same way on all five motions.
4. The two women disagreed in their voting more often than they agreed.
5. Mr. Chatham never made two yes votes on consecutive motions.
6. Mr. Akerman and Ms. Baird both voted in favor of motion 4.

[27] This exercise is from http://www.f1compiler.com/default.html

7. Motion 1 received two more yes votes than motion 2 did.
8. Motion 3 received twice as many yes votes as motion 4 did.

DJ contest

During a recent music festival, four DJs entered the mixing contest. Each wore a number, either 1, 2, 3 or 4 and their decks were different colors. DJ Skinf Lint came first, and only one DJ wore the same number as the position he finished in. DJ Slam Dunk wore number 1. The DJ who wore number 2 had a red deck and DJ Jam Jar didn't have a yellow deck. The DJ who came last had a blue deck. DJ Park'n Ride beat DJ Slam Dunk. The DJ who wore number 1 had a green deck and the DJ who came second wore number 3. Can you determine who came where, which number they wore and the color of their deck?

A knight, a knave and a spy

There are three people (Alex, Brook and Cody), one of whom is a knight, one a knave, and one a spy[28]. The knight always tells the truth, the knave always lies, and the spy can either lie or tell the truth. Alex says: "Cody is a knave." Brook says: "Alex is a knight." Cody says: "I am the spy." Who is the knight, who the knave, and who the spy?

Sum

Write a program which replaces all the letters with the respective digits in such a way that the following sum is correct:

```
    AND
     TO
    ALL
      A
   GOOD
 -------
  NIGHT
```

The same letters in this sum mean the same digit.

Magic square

Consider the Magic Square of order three:
A B C
D E F
G H I

[28] This exercise is from http://www.mathsisfun.com/puzzles.

Write a program for the following pattern of non-zero digits to be instantiated to add up to the same sum along each row, column and diagonal.

Books

Eight married couples meet to land one another some books[29]. Couples have the same surname, employment and a car. Eight couple has a favorite color. Furthermore we know the following facts:

(1) Danielle Black and her husband work as Shop-Assistants. (2) The book "The Death of the West" was brought by a couple who drive a Fiat and love the color red. (3) Owen and his wife Victoria like the color brown. (4) Stan Horricks and his wife Hannah like the color white. (5) Jenny Smith and her husband work as Warehouse Managers and they drive a Ford. (6) Monica and her husband Alexander borrowed the book "Economy in One Lesson". (7) Mathew and his wife like the color pink and brought the book "Archipelag Gulag". (8) Irene and her husband Oto work as Accountants. (9) The book "The Fatal Conceit" was borrowed by a couple driving a Chrysler. (10) The Cermaks are both Ticket-Collectors who brought the book "The Art of Worldly Wisdom". (11) Mr and Mrs Kuril are both Doctors who borrowed the book "Atlas Shrugged". (12) Paul and his wife like the color green. (13) Veronica Dvorak and her husband like the color blue. (14) Rick and his wife brought the book "Atlas Shrugged" and they drive a Volkswagen. (15) One couple brought the book "The Oxford Book of Humorouse Prose" and borrowed the book "Archipelag Gulag". (16) The couple who drive a Toyota, love the color violet. (17) The couple who work as Teachers borrowed the book "The Oxford Book of Humorouse Prose". (18) The couple who work as Agriculturalists drive a Moskvic. (19) Pamela and her husband drive a Renault and brought the book "Economy in One Lesson". (20) Pamela and her husband borrowed the book that Mr and Mrs Zajac brought. (21) Robert and his wife like the color yellow and borrowed the book "The Enlarged Devil's Dictionary". (22) Mr and Mrs Swain work as Shoppers. (23) "The Enlarged Devil's Dictionary" was brought by a couple driving a Audi.

Write a program to determine who likes violet and to find out everything about everyone from this.

Dinner [30]

Last weekend, five friends gathered for dinner at their favorite steak and

[29] This exercise is from http://www.mathsisfun.com/puzzles

[30] This exercise is from http://brownbuffalo.sourceforge.net/

seafood restaurant. Each friend (two men named George and Oliver, and three women named Colleen, Patti, and Theresa) ordered a different main courses (crab, filet mignon, ribs, shrimp, or sirloin steak), and a different type of potatoes (baked, French-fried, lyonnaise, mashed or scalloped). To wash down his or her meal, each friend selected a different beverage (ginger ale, iced tea, lemonade, root beer, or water). From the following clues, can you match each friend with his or her surname (two of which are Gold and Orlando), main course, side dish, and beverage? 1) The only person with the same first-name and last-name initials ordered the ribs. 2) The one surnamed Petroski and the person who had the shrimp are the person who had the lyonnaise potatoes and the one who ordered the root beer, in some order. 3) The one who selected the filet mignon didn't have the lemonade. 4) The one who had the scalloped potatoes (which didn't come with the sirloin steak) didn't drink the water. 5) The first-name initial of the one who had root beer is the same as George's last-name initial. 6) Theresa didn't order the water. 7) The ones who chose the shrimp and the baked potato are of opposite gender. 8) The first-name initial of the woman who ordered the crab is the same as the last-name initial of the person who chose the mashed potatoes. 9) The first-name initial of the person who ordered the lemonade is the same as the last-name initial of the one who ordered lyonnaise potatoes. 10) The one surnamed Chiasson (who isn't Patti) didn't order French-fried or lyonnaise potatoes. 11) The one surnames Truang (who didn't order French-fried or mashed potatoes) didn't choose the ginger ale. 12) Colleen ordered either the filet mignon or the sirloin steak.

Write a program to determine: First name - Last name - Main course - Side dish - Beverage.

Soup Selections

Each of six friends who met in cooking school is now an established chef at a different, notable restaurant in the area. Every few weeks, the friends like to get together to trade secrets of their field and share some of their favorite creations. This past Tuesday night, each chef arrived at the group's favorite gathering spot with a different kind of soup that he or she had prepared for the evening's taste-test. From the following information, write a program to match the full name of each chef (one surname is Earle) with his or her seat (labeled one through six in the illustration) at the table at which the group gathered and determine the restaurant where each works and the type of soup that he or she prepared?

1. Gloria (who works for either the Apple Orchard Inn or Hennigan's Place) prepared either the French onion or split pea soup. 2. The one who made the minestrone sat in a lower-numbered seat than Marvin. 3. The one surnamed Anderson sat directly across from the chef who works at Michel's Cafe. 4. Marvin and the one who prepared the asparagus soup are the one who sat in seat five and the person who sat directly across from the chef who made the chicken noodle soup, in some order. 5. Norville sat next to the one who cooks for the Country Kitchen. 6. Quincy and the chef who works for the Village Smorgasbord are the one surnamed Dugan and a person who didn't sit directly across from the chef who made the asparagus soup, in some order. 7. The chef who works at the Pine Cove Restaurant and the chef who sat in seat four are the one surnamed Anderson and someone who didn't prepare the minestrone, in some order. 8. The one surnamed Burns (who works for the Apple Orchard Inn) didn't prepare the split pea soup. 9. The six chefs are Jenna, the chef surnamed Dugan, the person who works for the Pine Cove Restaurant, the person who prepared the clam chowder, the chef who sat in seat three, and the chef who sat directly across from the one surnamed Dugan. 10. Isabel and the one surnamed Friedman are the chef who works at Michel's Cafe and the one who made the chicken noodle soup, in some order. 11. Marvin didn't prepare the clam chowder. 12. Jenna (who sat in an odd-numbered seat) sat next to the one surnamed Caruso. 13. The chef who works for the Pine Cove Restaurant didn't occupy chair number six.

Killer Sudoku [31]

Write a program to solve the Killer Sudoku from Figure 4.8a: The objective is to fill the grid with numbers from 1 to 9 in a way that the following conditions are met:

- Each row, column, and nonet[32] contains each number exactly once.
- The sum of all numbers in a cage must match the small number printed in its corner.
- No number appears more than once in a cage. The solution of Killer Sudoku is given by Figure 4.8b).

[31] This exercise is from http://en.wikipedia.org/wiki/Killer sudoku

[32] A 3 x 3 grid of cells, as outlined by the bolder lines in the diagram

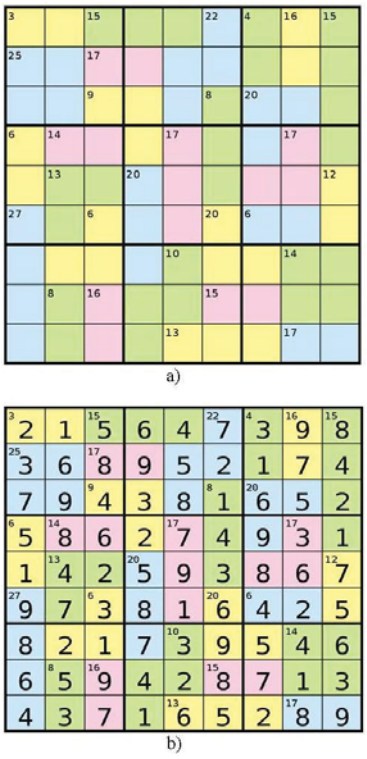


Figure 4.8: Killer Sudoku problem a) and solution b)

Pi-Day Sudoku [33]

Write a program to solve the Pi-Day Sudoku from Figure 4.9a). Each row, column, and jigsaw region must contain exactly the first twelve digits of pi, including repeats: 3.14159265358. Notice that each region will contain two 1's, two 3's, three 5's, and no 7's. The solution of Pi-Day Sudoku is given by Figure 4.9b).

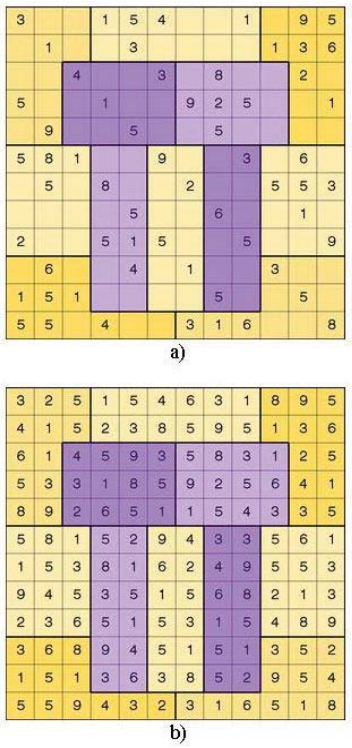


Figure 4.9: Pi-Day Sudoku problem a) and solution b)

[33] This exercise is from http://www.brainfreezepuzzles.com/main/piday2008.html

Chapter 5

CLP with elementary constraints for optimal solutions

5.1 General optimization approaches

The origin of combinatorial optimization may be traced to *Operation Research* (OR). There a number of effective combinatorial optimization approaches was developed under the heading of *Integer Programming*. Its distinctive feature is the encoding of all combinatorial variables by means of `0 - 1` binary variables. Such decoding can be used for *Constraint Optimization Problems* as well, although it is not recommended because of the explosive growth of the number of variables needed to define *COP*. What's more, it may sometimes destroy declarativity and create a troublesome *semantic gap* between the original problem formulation and the program. However, for tutorial reason this approach (termed as *OR* approach) will be illustrated by a number of examples, distinguished by naming them with an *OR* postfix.

The CLP community has developed another approach to combinatorial optimization, which does the job without transforming the original (problem specific) integer variables into (more or less) vague binary variables. The approach, preferred in the sequel, will be distinguished by naming the examples with a *CLP* postfix.

5.2 Branch-and-bound

The basic optimization method used here and in the next section is *branch-and-bound*. A standard version of this method has already been used in Section 2.3.1. However, it would be worthwhile to have a closer look at that method.

To begin with, let us stress that there is a close correspondence between standard[1] *Depth-First Backtracking Search* and standard *Branch-and-Bound*: what for CSP is standard *Depth-First Backtracking Search*, for COP is standard *Branch-and-Bound*, see Figure 5.1.

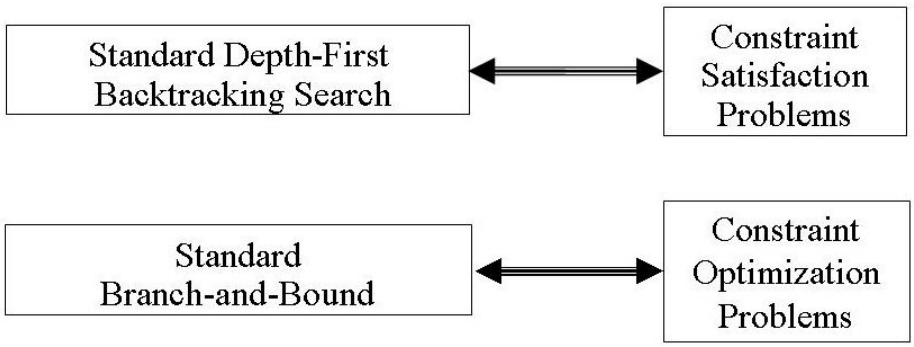


Figure 5.1: Analogy between standard *Depth-First Backtracking Search* and standard *Branch-and-Bound*

The main difference between them is that for *branch-and-bound* search an additional constraint is tested, namely the relation between the *current objective function value* (COFV) and the best objective function value so far (BOFVSF). For minimization, which is a standard optimization mode for ECL^iPS^eCPS, the details are as follows:

- if `COFV < BOFVSF`, then the stored `BOFVSF` is swapped for `COFV`, and the stored set of decision variables corresponding to former `BOFVSF` is swapped for the set of decision variables corresponding to `COFV`;
- if `COFV > BOFVSF`, nothing is changed;

[1] The backtracking discussed so far is considered to be 'standard'.

- if `COFV = BOFVSF`, two approaches are used: 1)nothing is changed - this approach is used by ECL^iPS^eCLP; 2)the set of decision variables corresponding to `COFV` is stored alongside with this for `BOFVSF`; this enables to find multiple optimum solutions as shown in example `2_7_conf_opt.pl`.

Strictly speaking, *branch-and-bound* is not an optimization algorithm but a general methodology (a *paradigm*) of combinatorial optimization, able in principle to find global optima for nonlinear objective functions under nonlinear constraints. Practically - for numerical reasons - *branch-and-bound* is in most implementations (including ECL^iPS^e)) applicable only for linear objective functions under linear constraints.

5.3 Upgrading *Branch-and-Bound*

From similarities between Standard *Branch-and-Bound* and Standard *Depth First Backtracking Search* follows that Standard *Branch-and-Bound* may be updated by introducing search mechanisms discussed in Section 3.2, i.e. *Forward Checking* and *Looking Ahead*. The discussion of search heuristics from Section 3.3 is also relevant for *Branch-and-Bound*.

5.3.1 Optimum queens - standard *Branch-and-Bound*

In order to better understand what should be done to upgrade *Branch-and-Bound*, let's consider its standard version for the simple problem of optimally placing four queens on a 4×4 chessboard. The objective function (quite artificial) is:

```
J = 1*X1+0*X2+1*X3+1*X4 ,
```

where - as previously - `Xi` is the row number occupied by the queen in column `i`. Figure 5.2 shows two feasible placement of four queens, one of which is optimum.

Branch-and-Bound may be characterized by naming states, for which backtracking is initiated. For the standard *Branch-and-Bound* this happens:

- when a worse objective function value has been computed (*Branch-and-Bound* backtracking);
- when some constraint is violated (constraint violation backtracking).

This is shown by the search tree from Figure 5.3.

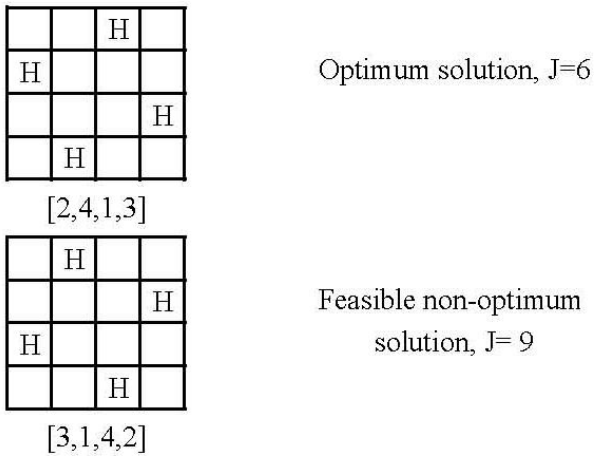


Figure 5.2: Two feasible placements for four queens

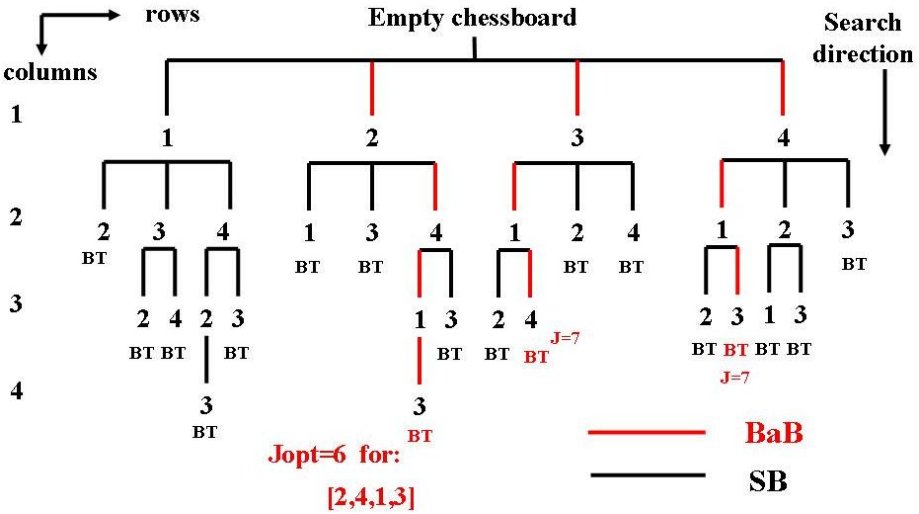


Figure 5.3: Search tree for standard *Branch-and-Bound* for 4 queens

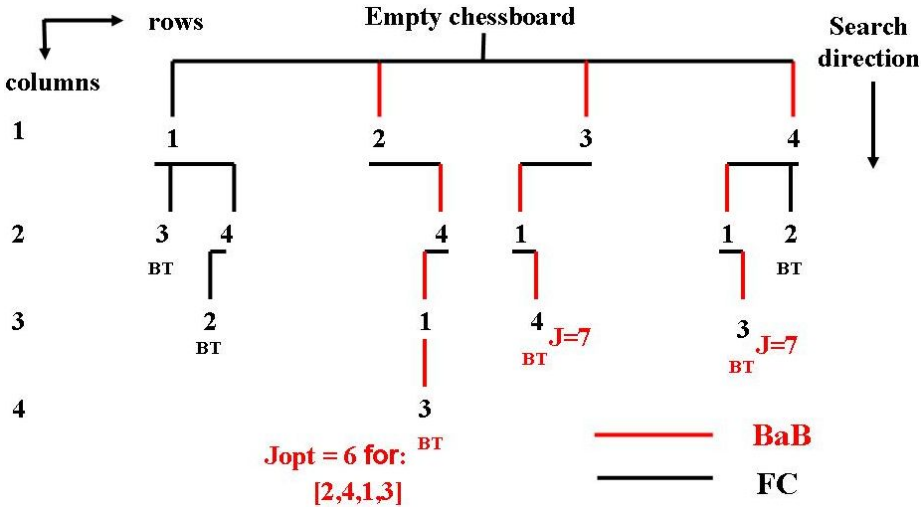


Figure 5.4: Search tree for *Branch-and-Bound+Forward Checking* for 4 queens

5.3.2 Optimum queens - *Forward Checking*

For *Branch-and-Bound + Forward Checking* backtracking is initiated for following states:

- when a worse objective function value has been computed (*Branch-and-Bound* backtracking);
- when the domain of any variable is emptied (*Forward Checking* backtracking).

This is shown by the search tree from Figure 5.4.

5.3.3 Optimum queens - *Looking Ahead + Forward Checking*

For *Branch-and-Bound + Looking Ahead + Forward Checking* backtracking is initiated for following states:

- when a worse objective function value has been computed (*Branch-and-Bound* backtracking);

- when non-empty domains contain no feasible values (*Looking Ahead* backtracking);
- when the domain of any variable is emptied (*Forward Checking* backtracking).

This is shown by the search tree from Figure 5.5.

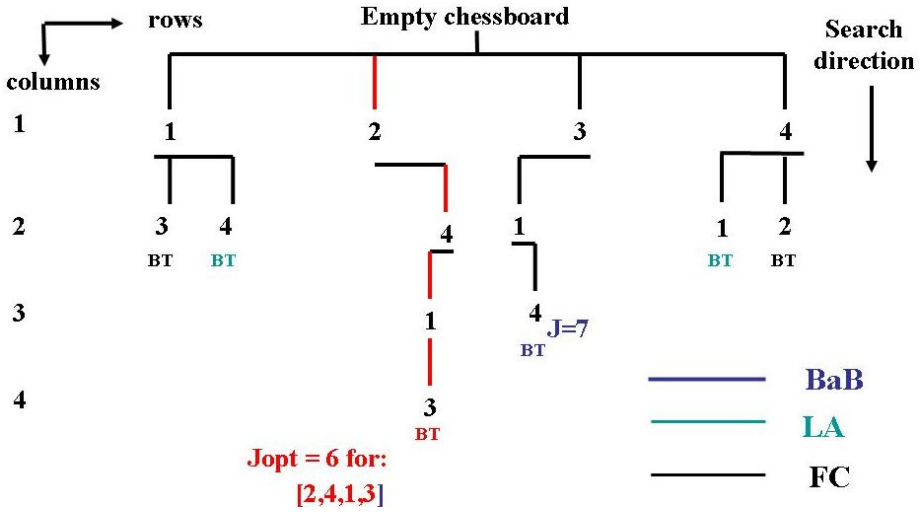


Figure 5.5: Search tree for *Branch-and-Bound+Looking Ahead+Forward Checking* for 4 queens

To end this Section, lets state this:

- the ECL^iPS^e user is not expected to deal explicitly with the described backtracking enhancements;
- they are automatically provided by the mere declaration of optimizing some objective function.

The above discussion just aims to give the ECL^iPS^e user some idea about how to make standard *branch-and-bound* more efficient.

5.4 Basic built-ins

Now two important built-ins will be introduced:

1. `bb_min/3` for *Branch-and-Bound*.
2. `search/6` for parameterizing any search or *Branch-and-Bound* - related search.

A detailed description of both built-ins is available in ECL^iPS^e" documentation, see Figure 5. Because of their importance, their properties will be shortly summarized.

5.4.1 The 'bb min/3' built-in

It is used for initiating *Branch-and-Bound* search. Its simplest version is:

```
bb_min(+Goal, ?Cost, ?Options)
```

where:

- `Goal` is a (nondeterministic) search goal, i.e, a predicate with the optimization problem decision variables as arguments;
- `Cost` is the objective function minimized by grounding decision variables;
- `Options` (the most important) may be as follows:
  - `strategy`:
    - `continue` (default): after finding a solution, continue search with the newly found bound imposed on `Cost`;
    - `restart`: after finding a solution, restart the whole search with the newly found bound imposed on `Cost`;
    - `dichotomic`: after finding a solution, split the remaining cost range and restart search to find a solution in the lower sub-range. If that fails, assume the upper sub-range as the remaining cost range and split again; The new bound or the split point, respectively, are computed from the current best solution, while taking into account the parameters `delta` and `factor`, see below.
  - `from` : number - an initial lower bound for the cost, (default -1.0Inf);
  - `to:` number - an initial upper bound for the cost (default +1.0Inf);

- `delta`: number - minimal absolute improvement required for each step (default 1.0), applies to all strategies;
- `factor`: number - minimal improvement ratio (with respect to the lower cost bound) for strategies 'continue' and 'restart' (default 1.0), or split factor for strategy 'dichotomic', (default 0.5);
- `timeout`: number - maximum seconds of cpu time to spend (default: no limit).

5.4.2 The 'search/6' built-in

This is a more general version of the already discussed `labeling/1` built-in, see Section 3.2. The version supported by the *ic* library is:

```
search(+List, ++Arg, ++Select, +Choice, ++Method, +Option)
```

where:

- `List` is a list of domain variables (for `Arg = 0`) or of terms (for `Arg > 0`);
- `Arg` is an integer, which is `0` if the list is a list of domain variables, or greater than `0`. If the list consists of terms of arity greater than Arg, the value Arg indicates the selected argument of the term;
- `Select` is a predefined *variable choice heuristic*:
  - `input_order` - the first entry in the list is selected;
  - `first_fail` - the entry with the smallest domain size is selected;
  - `anti_first_fail` - the entry with the largest domain size is selected;
  - `smallest` - the entry with the smallest value in the domain is selected;
  - `largest` - the entry with the largest value in the domain is selected;
  - `occurrence` - the entry with the largest number of associated constraints is selected;
  - `most_constrained` - the entry with the smallest domain size is selected. If several entries have the same domain size, the entry with the largest number of attached constraints is selected;
  - `max_regret` - the entry with the largest difference between the smallest and second smallest value in the domain is selected.

- `Choice` is a predefined *value choice heuristic* for variables determined by `Select`:
  - `indomain` - uses the built-in indomain/1. Values are tested in increasing order. On failure, the previously tested value is not removed from the domain;
  - `indomain_min` - values are tested in increasing order. On failure, the previously tested value is removed. The values are tested in the same order as for `indomain/1`, but backtracking may occur earlier;
  - `indomain_max` - values are tested in decreasing order. On failure, the previously tested value is removed;
  - `indomain_reverse_min` - like `indomain_min`, but the lues are tested in reverse order, i.e. the smallest value is first removed from the domain, and only if that fails, the value is assigned;
  - `indomain_reverse_max` - like `indomain_max`, but the values are tested in reverse order, i.e. the largest value is first removed from the domain, and only if that fails, the value is assigned;
  - `indomain_middle` - values are tested beginning from the middle of the domain. On failure, the previously tested value is removed;
  - `indomain_median` - values are tested beginning from the median value of the domain. On failure, the previously tested value is removed.;
  - `indomain_split` - values are tested by successive domain splitting, testing the lower half of the domain first. On failure, the tested interval is removed. This enumerates values in the same order as `indomain/1` or `indomain_min`, but may fail earlier;
  - `indomain_reverse_split` - values are tested by successive domain splitting, trying the upper half of the domain first. On failure, the tested interval is removed. This enumerates values in the same order as `indomain/1` or `indomain_max`, but may fail earlier.
  - `indomain_random` - values are tested in a random order. On backtracking, the previously tested value is removed;
  - `indomain_interval` - if the domain consists of several intervals, we first branch on the choice of the interval. For one interval, we use `indomain_split`.

- `Method` denotes one of ten search methods. The basic are:
  - `complete` - a complete search routine, which is testing all variable groundings;
  - `bbs(Backtracking_steps)` - a bounded backtracking search, which allows only `Backtracking_steps` steps;
  - `sbds` - a complete search routine, which uses the SBDS symmetry breaking library (`lib(ic_sbds)` or `lib(fd_sbds)`) to exclude symmetric parts of the search tree from consideration.
- `Options` denotes one of four options. The basic are:
  - `backtrack(-N)` - returns the number of backtracking steps used in the search routine;
  - `nodes(++N)` - sets an upper limit on the number of nodes explored in the search. If the given limit is exceeded, the search routine stops the exploration of the search tree.

5.5 A simple example

Consider a small computer assembly plant, which has available `60` motherboards of type A, `50` motherboards of type B and `120` SSD (solid state drives). Computers with type A motherboard (which need a singleSSD) may be sold with profit 300 JP. Computers with type B motherboard (which need a two SSDs) may be sold with profit 500 JP. How many computers of type A and B should be produced to maximize profit? This is an integer optimiziation problem which luckily could be solved graphically as shown in Figure 5.6. A program doing this (`5_1_PL.ecl`) is as follows:

```
/*1*/   :- lib(ic).
/*2*/   :- lib(branch_and_bound).
/*3*/   top :-
/*4*/       Boards=[A,B],
/*5*/       Boards :: 1..60,
/*6*/       Profit :: 30000..40000,
/*7*/       A #=< 60,
/*8*/       B #=< 50,
/*9*/       A + 2*B #=< 120,
```

```
/*10*/      Profit #= 300*A + 500*B,
/*11*/      Negative_profit #= - Profit,
/*12*/      minimize(labeling([A,B]),Negative_profit),
/*13*/      writeln("Maximum profit":Profit),nl,
/*14*/      write("A_opt = "), write(A), nl,
/*15*/      write("B_opt = "), write(B),nl.
```

Because maximization has to be done, and ECL^iPS^e makes available only predicates for minimization, maximization is performed for negative profit.

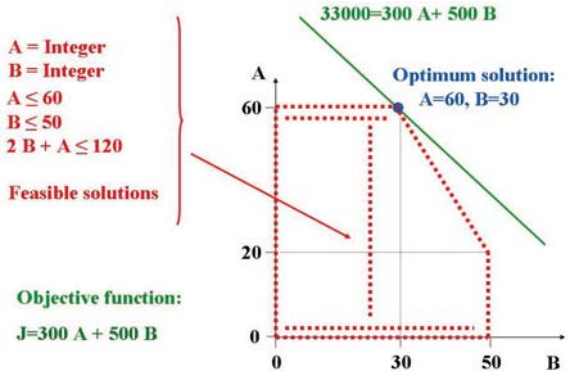


Figure 5.6: Graphical solution to the simple optimization problem

The program displays all intermediate solutions from the search tree:

```
Found a solution with cost -30100 Found a solution with cost -30400
Found a solution with cost -30700 Found a solution with cost -31000
Found a solution with cost -31100 Found a solution with cost -31200
Found a solution with cost -31300 Found a solution with cost -31400
Found a solution with cost -31500 Found a solution with cost -31600
Found a solution with cost -31700 Found a solution with cost -31800
Found a solution with cost -31900 Found a solution with cost -32000
Found a solution with cost -32100 Found a solution with cost -32200
Found a solution with cost -32300 Found a solution with cost -32400
Found a solution with cost -32500 Found a solution with cost -32600
Found a solution with cost -32700 Found a solution with cost -32800
Found a solution with cost -32900 Found a solution with cost -33000
```

```
Found no solution with cost -40000.0 .. -33001.0
Maximum profit :  33000
A_opt = 60 B_opt = 30
```

The problems discussed and solved below conform to the classification presented in Section 1.7.

5.6 Optimum configuration problems

5.6.1 Optimum configuration - OR approach

Next - let's solve the optimum system configuration problem from Section 2.3.1 using an OR approach. This is done by program `5_2_configuration_OR.ecl`[2]:

```
/*1*/   :- lib(ic).
/*2*/   :- lib(branch_and_bound).
/*3*/   top :-
/*4*/       Components=[A_1,A_2,A_3,B_1,B_2,B_3,B_4,C_1,C_2],
  % A_1 = 1 means element A_1 belongs to the configuration
  % A_1 = 0 means element A_1 does not belong to the configuration
/*5*/       Components :: 0..1,
/*6*/       Price:: 1..3000,

/*7*/       A_1 + A_2 + A_3 #= 1, % only one A-type element is needed
/*8*/       B_1 + B_2 + B_3 + B_4 #= 1, % only one B-type element is needed
/*9*/       C_1 + C_2 #= 1, % only one C-type element is needed

/*10*/      C_1 + A_2 #=< 1,  %  C_1 and A_2 should not appear both in a system
/*11*/      B_2 + C_2 #=< 1,
/*12*/      C_2 + B_3 #=< 1,
/*13*/      B_4 + A_2 #=< 1,
/*14*/      B_3 + A_1 #=< 1,
/*15*/      A_3 + B_3 #=< 1,

/*16*/      Price #= A_1 * 1900 + A_2 * 750 +
                   A_3 * 900 + B_1 * 300 + B_2 * 500 + B_3 * 450 +
                         B_4 * 600 + C_1 * 700 + C_2 * 850,

/*17*/      bb_min(labeling(Components),Price,bb_options with [strategy:step]),

/*18*/      writeln("Minimum configuration price":Price),nl,nl,
/*19*/      write("Optimum configuration:"),nl,
/*20*/      write([A_1,A_2,A_3,B_1,B_2,B_3,B_4,C_1,C_2]),nl,
```

[2]This is an OS-type problem.

```
/*21*/      write(["A_1","A_2","A_3","B_1","B_2","B_3","B_4","C_1","C_2"]),nl,
/*22*/      write_configuration([A_1,A_2,A_3,B_1,B_2,B_3,B_4,C_1,C_2],
                    ["A_1","A_2","A_3","B_1","B_2","B_3","B_4","C_1","C_2"]),nl,nl,
/*23*/      fail.

/*24*/   top :-
/*25*/      write("Those are all optimum configurations.").

/*26*/   write_configuration([H1|T1],[H2|T2]):-
/*27*/      H1 is 1, write(H2),write("  "),
/*28*/      write_configuration(T1,T2).

/*29*/   write_configuration([H1|T1],[_|T2]):-
/*30*/      H1 is 0,
/*31*/      write_configuration(T1,T2).
/*32*/   write_configuration([],[]).
```

The message is:

```
Found a solution with cost 2350
Found a solution with cost 2200
Found a solution with cost 2100
Found a solution with cost 2050
Found a solution with cost 1900
Found no solution with cost 1.0 .. 1899.0

Minimum configuration price : 1900

Optimum configuration:
[0, 0, 1, 1, 0, 0, 0, 1, 0]
[A_1, A_2, A_3, B_1, B_2, B_3, B_4, C_1, C_2]

   A_3 B_1 C_1

Those are all optimum configurations.
```

It can be seen that `fail` in line `/*26*/` did not initiate backtracking to determine the second optimum solution, which is known to exist as demonstrated by program `2_9_conf_opt.pl`. This is a serious limitation that however may be bypassed as follows: in order to get all optimum solutions, a single one has to be determined first. Next, the optimum solution data is used to constrict the domains of variables (see line `/*5*/` below) for a program that just determines all feasible solutions. This program is given by `5_3_configuration_all_OR.ecl`:

```
/*1*/   :- lib(ic).
/*2*/   top :-
/*3*/       Components=[A_1,A_2,A_3,B_1,B_2,B_3,B_4,C_1,C_2],
/*4*/       Components :: 0..1,
/*5*/       Price is 1900,

/*6*/       A_1 + A_2 + A_3 #= 1,
/*7*/       B_1 + B_2 + B_3 + B_4 #= 1,
/*8*/       C_1 + C_2 #= 1,
/*9*/       C_1 + A_2 #=< 1,
/*10*/      B_2 + C_2 #=< 1,
/*11*/      C_2 + B_3 #=< 1,
/*12*/      B_4 + A_2 #=< 1,
/*13*/      B_3 + A_1 #=< 1,
/*14*/      A_3 + B_3 #=< 1,

/*15*/      Price #=  A_1 * 1900 + A_2 * 750 + A_3 * 900 +
                    B_1 * 300 + B_2 * 500 + B_3 * 450 + B_4 * 600 +
                            C_1 * 700 + C_2 * 850,

/*16*/      labeling(Components),

/*17*/      writeln("Minimum configuration price": Price),nl,nl,
/*18*/      write("Search result:"),nl,
/*19*/      write([A_1,A_2,A_3,B_1,B_2,B_3,B_4,C_1,C_2]),nl,
/*20*/      write(["A_1","A_2","A_3","B_1","B_2","B_3","B_4","C_1","C_2"]),nl,
/*21*/      write("Optimum configuration:"),nl,
/*22*/      write_configuration([A_1,A_2,A_3,B_1,B_2,B_3,B_4,C_1,C_2],
                    ["A_1","A_2","A_3","B_1","B_2","B_3","B_4","C_1","C_2"]),nl,nl,
/*23*/      fail.

/*24*/   top :-
/*25*/      write("Those are all optimum configurations.").

/*26*/   write_configuration([H1|T1],[H2|T2]):-
/*27*/      H1 is 1, write(H2),write("  "),
/*28*/      write_configuration(T1,T2).

/*29*/   write_configuration([H1|T1],[_|T2]):-
/*30*/      H1 is 0,
/*31*/      write_configuration(T1,T2).
/*32*/   write_configuration([],[]).
```

The message is:

```
Minimum configuration price:1900
Search result:
```

```
[0, 0, 1, 1, 0, 0, 0, 1, 0]
[A_1,A_2,A_3,B_1,B_2,B_3,B_4,C_1,C_2]
Optimum configuration:
A_3  B_1  C_1

Minimum configuration price:1900
Search result:
[0, 1, 0, 1, 0, 0, 0, 0, 1]
[A_1,A_2,A_3,B_1,B_2,B_3,B_4,C_1,C_2]
Optimum configuration:
A_2  B_1  C_2

Those are all optimum configurations.
```

5.6.2 Optimum configuration - CLP approach

Next the optimum configuration problem from Section 2.3.1 will be solved using the CLP approach. The program `5_4_configuration_CLP.ecl`[3] is as follows:

```
/*1*/   :- lib(ic).
/*2*/   :- lib(branch_and_bound).
/*4*/   top:-
      %  CA - cost of element A
      %  NA - number of element A
/*4*/        NA :: 1..3,
/*5*/        NB :: 1..4,
/*6*/        NC :: 1..2,
/*7*/        [CA,CB,CC] :: 300..1900,
/*8*/        Cost :: 1800..2600,
/*9*/        element(NA,[1900,750,900],CA),
/*10*/       element(NB,[300,500,450,600],CB),
/*11*/       element(NC,[700,850],CC),
/*12*/       ~incompatible_NB_NC(NB,NC),
/*13*/       ~incompatible_NA_NB(NA,NB),
/*14*/       ~incompatible_NA_NC(NA,NC),
% ~Goal is the sound negation operator, which delays if Goal is not grounded.+

/*15*/       Cost #= CA + CB + CC,
/*16*/       bb_min(labeling([NA,NB,NC]),Cost,bb_options with [strategy:step]),

/*17*/       writeln("Optimum configuration:"),
/*18*/       write("("),write("A"),write(NA),write(","),
/*19*/       write("B"),write(NB),write(","),write("C"),write(NC),writeln(")"),
```

[3] This is an OS-type problem.

```
/*20*/      write("priced at "),write(Cost), writeln("."),nl,fail.

/*21*/  top:-
/*22*/      writeln("That's all!").

/*23*/  incompatible_NA_NB(2,4).
/*24*/  incompatible_NA_NB(1,3).
/*25*/  incompatible_NA_NB(3,3).

/*26*/  incompatible_NA_NC(2,1).

/*27*/  incompatible_NB_NC(2,2).
/*28*/  incompatible_NB_NC(3,2).
```

The message is:

```
Found a solution with cost 1900
Found no solution with cost 1800.0 .. 1899.0
Optimum configuration:
(A2,B1,C2)
priced at 1900.

That's all!
```

In order to generate all optimum solution the same trick as for example `5_3_configuration_all_OR.ecl` has to be used. This is done in example `5_5_configuration_all_CLP.ecl`[4] :

```
/*1*/   :- lib(ic).
/*2*/   :- lib(branch_and_bound).

/*3*/   top:-
/*4*/       NA :: 1..3,
/*5*/       NB :: 1..4,
/*6*/       NC :: 1..2,
/*7*/       [CA,CB,CC] :: 300..1900,
/*8*/       Cost is 1900,
/*9*/       element(NA,[1900,750,900],CA),
/*10*/      element(NB,[300,500,450,600],CB),
/*11*/      element(NC,[700,850],CC),
/*12*/      ~incompatible_NB_NC(NB,NC),
/*13*/      ~incompatible_NA_NB(NA,NB),
/*14*/      ~incompatible_NA_NC(NA,NC),
```

[4]This is an OS-type problem.

```
/*15*/       Cost #= CA + CB + CC,
/*16*/       labeling([NA,NB,NC]),

/*17*/       writeln("Optimum configuration:"),
/*18*/       write("("),write("A"),write(NA),write(","),
/*19*/       write("B"),write(NB),write(","),write("C"),write(NC),writeln(")"),
/*20*/       write("priced at "),write(Cost), writeln("."),nl,fail.

/*21*/  top:-
/*22*/       writeln("That's all!").

/*23*/  incompatible_NA_NB(2,4).
/*24*/  incompatible_NA_NB(1,3).
/*25*/  incompatible_NA_NB(3,3).

/*26*/  incompatible_NA_NC(2,1).

/*27*/  incompatible_NB_NC(2,2).
/*28*/  incompatible_NB_NC(3,2).
```

The message is:

```
Optimum configuration:
(A2,B1,C2)
priced at 1900.

Optimum configuration:
(A3,B1,C1)
priced at 1900.

That's all!
```

5.6.3 Knapsack problem 1

The *knapsack problem* is a classical optimization problem that derives its name from a fixed-size *smuggler knapsack*, which must be filled with the most valuable items. It may be formulated as follows: given a set of items, each with a dimension (length, area, volume or weight) and a value, determine the items to include in a collection so that the total dimension is less than a given limit and the total value is maximized. The problem is known to exhibit *combinatorial explosion*.

The most simple knapsack problem - a length-constrained knapsack problem - can be solved using the `scalar_product/3` predicate as shown in program

5_6_knapsack_1.ecl[5]:

```
/*1*/  :- lib(ic).
/*2*/  :- lib(branch_and_bound).
/*3*/  top:-
/*4*/       knapsack([52,23,35,15,7],[100,60,70,15,15],60,[_,_,_,_,_]).

/*5*/  knapsack(Sizes,Values,Knapsack_size,[X1,X2,X3,X4,X5]):-
/*6*/       X = [X1,X2,X3,X4,X5],
/*7*/       X :: 0..1,
/*8*/       scalar_product(Sizes,X,Size),
/*9*/       Size #=< Knapsack_size,
/*10*/      scalar_product(Values,X,Value),
/*11*/      Cost #= -Value,
/*12*/      minimize(labeling(X),Cost),nl,
/*13*/      Value is -Cost,
/*14*/      write("Value = "),writeln(Value),
/*15*/      write("Knapsack = "), writeln(X),
/*16*/      write("Size ="),writeln(Size).

/*17*/ scalar_product(List_1,List_2,Scalar_product):-
/*18*/      (
/*19*/      foreach(V1, List_1),
/*20*/      foreach(V2, List_2),
/*21*/      foreach(Product,List_of_products)
/*22*/      do
/*232*/      Product = V1 * V2
/*24*/      ),
/*25*/      Scalar_product #= sum(List_of_products).
```

The message is:

```
Found a solution with cost 0
Found a solution with cost -15
Found a solution with cost -30
Found a solution with cost -70
Found a solution with cost -85
Found a solution with cost -100
Found a solution with cost -130
Found no solution with cost -260.0 .. -131.0

Value = 130
Knapsack = [0, 1, 1, 0, 0]
```

[5]This is an OS-type problem.

```
  Size = 58,
```

So the optimum knapsack loading comprises items 2 and 3 from the list, of corresponding sizes 23 and 35 amounting to 58, and of corresponding values 60 and 70 amounting to 130.

5.6.4 Reified constraints

Often it is desirable that the satisfaction of some constraint makes a Boolean variable (further referred to as `Index`) bounded to `1`; the failing of the constraint makes this variable bounded to `0`. The `Index` may be useful to formulate other constraints. This may be done by *reifying* the constraint with respect to the `Index`, as illustrated by following examples:

```
 This is a command:
 [eclipse 1]: Number = 0, #>(Number,0,Index).

 This is the response:
  Number = 0
  Index = 0,
```

i.e. `Number > 0` is false.

```
 This is a command:
  [eclipse 2]: Number = 6, #>(Number,0,Index).

 This is the response:
  Number = 6
  Index = 1,
```

i.e. `Number > 0` is true.

All elementary constraints can be changed into reified forms. E.g. the implication constraint `+constraint(X) => +constraint(Y),`
for which the satisfaction of "`constraint(X)`" implies the satisfaction of "`constraint(Y)`", is functioning as follows:

```
 This is a command for a non-reified form:
  [eclipse 3]: X is 9, Y is 8, X#<10 => Y+2#<12.
```

```
This is the response for a non-reified form:
X = 9
Y = 8
Yes
```

After reifying we get:

```
This is a command for a reified form:
[eclipse 4]: X is 9, Y is 8, =>(X#<10,Y+2#<12,Index).

This is the response for a reified form:
X = 9
Y = 8
Index = 1
```

If the implication is false:

```
This is a command:
[eclipse 5]: X is 9, Y is 8, =>(X#<10,Y+2#>15,Index).

This is the response:
X = 9
Y = 8
Index = 0
```

The implication may be true for any value from the domain, e.g.:

```
This is a command:
[eclipse 6]: X is 12, Y::14..18,=>(X#=5,Y+2#>15,Index).

This is the response:
X = 12
Y = Y{14 .. 18}
Index = 1,
```

then `Index` is bounded to a unique value.

If the implication is true only for a subset of domain values, e.g.:

```
This is a command:
[eclipse 7]: X is 12, Y::12..17,=>(X#=12,Y+2#>15,Index).

This is the response:
X = 12
Y = Y{12 .. 17}
Index = Index{[0, 1]},
```

then `Index` remains free.

5.6.5 Constraints for sets

A useful feature of ECL^iPS^e *CPS* is a the possibility of formulating constraints for domains given by sets of integers. To use this feature the library `ic_sets` needs to be loaded.
Sets of integers in ECL^iPS^e *CPS* are *ordered n-tuples* of *unique integers*, e.g.:

```
set_of_four_integers = [41,42,43,44],   empty_set =[].
```

Set variables are variables that may be grounded to sets of integers. They are declared as follows:

```
Set_variable :: []..[1,2,3,4,5,6]
```

where the empty set is the lower bound, and the set `[1,2,3,4,5,6]` is the upper bound for the `Set_variable`. Let's check some of its properties:

```
This is a command:
[eclipse 1]::-lib(ic_sets). Set_variable :: []..[1,2,3,4,5,6],
        Set_variable = [3,2,1].

This is the response:
[eclipse 2]:
No (0.00s cpu)

This is a command:
[eclipse 3]::-lib(ic_sets). Set_variable :: []..[1,2,3,4,5,6],
        Set_variable = [1,4,6].

This is the response:
[eclipse 4]:
Set_variable = [1, 4, 6]
```

```
Yes (0.00s cpu)

This is a command:
[eclipse 5]::-lib(ic_sets). Set_variable :: []..[1,2,3,4,5,6],
        Set_variable = [].

This is the response:
[eclipse 6]:
Set_variable = []
Yes (0.00s cpu)
```

For ECL^iPS^e CPS the empty set `[]` does not belong to the set domain if it has not been *explicitly* declared:

```
This is a command:
[eclipse 7]::-lib(ic_sets). Set_variable :: [4]..[5,6,7],
        Set_variable = [].

This is the response:
[eclipse 8]:
No (0.00s cpu)
```

However, the empty set `[]` is a subset of any set, e.g.:

```
This is a command:
[eclipse 9]::-lib(ic_sets). [4,5,6,7] includes X.
        Set_variable = [].

This is the response:
[eclipse 8]:
X = X{([] .. [4, 5, 6, 7]) : _358{0 .. 4}},
```

where the `_358` is the range of cardinal numbers for the `X` set.

The lower bound does not belong to any set containing also elements of the upper bound as illustrated below:

```
This is a command:
[eclipse 9]::-lib(ic_sets). Set_variable :: [4]..[5,6,7],
```

```
        Set_variable = [4,5].

This is the response:
 [eclipse 8]:
 No (0.00s cpu)
```

In order for the lower bound to belong to some set containing (beside the lower bound) also elements from the upper bound, it has to be included in the upper bound:

```
This is a command:
 [eclipse 10]: :-lib(ic_sets). Set_variable :: [4]..[4,5,6,7],
        Set_variable=[4,7].

This is the response:
 [eclipse 11]:
 Set_variable = [4, 7]
 Yes (0.00s cpu)
```

An important built-in for connecting sets and arrays is:

```
weight(?Set, ++Array_of_Set_Element_Weights, ?Weight_of_set),
```

for which `?Weight_of_set` is the sum of all those elements from `Array_of_Set_Element_Weights` that in the array are at positions given by elements of `Set`. This is illustrated by program `5_7_weight_of_set_1.ecl`[6] :

```
/*1*/  :- lib(ic_sets).
/*2*/  top:-
/*3*/       Set = [1,3],
/*4*/       weight(Set,[](10,20,30,40,50),Weight_of_set),
/*5*/       write("Weight of set = "),write(Weight_of_set),nl.
```

The message is:

```
        Weight of set = 40
```

[6]This is an FS-type problem.

The built-in `weight/3` may be used to determine sets with constraint weights as shown in the program `5_8_weight_of_set_2.ecl`:

```
/*1*/  :- lib(ic).
/*2*/  :- lib(ic_sets).
/*3*/  top:-
/*4*/       ic_sets:(Set :: [].. [1, 2, 3, 4, 5]),
/*5*/       weight(Set,[](10,20,30,40,50),Weight_of_set),
/*6*/       Weight_of_set #=< 50,
/*7*/       Weight_of_set #>= 35,
/*8*/       insetdomain(Set,_,_,_),
/*9*/       write("Weight of set = "),write(Weight_of_set),
/*10*/      write("    Set = "),write(Set),nl.
```

The built-in `insetdomain/4` is a *set-wise* correspondent of the built-in `indomain/1` for integer domains.
The message presents a number of solutions:

```
Weight of set = 40    Set = [1, 3]
Weight of set = 50    Set = [1, 4]
Weight of set = 50    Set = [2, 3]
Weight of set = 40    Set = [4]
Weight of set = 50    Set = [5]
```

5.6.6 Knapsack problem 2

The length-constrained knapsack problem may also be solved using the `weight/3` built-in, as shown in `5_9_knapsack_2.ecl`[7]:

```
/*1*/  :- lib(ic).
/*2*/  :- lib(ic_sets).
/*3*/  :- lib(branch_and_bound).
/*4*/  top:-
/*5*/       array_of_sizes(Sizes),
/*6*/       array_of_values(Values),
/*7*/       knapsack_size(Knapsack_size),
/*8*/       ic_sets:(Set :: [].. [1, 2, 3, 4, 5]),
/*9*/       weight(Set,Sizes,Knapsack_load),
```

[7] This is an OS-type problem.

```
/*10*/      Knapsack_load #=< Knapsack_size,
/*11*/      weight(Set,Values,Value),
/*12*/      Cost #= -Value,

/*13*/      minimize(insetdomain(Set,decreasing,_,_),Cost),

/*14*/      write("Value = "),writeln(Value),
/*15*/      write("Knapsack = "), writeln(Set),
/*16*/      write("Knapsack load = "),writeln(Knapsack_load).

/*17*/  array_of_sizes([](52,23,35,15,7)).
/*18*/  array_of_values([](100,60,70,15,15)).
/*19*/  knapsack_size(60).
```

The message is:

```
Found a solution with cost -90
Found a solution with cost -100
Found a solution with cost -115
Found a solution with cost -130
Found no solution with cost -260.0 .. -131.0
Value = 130
Knapsack = [2, 3]
Knapsack_load = 58,
```

The optimum knapsack load is thus 58, given by items z 2 and 3, with overall value 130.

5.6.7 How to cut optimally?

The range of different optimum configuration problems is broad indeed. As yet another example may serve the one-dimensional rod-cutting problem:

A number of 100 cm long rods should be cut into 36 rods of 28 cm and 24 rods of 45 cm so as to minimize the total waste. There are only 3 feasible cutting strategies for a 100 cm long rod and the demanded smaller rods, illustrated by Figure 5.7. An overall optimum cutting strategy that minimizes waste for the given order of small rods is given by program `5_10_cutting.ecl`[8], where variables `Strategy_1`, `Strategy_2` and `Strategy_3` denote numbers of 100 cm

[8] This is an OS-type problem.

rods cut using correspondingly strategy 1, strategy 2 and strategy 3:

```
/*1*/  :-lib(ic).
/*2*/  :-lib(branch_and_bound).
/*3*/  top :-
/*4*/  Variables = [Strategy_1,Strategy_2,Strategy_3],
/*4*/  Variables :: 0..60,

/*5*/  3*Strategy_1 + 1*Strategy_2 + 0*Strategy_3 #>=36,
/*6*/  0*Strategy_1 + 1*Strategy_2 + 2*Strategy_3 #>= 24,

/*7*/  Cost #= 10*Strategy_1 + 25*Strategy_2 + 10*Strategy_3,

/*8*/  minimize(search(Variables,0,first_fail,indomain,complete,[]),Cost),
/*9*/  writeln("Variables":Variables ),
/*10*/ writeln("Cost":Cost).
```

The message is:

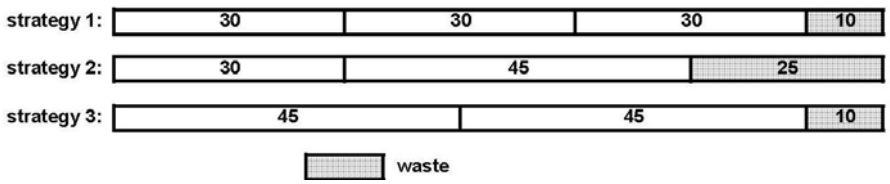


Figure 5.7: Feasible cutting strategies for a 100 cm long rod

```
Found a solution with cost 900
  Found a solution with cost 835
  Found a solution with cost 770
  Found a solution with cost 705
  Found a solution with cost 640
  Found a solution with cost 595
  Found a solution with cost 540
  Found a solution with cost 495
  Found a solution with cost 440
  Found a solution with cost 395
  Found a solution with cost 340
  Found a solution with cost 295
  Found a solution with cost 240
  Found no solution with cost 0.0 .. 239.0
```

```
Variables : [12, 0, 12]
Cost : 240
```

It means that 12 rods should be cut using strategy 1 and 12 rods using strategy 3.

5.6.8 Appointing a parliamentary committee

A common optimization problem is concerned with set representation: find the smallest set, which contains elements of other sets. A minimization is possible if the sets contain some shared elements. This is illustrated by the following example:

The ruling Absurdoland's coalition of two immensely popular parties, "Spreading Wealth" and "Paradise on Earth", is facing the problem of delegating four parliamentarians to a newly established (under electoral pressure) parliamentary committee for the investigation of illegal lobbying activities aimed at influencing the outcome of legislative processes. Each coalition party appointed five parliamentarians as candidates to the committee; out of the team of ten parliamentarians available, four parliamentarians have to be finally chosen to serve on the committee. Obviously, the parties were interested in having on the committee parliamentarians representing all active main streams of political and social thought cultivated in both parties. A close look at the initially appointed ten parliamentarians (which would be referred to by numbers in order not to compromise party secrets), 1, 2, 3,...,10, assured both parties that they represent all active main streams of political and social thought. What's more, a yet closer look disclosed that some of the appointed parliamentarians have such extraordinary high intellectual capacity to make them contribute in the past to more than one active main stream, as shown in Table 5.1.

Therefore it was justifiably concluded that they should represent in the committee all active main streams, to which they contributed.

However, the question is still open whether a team of four parliamentarians representing all active main streams could be selected out of the ten candidates. To answer that question a program `5_11_committee.ecl`[9] has been designed to establish the minimum number of parliamentarians representing all active main streams. The program shown below has been inspired by one presented at the website [Kjellerstrand-13]:

[9]This is an OS-type problem.

Parliamentarians	Coalition parties
1, 2, 3, 4, 5	Spreading Wealth
6, 7, 8, 9, 10	Paradise on Earth
	Main streams of political and social thought
3, 8, 9	Agents of Influence 1
1, 6, 7	Agents of Influence 2
3, 4	Mafia 1 Supporters
2, 6	Mafia 2 Supporters
7, 10	Gambling Business Advocates
3, 6	Anthropogenic Global Warming Believers
7	Big Bank Advocates
2	LGBT Supporters
5, 10	Useful Idiots

Table 5.1: Parliamentarians, their affiliation to parties and contributions to main streams

```
/*1*/  :-lib(ic).
/*2*/  :-lib(branch_and_bound).
/*3*/  top :-
  % First a single minimum number of parliamentarians is determined,
  % next all lists of parliamentarians for this minimum are determined.
/*4*/          writeln("Finding a single optimum solution:"),
/*5*/          data(WhoWhere),
/*6*/          select_from_sets(WhoWhere, Minimum,_),
/*7*/          writeln("\nFinding all optimum solutions:"),
/*8*/          findall(X, select_from_sets(WhoWhere, Minimum,X), L),
/*9*/          length(L, Len),
/*10*/         printf("%d optimum solutions have been found.\n", [Len]).
 /*11*/  select_from_sets(WhoWhere, Minimum, X) :-
/*12*/         dim(WhoWhere,[NumberOfGroups,NumberOfMembers]),
 % Creating a list of parliamentarians:
/*13*/         dim(X,[NumberOfMembers]),
/*14*/         X :: 0..1,

 % Choosing parliamentarians from each main stream:
/*15*/         (
/*16*/         for(I,1,NumberOfGroups),
/*17*/         param(NumberOfMembers,X,WhoWhere)
/*18*/         do
/*19*/            (
/*20*/            for(J,1,NumberOfMembers),
/*21*/            fromto(0,In,Out,Sum),
```

```
/*22*/            param(X,WhoWhere,I)
/*23*/                  do
/*24*/                  Out #= In + X[J]*WhoWhere[I,J]
/*25*/            ),
/*26*/         Sum #>= 1
/*27*/         ),

 % Minimizing the number of parliamentarians:
/*28*/         flatten_array(X, Variables),
/*29*/         Z #= sum(Variables),
/*30*/         Z #= Minimum,

 % Depending whether the minimum is or is not known,
 % all minimal solutions are determined or
 % a single minimum solution is determined:
/*31*/         (
/*32*/         ground(Minimum)
/*33*/         ->
/*34*/         search(Variables,0,first_fail,indomain,complete, [])
/*35*/         ;
/*36*/         minimize(search(Variables,0,first_fail,indomain,complete,[]),Z)
/*37*/         ),
/*38*/         writeln("Minimum number of parliamentarians":Z),
/*39*/         writeln("Selected parliamentarians":X).

/*40*/         data([](
   [](1, 1, 1, 1, 1, 0, 0, 0, 0, 0),  % Spreading Wealth
   [](0, 0, 0, 0, 0, 1, 1, 1, 1, 1),  % Paradise on Earth
   [](0, 0, 1, 0, 0, 0, 0, 1, 1, 0),  % Agents of Influence 1
   [](1, 0, 0, 0, 0, 1, 1, 0, 0, 0),  % Agents of Influence 2
   [](0, 0, 1, 1, 0, 0, 0, 0, 0, 0),  % Mafia 1 Supporters
   [](0, 1, 0, 0, 0, 1, 0, 0, 0, 0),  % Mafia 2 Supporters
   [](0, 0, 0, 0, 0, 0, 1, 0, 0, 1),  % Gambling Business Advocates
   [](0, 0, 1, 0, 0, 1, 0, 0, 0, 0),  % Anthropogenic Global Warming Believers
   [](0, 0, 0, 0, 0, 0, 1, 0, 0, 0),  % Big Bank Advocates
   [](0, 1, 0, 0, 0, 0, 0, 0, 0, 0),  % LGBT Supporters
   [](0, 0, 0, 0, 1, 0, 0, 0, 0, 1))).% Useful Idiots
```

The message is:

```
Finding a single optimum solution:
Found a solution with cost 6
Found a solution with cost 4
Found no solution with cost 2.0 .. 3.0
Minimum number of parliamentarians : 4
```

```
Selected parliamentarians : [](0, 1, 1, 0, 0, 0, 1, 0, 0, 1)

Finding all optimum solutions:
Minimum number of parliamentarians : 4
Selected parliamentarians : [](0, 1, 1, 0, 0, 0, 1, 0, 0, 1)
Minimum number of parliamentarians : 4
Selected parliamentarians : [](0, 1, 1, 0, 1, 0, 1, 0, 0, 0)
2 optimum solutions have been found.
```

The meaning of this results is obvious: only if the `i`-th element of the one-dimensional array `Selected parliamentarians` is equal `1`, then the `i`-th parliamentarian may be chosen to be a committee member.

The result is both good and bad news. The good news is that there are four parliamentarians representing all main streams of political and social thought cultivated in both parties. The bad news is that there are two teams of such parliamentarians, which means that there will be much arguing in the coalition.

5.6.9 Ambulance Service Stations

A Town Council is analyzing possible locations for the newly established large and modern Ambulance Service Stations (ASS). The Town consists of 11 districts as shown in Figure 5.8.

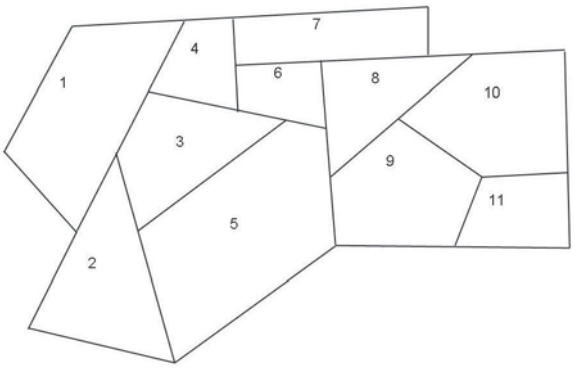


Figure 5.8: District maps

An Ambulance Service Station may be located in any district and provide

its services to its native and all adjacent districts. The conservative majority in the Town Council successfully defended a motion about minimizing the number of ASS while providing all districts with their services. It also put forward some additional suggestions about avoiding the establishment of ASS in adjacent districts and favoured a location plan for which any district having no ASS is adjacent to only one district with ASS.

The program `5_12_ambulance_service.ecl` explains how was it done:

```
/*1*/  :- lib(ic).
/*2*/  :- lib(branch_and_bound).

/*3*/  top :-

/*4*/  [S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,S11] :: 0..1,
% Si = 1 - an ASS is located in i-th district
% Si = 0 - an ASS  is not located in i-th district

  % if an ASS is located in district 1, then no ASS is needed
  % for districts 2, 3 and 4;
  % if no ASS is located in district 1, then one of the
  % districts  2,  3 or 4 should have an ASS:
/*5*/   S1+S2+S3+S4 #= 1,

  % if an ASS is located in district 2, then no ASS is needed
  % for districts 1, 3 and 5;
  % if no ASS is located in district 2, then one of the
  % districts  1, 3 and 5should have an ASS:
/*6*/   S1+S2+S3+   S5                      #= 1,

  % if an ASS is located in district 3, then no ASS is needed
  % for districts 1, 2, 4, 5 and 6;
  % if no ASS is located in district 3, then one of the
  % districts  1, 2, 4, 5 and 6 should have an ASS:
/*7*/   S1+S2+S3+S4+S5+S6                   #= 1,

  % if an ASS is located in district 4, then no ASS is needed
  % for districts 1, 3, 6 and 7;
  % if no ASS is located in district 4, then one of the
  % districts  1, 3, 6 and 7 should have an ASS:
/*8*/   S1+   S3+S4+   S6+S7                #= 1,

  % if an ASS is located in district 5, then no ASS is needed
  % for districts 2, 3, 6, 8 and 9;
  % if no ASS is located in district 5, then one of the
  % districts  2, 3, 6, 8 and 9 should have an ASS:
/*9*/       S2+S3+   S5+S6+   S8+S9          #= 1,
```

```
  % if an ASS is located in district 6, then no ASS is needed
  % for districts 3, 4, 5, 7 and 8;
  % if no ASS is located in district 6, then one of the
  % districts  3, 4, 5, 7 and 8 should have an ASS:
/*10*/        S3+S4+S5+S6+S7+S8              #= 1,

  % if an ASS is located in district 7, then no ASS is needed
  % for districts 4, 6 and 8;
  % if no ASS is located in district 7, then one of the
  % districts  4, 6 and 8 should have an ASS:
/*11*/           S4+   S6+S7+S8              #= 1,

  % if an ASS is located in district 8, then no ASS is needed
  % for districts 5, 6, 7, 9 and 10;
  % if no ASS is located in district 8, then one of the
  % districts  5, 6, 7, 9 and 10 should have an ASS:
/*12*/              S5+S6+S7+S8+S9+S10       #= 1,

  % if an ASS is located in district 9, then no ASS is needed
  % for districts 5, 8, 10 and 11;
  % if no ASS is located in district 9, then one of the
  % districts  5, 8, 10 and 11 should have an ASS:
*13*/              S5+      S8+S9+S10+S11   #= 1,

  % if an ASS is located in district 10, then no ASS is needed
  % for districts 8, 9 and 11;
  % if no ASS is located in district 10, then one of the
  % districts  8, 9 and 11 should have an ASS:
/*14*/                       S8+S9+S10+S11   #= 1,

  % if an ASS is located in district 11, then no ASS is needed
  % for districts 9 and 10;
  % if no ASS is located in district 11, then one of the
  % districts 9 and 10 should have an ASS:
/*15*/                           S9+S10+S11   #= 1,

/*16*/ Number_of_ASS #= S1+S2+S3+S4+S5+S6+S7+S8+S9+S10+S11,

/*17*/ bb_min(search([S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,S11],0,
            first_fail,indomain,complete,[]),Number_of_ASS,
                     bb_options with [strategy:step]),

/*18*/ write([S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,S11]),nl,
/*19*/ final_message([S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,S11]),nl,nl.

/*20*/final_message([S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,S11]):-
/*21*/ print("ASS should be located in districts:"),nl,
/*22*/ ((S1 is 1) -> print(" 1, ") ; print("")),
```

```
/*23*/ ((S2 is 1) -> print(" 2, ") ; print("")),
/*24*/ ((S3 is 1) -> print(" 3, ") ; print("")),
/*25*/ ((S4 is 1) -> print(" 4, ") ; print("")),
/*26*/ ((S5 is 1) -> print(" 5, ") ; print("")),
/*27*/ ((S6 is 1) -> print(" 6, ") ; print("")),
/*28*/ ((S7 is 1) -> print(" 7, ") ; print("")),
/*29*/ ((S8 is 1) -> print(" 8, ") ; print("")),
/*30*/ ((S9 is 1) -> print(" 9, ") ; print("")),
/*31*/ ((S10 is 1) -> print(" 10, ") ; print("")),
/*32*/ ((S11 is 1) -> print(" 11, ") ; print("")).
```

The solution is as follows:

```
Found a solution with cost 3
Found no solution with cost 0.0 .. 2.0
[0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1]
ASS should be located in districts:
 2,  7,  11,
```

It is depicted on Figure 5.9.

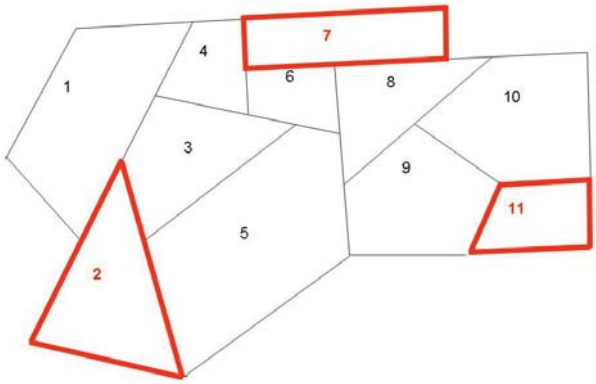


Figure 5.9: Optimum location of ASS

Let's check if there are other optimum solutions. This is done by program

5_13_ambulance_service_all.ecl:

```
/*1*/  :- lib(ic).
/*2*/  :- lib(ic_global).

/*3*/  top :-

/*4*/ Stations =  [S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,S11] ,
/*5*/ Stations :: 0..1,
/*6*/ Number_of_stations is 3,
% Si = 1 - an ASS is located in i-th district
% Si = 0 - an ASS  is not located in i-th district

  % if an ASS is located in district 1, then no ASS is needed
  % for districts 2, 3 and 4;
  % if no ASS is located in district 1, then one of the
  % districts  2,  3 or 4 should have an ASS:
/*7*/   S1+S2+S3+S4 #= 1,

  % if an ASS is located in district 2, then no ASS is needed
  % for districts 1, 3 and 5;
  % if no ASS is located in district 2, then one of the
  % districts  1, 3 and 5should have an ASS:
/*8*/   S1+S2+S3+   S5                      #= 1,

  % if an ASS is located in district 3, then no ASS is needed
  % for districts 1, 2, 4, 5 and 6;
  % if no ASS is located in district 3, then one of the
  % districts  1, 2, 4, 5 and 6 should have an ASS:
/*9*/   S1+S2+S3+S4+S5+S6                   #= 1,

  % if an ASS is located in district 4, then no ASS is needed
  % for districts 1, 3, 6 and 7;
  % if no ASS is located in district 4, then one of the
  % districts  1, 3, 6 and 7 should have an ASS:
/*10*/   S1+   S3+S4+   S6+S7                #= 1,

  % if an ASS is located in district 5, then no ASS is needed
  % for districts 2, 3, 6, 8 and 9;
  % if no ASS is located in district 5, then one of the
  % districts  2, 3, 6, 8 and 9 should have an ASS:
/*11*/      S2+S3+   S5+S6+   S8+S9           #= 1,

  % if an ASS is located in district 6, then no ASS is needed
  % for districts 3, 4, 5, 7 and 8;
  % if no ASS is located in district 6, then one of the
  % districts  3, 4, 5, 7 and 8 should have an ASS:
/*12*/        S3+S4+S5+S6+S7+S8              #= 1,
```

```
  % if an ASS is located in district 7, then no ASS is needed
  % for districts 4, 6 and 8;
  % if no ASS is located in district 7, then one of the
  % districts  4, 6 and 8 should have an ASS:
/*13*/           S4+   S6+S7+S8              #= 1,

  % if an ASS is located in district 8, then no ASS is needed
  % for districts 5, 6, 7, 9 and 10;
  % if no ASS is located in district 8, then one of the
  % districts  5, 6, 7, 9 and 10 should have an ASS:
/*14*/              S5+S6+S7+S8+S9+S10       #= 1,

  % if an ASS is located in district 9, then no ASS is needed
  % for districts 5, 8, 10 and 11;
  % if no ASS is located in district 9, then one of the
  % districts  5, 8, 10 and 11 should have an ASS:
*15*/              S5+      S8+S9+S10+S11   #= 1,

  % if an ASS is located in district 10, then no ASS is needed
  % for districts 8, 9 and 11;
  % if no ASS is located in district 10, then one of the
  % districts  8, 9 and 11 should have an ASS:
/*16*/                       S8+S9+S10+S11   #= 1,

  % if an ASS is located in district 11, then no ASS is needed
  % for districts 9 and 10;
  % if no ASS is located in district 11, then one of the
  % districts 9 and 10 should have an ASS:
/*17*/                           S9+S10+S11   #= 1,

/*18*/ Number_of_stations #= S1+S2+S3+S4+S5+S6+S7+S8+S9+S10+S11,

/*19*/ sumlist(Stations,Number_of_stations),
/*20*/ labeling(Stations),

/*21*/ write("ASS should be located in districts:"),
/*22*/   (foreach(Station,Stations),
/*23*/   count(I,1,11)
/*24*/   do
/*25*/   (Station #= 1 -> (write(" "),write(I)); true)
/*26*/    ),nl, fail.

/*27*/ top:-
/*28*/   writeln("Those are all solutions.").
```

The solution generated is the same as before:

```
ASS should be located in districts: 2 7 11
Those are all solutions.
```

So there is only one optimum solution to the ASS location problem.

5.7 Optimum assignment problems

5.7.1 Tasks allocation for 7 machines - OR approach

Tasks allocation (as any allocation) may sometimes be also optimized, as shown by the following example:

Any one of seven machines may perform any one of seven different tasks, but at different costs, as shown in Table 5.2.

Machine	Task						
	1	2	3	4	5	6	7
1	15	23	43	27	76	43	91
2	45	76	32	39	72	37	48
3	56	45	87	75	34	76	29
4	13	45	34	51	52	21	76
5	45	49	18	48	58	98	23
6	23	25	29	39	52	41	12
7	76	98	86	41	34	76	77

Table 5.2: Task costs for machines

The tasks should be allocated between machines in a way minimizing the overall cost of performing all of them[10].
A program for doing this (`5_14_opty77_OR.ecl`) is as follows:

```
/*1*/   :- lib(ic).
/*2*/   :- lib(branch_and_bound).
      % Uij - Usage of machine i for operation j:
      % Uij = 1 - machine i is used for operation j.
      % Uij = 0 - machine i is not used for operation j.
/*3*/   top :-
/*4*/     Machine_usage =
```

[10]This is an OS-type problem.

```
            [U11,U12,U13,U14,U15,U16,U17,
            U21,U22,U23,U24,U25,U26,U27,
            U31,U32,U33,U34,U35,U36,U37,
            U41,U42,U43,U44,U45,U46,U47,
            U51,U52,U53,U54,U55,U56,U57,
            U61,U62,U63,U64,U65,U66,U67,
            U71,U72,U73,U74,U75,U76,U77],
/*5*/     Machine_usage :: 0..1,
/*6*/     Cost :: 1..700,

/*7*/     U11+U21+U31+U41+U51+U61+U71 #= 1,
/*8*/     U12+U22+U32+U42+U52+U62+U72 #= 1,
/*9*/     U13+U23+U33+U43+U53+U63+U73 #= 1,
/*10*/    U14+U24+U34+U44+U54+U64+U74 #= 1,
/*11*/    U15+U25+U35+U45+U55+U65+U75 #= 1,
/*12*/    U16+U26+U36+U46+U56+U66+U76 #= 1,
/*13*/    U17+U27+U37+U47+U57+U67+U77 #= 1,

/*14*/    U11+U12+U13+U14+U15+U16+U17 #= 1,
/*15*/    U21+U22+U23+U24+U25+U26+U27 #= 1,
/*16*/    U31+U32+U33+U24+U35+U36+U37 #= 1,
/*17*/    U41+U42+U43+U44+U45+U46+U47 #= 1,
/*18*/    U51+U52+U53+U54+U55+U56+U57 #= 1,
/*19*/    U61+U62+U63+U64+U65+U66+U67 #= 1,
/*20*/    U71+U72+U73+U74+U75+U76+U77 #= 1,

/*21*/    Cost #= U11*15+U12*23+U13*43+U14*27+U15*76+U16*43+U17*91 +
            U21*45 + U22*76 + U23*32 + U24*39 + U25*72 + U26*37 + U27*48 +
            U31*56 + U32*45 + U33*87 + U34*75 + U35*34 + U36*76 + U37*29 +
            U41*13 + U42*45 + U43*34 + U44*51 + U45*52 + U46*21 + U47*76 +
            U51*45 + U52*49 + U53*18 + U54*48 + U55*58 + U56*98 + U57*23 +
            U61*23 + U62*25 + U63*29 + U64*39 + U65*52 + U66*41 + U67*12 +
            U71*76 + U72*98 + U73*86 + U74*41 + U75*34 + U76*76 + U77*77,

/*22*/    bb_min(labeling(
            [U11,U12,U13,U14,U15,U16,U17,
            U21,U22,U23,U24,U25,U26,U27,
            U31,U32,U33,U34,U35,U36,U37,
            U41,U42,U43,U44,U45,U46,U47,
            U51,U52,U53,U54,U55,U56,U57,
            U61,U62,U63,U64,U65,U66,U67,
            U71,U72,U73,U74,U75,U76,U77]),
            Cost,bb_options with [strategy:step]),

/*23*/    write("Overall cost: "),writeln(Cost),
/*24*/    display_results(1,[U11,U12,U13,U14,U15,U16,U17],[15,23,43,27,76,43,91]),
/*25*/    display_results(2,[U21,U22,U23,U24,U25,U26,U27],[45,76,32,39,72,37,48]),
/*26*/    display_results(3,[U31,U32,U33,U34,U35,U36,U37],[56,45,87,75,34,76,29]),
/*27*/    display_results(4,[U41,U42,U43,U44,U45,U46,U47],[13,45,34,51,52,21,76]),
```

```
/*28*/    display_results(5,[U51,U52,U53,U54,U55,U56,U57],[45,49,18,48,58,98,23]),
/*29*/    display_results(6,[U61,U62,U63,U64,U65,U66,U67],[23,25,29,39,52,41,12]),
/*30*/    display_results(7,[U71,U72,U73,U74,U75,U76,U77],[76,98,86,41,34,76,77]),
/*31*/     fail.

/*32*/  top:-
/*33*/    writeln("That's all!").

/*34*/  display_results(M,U,C):-
/*35*/    element(N, U, 1),
/*36*/    element(N, C, Op_Cost),
/*37*/    write("Machine "),write(M),write(" is performing operation "),write(N),
                    write(" costing "),write(Op_Cost),writeln(".").
```

The message is:

```
Found a solution with cost 332
Found a solution with cost 309
Found a solution with cost 307
Found a solution with cost 289
Found a solution with cost 259
Found a solution with cost 252
Found a solution with cost 222
Found a solution with cost 211
Found a solution with cost 183
Found a solution with cost 178
Found no solution with cost 1.0 .. 177.0
Overall cost: 178
Machine 1 is performing operation 2 costing 23.
Machine 2 is performing operation 6 costing 37.
Machine 3 is performing operation 5 costing 34.
Machine 4 is performing operation 1 costing 13.
Machine 5 is performing operation 3 costing 18.
Machine 6 is performing operation 7 costing 12.
Machine 7 is performing operation 4 costing 41.
That's all!
```

In order to check whether there are more optimum solutions, program `5_15_opty77_all_OR.ecl` may be used. The cost is fixed at the optimum cost `178` and no optimization is performed:

```
/*1*/   :- lib(ic).
/*2*/   :- lib(branch_and_bound).
      % Uij - Usage of machine i for operation j:
      % Uij = 1 - machine i is used for operation j.
      % Uij = 0 - machine i is not used for operation j.

/*3*/   top :-
/*4*/     Machine_usage =
             [U11,U12,U13,U14,U15,U16,U17,
             U21,U22,U23,U24,U25,U26,U27,
             U31,U32,U33,U34,U35,U36,U37,
             U41,U42,U43,U44,U45,U46,U47,
             U51,U52,U53,U54,U55,U56,U57,
             U61,U62,U63,U64,U65,U66,U67,
             U71,U72,U73,U74,U75,U76,U77],
/*5*/     Machine_usage :: 0..1,
/*6*/     Cost is 178,

/*7*/     U11+U21+U31+U41+U51+U61+U71 #= 1,
/*8*/     U12+U22+U32+U42+U52+U62+U72 #= 1,
/*9*/     U13+U23+U33+U43+U53+U63+U73 #= 1,
/*10*/    U14+U24+U34+U44+U54+U64+U74 #= 1,
/*11*/    U15+U25+U35+U45+U55+U65+U75 #= 1,
/*12*/    U16+U26+U36+U46+U56+U66+U76 #= 1,
/*13*/    U17+U27+U37+U47+U57+U67+U77 #= 1,

/*14*/    U11+U12+U13+U14+U15+U16+U17 #= 1,
/*15*/    U21+U22+U23+U24+U25+U26+U27 #= 1,
/*16*/    U31+U32+U33+U24+U35+U36+U37 #= 1,
/*17*/    U41+U42+U43+U44+U45+U46+U47 #= 1,
/*18*/    U51+U52+U53+U54+U55+U56+U57 #= 1,
/*19*/    U61+U62+U63+U64+U65+U66+U67 #= 1,
/*20*/    U71+U72+U73+U74+U75+U76+U77 #= 1,

/*21*/    Cost #= U11*15+U12*23+U13*43+U14*27+U15*76+U16*43+U17*91 +
             U21*45 + U22*76 + U23*32 + U24*39 + U25*72 + U26*37 + U27*48 +
             U31*56 + U32*45 + U33*87 + U34*75 + U35*34 + U36*76 + U37*29 +
             U41*13 + U42*45 + U43*34 + U44*51 + U45*52 + U46*21 + U47*76 +
             U51*45 + U52*49 + U53*18 + U54*48 + U55*58 + U56*98 + U57*23 +
             U61*23 + U62*25 + U63*29 + U64*39 + U65*52 + U66*41 + U67*12 +
             U71*76 + U72*98 + U73*86 + U74*41 + U75*34 + U76*76 + U77*77,

/*22*/    labeling(
             [U11,U12,U13,U14,U15,U16,U17,
             U21,U22,U23,U24,U25,U26,U27,
             U31,U32,U33,U34,U35,U36,U37,
             U41,U42,U43,U44,U45,U46,U47,
             U51,U52,U53,U54,U55,U56,U57,
```

```
            U61,U62,U63,U64,U65,U66,U67,
            U71,U72,U73,U74,U75,U76,U77]),

/*22*/    write("Overall cost: "),writeln(Cost),
/*23*/    display_results(1,[U11,U12,U13,U14,U15,U16,U17],[15,23,43,27,76,43,91]),
/*24*/    display_results(2,[U21,U22,U23,U24,U25,U26,U27],[45,76,32,39,72,37,48]),
/*25*/    display_results(3,[U31,U32,U33,U34,U35,U36,U37],[56,45,87,75,34,76,29]),
/*26*/    display_results(4,[U41,U42,U43,U44,U45,U46,U47],[13,45,34,51,52,21,76]),
/*27*/    display_results(5,[U51,U52,U53,U54,U55,U56,U57],[45,49,18,48,58,98,23]),
/*28*/    display_results(6,[U61,U62,U63,U64,U65,U66,U67],[23,25,29,39,52,41,12]),
/*29*/    display_results(7,[U71,U72,U73,U74,U75,U76,U77],[76,98,86,41,34,76,77]),
/*30*/    fail.

/*31*/  top:-
/*32*/    writeln("That's all!").

/*33*/  display_results(M,U,C):-
/*34*/    element(N, U, 1),
/*35*/    element(N, C, Op_Cost),
/*36*/    write("Machine "),write(M),write(" is performing operation "),write(N),
                    write(" costing "),write(Op_Cost),writeln(".").
```

There is only a single optimum solution. The message generated is:

```
Overall cost: 178
Machine 1 is performing operation 2 costing 23.
Machine 2 is performing operation 6 costing 37.
Machine 3 is performing operation 5 costing 34.
Machine 4 is performing operation 1 costing 13.
Machine 5 is performing operation 3 costing 18.
Machine 6 is performing operation 7 costing 12.
Machine 7 is performing operation 4 costing 41.
That's all!
```

5.7.2 Tasks allocation for 7 machines - CLP approach

As before, the CLP approach is more parsimonious than the OR approach with respect to the number of variables needed to solve the problem. This is well demonstrated by program `5_16_opty77_CLP.ecl`[11]:

[11] This is an OS-type problem.

```
/*1*/   :- lib(ic).
/*2*/   :- lib(branch_and_bound).

/*3*/   top :-
/*4*/         [O1,O2,O3,O4,O5,O6,O7] :: 1..7,
/*5*/         [C1,C2,C3,C4,C5,C6,C7] :: 1..100,
/*6*/         Cost :: 1..700,
/*7*/         alldifferent([O1,O2,O3,O4,O5,O6,O7]),

/*8*/         element(O1,[15,23,43,27,76,43,91],C1),
/*9*/         element(O2,[45,76,32,39,72,37,48],C2),
/*10*/        element(O3,[56,45,87,75,34,76,29],C3),
/*11*/        element(O4,[13,45,34,51,52,21,76],C4),
/*12*/        element(O5,[45,49,18,48,58,98,23],C5),
/*13*/        element(O6,[23,25,29,39,52,41,12],C6),
/*14*/        element(O7,[76,98,86,41,34,76,77],C7),

/*15*/        Cost #= C1+C2+C3+C4+C5+C6+C7,
/*16*/        bb_min(labeling([O1,O2,O3,O4,O5,O6,O7]),Cost,
                    bb_options with [strategy:step]),
/*17*/        display_results([O1,C1,O2,C2,O3,C3,O4,C4,O5,C5,O6,C6,O7,C7],1),
/*18*/        write("Overall cost: "),write(Cost).

/*19*/    display_results([],_).
/*20*/    display_results([A,B|R],N):-
/*21*/        write("Machine "),write(N),write(" is performing operation "),
/*22*/        write(A),write(" costing "),write(B),write("."),nl,
/*23*/        M is N1, +
/*24*/        display_results(R,M).
```

The message generated is exactly the same as for program `5_14_opty77_OR.ecl`.

To check for multiple optimum solutions the program `5_17_opty77_all_CLP.ecl` is used:

```
/*1*/   :- lib(ic).
/*2*/   :- lib(branch_and_bound).

/*3*/   top :-
/*4*/         [O1,O2,O3,O4,O5,O6,O7] :: 1..7,
/*5*/         [C1,C2,C3,C4,C5,C6,C7] :: 1..100,
/*6*/         Cost is 178,
/*7*/         alldifferent([O1,O2,O3,O4,O5,O6,O7]),
```

```
/*8*/         element(O1,[15,23,43,27,76,43,91],C1),
/*9*/         element(O2,[45,76,32,39,72,37,48],C2),
/*10*/        element(O3,[56,45,87,75,34,76,29],C3),
/*11*/        element(O4,[13,45,34,51,52,21,76],C4),
/*12*/        element(O5,[45,49,18,48,58,98,23],C5),
/*13*/        element(O6,[23,25,29,39,52,41,12],C6),
/*14*/        element(O7,[76,98,86,41,34,76,77],C7),

/*15*/        Cost #= C1+C2+C3+C4+C5+C6+C7,
/*16*/        labeling([O1,O2,O3,O4,O5,O6,O7]),
/*17*/        display_results([O1,C1,O2,C2,O3,C3,O4,C4,O5,C5,O6,C6,O7,C7],1),
/*18*/        write("Overall cost: "),write(Cost),
/*19*/        fail.

/*20*/    top:-
/*21*/        writeln("That's all!").

/*22*/    display_results([],_).
/*23*/    display_results([A,B|R],N):-
/*24*/         write("Machine "),write(N),write(" is performing operation "),
/*25*/         write(A),write(" costing "),write(B),write("."),nl,
/*26*/         M is N+1,
/*27*/         display_results(R,M).
```

Obviously, the message generated is exactly the same as for the already discussed program `5_14_opty77_all_OR.ecl`.

5.7.3 Delivering mining output 1

Transport- and production problems, which have been from the beginning of OR successfully solved by OR techniques, are also rewarding problems for CLP techniques. Consider the following example:

Three mines m1, m2 and m3 deliver their output to five stockyards s1, s2, s3, s4 i s5 at different locations. The capacity of each stockyard equals 400 ton of output per month, while the monthly outputs equals 600 ton for mine m1 and 700 ton for mines m2 and m3. The production cost for one ton of output are respectively 108, 96 i 102 MU. The delivery costs for one ton of output are shown in Table 5.3.

How large should the output of mines be and how much output should the mines deliver to the stockyard in order to minimize the overall cost of production and transportation? This problem is solved by program `5_18_mines_1.ecl`[12]:

[12]This is an OS-type problem.

Mine	Stockyard				
	s1	s2	s3	s4	s5
m1	14	5	9	24	15
m2	30	24	11	8	19
m3	9	22	15	7	18

Table 5.3: Deliver costs for mine outputs

```
  /*1*/   :- lib(ic).
/*2*/   :- lib(branch_and_bound).

/*3*/   top :-

 % Mine m1 is delivering A1 tons of output to stockyard s1,
 % A2 ton to stockyard s2,...:
/*4*/       [A1,A2,A3,A4,A5] :: 0..600,
 % Mine m2 is delivering B1 tons of output to stockyard s1,
 % B2 ton to stockyard s2,...:
/*5*/       [B1,B2,B3,B4,B5] :: 0..700,
 % Mine m3 is delivering C1 tons of output to stockyard s1,
 % C2 ton to stockyard s2,...:
/*6*/       [C1,C2,C3,C4,C5] :: 0..700,

/*7*/       Cost :: 0..300000,

/*8*/       A1+A2+A3+A4+A5 #=600,    % Output of mine m1
/*9*/       B1+B2+B3+B4+B5 #=700,    % Output of mine m2
/*10*/      C1+C2+C3+C4+C5 #=700,    % Output of mine m3

/*11*/      A1+B1+C1 #=400,         % Capacity of stockyard s1
/*12*/      A2+B2+C2 #=400,         % Capacity of stockyard s2
/*13*/      A3+B3+C3 #=400,         % Capacity of stockyard s3
/*14*/      A4+B4+C4 #=400,         % Capacity of stockyard s4
/*15*/      A5+B5+C5 #=400,         % Capacity of stockyard s5

% Overall cost (sum of production and deliver costs):
/*16*/      Cost #= 14*A1+5*A2+9*A3+24*A4+15*A5+
                    30*B1+24*B2+11*B3+8*B4+19*B5+
                    9*C1+22*C2+15*C3+7*C4+18*C5+
                    108*A1+108*A2+108*A3+108*A4+108*A5+
                    96*B1+96*B2+96*B3+96*B4+96*B5+
                    102*C1+102*C2+102*C3+102*C4+102*C5,

/*17*/      bb_min(search([A1,A2,A3,A4,A5,B1,B2,B3,B4,B5,
           C1,C2,C3,C4,C5],0,first_fail,indomain,complete,[]),
```

```
                    Cost,bb_options with [strategy:continue]),

/*18*/write("Mine m1 has to deliver to:"),nl,
/*19*/      write("stockyard s1 "),write(A1),write(" tons of output."),nl,
/*20*/      write("stockyard s2 "),write(A2),write(" tons of output."),nl,
/*21*/      write("stockyard s3 "),write(A3),write(" tons of output."),nl,
/*22*/      write("stockyard s4 "),write(A4),write(" tons of output."),nl,
/*23*/      write("stockyard s5 "),write(A5),write(" tons of output."),nl,nl,

/*24*/write("Mine m2 has to deliver to:"),nl,
/*25*/      write("stockyard s1 "),write(B1),write(" tons of output."),nl,
/*26*/      write("stockyard s2 "),write(B2),write(" tons of output."),nl,
/*27*/      write("stockyard s3 "),write(B3),write(" tons of output."),nl,
/*28*/      write("stockyard s4 "),write(B4),write(" tons of output."),nl,
/*29*/      write("stockyard s5 "),write(B5),write(" tons of output."),nl,nl,

/*30*/write("Mine m3 has to deliver to:"),nl,
/*31*/      write("stockyard s1 "),write(C1),write(" tons of output."),nl,
/*32*/      write("stockyard s2 "),write(C2),write(" tons of output."),nl,
/*33*/      write("stockyard s3 "),write(C3),write(" tons of output."),nl,
/*34*/      write("stockyard s4 "),write(C4),write(" tons of output."),nl,
/*35*/      write("stockyard s5 "),write(C5),write(" tons of output."),nl,nl,

/*36*/write("Overall minimum cost of production and delivery:"),
          write(Cost),nl,nl.
```

The (slowly) generated message is:

```
Found a solution with cost 233000
Found a solution with cost 232999
Found a solution with cost 232998
Found a solution with cost 232997
...
Found a solution with cost 232964
Found a solution with cost 232963
Found a solution with cost 232962
Found a solution with cost 232961
Found a solution with cost 232960
...
```

It takes a long time to et the solution:

```
...
Found a solution with cost 223100
Found no solution with cost 0.0 .. 223000.0

Mine m1 has to deliver to:
stockyard s1 000  tons of output.
stockyard s2 400  tons of output.
```

```
stockyard s3 000  tons of output.
stockyard s4 000  tons of output.
stockyard s5 200  tons of output.

Mine m2 has to deliver to:
stockyard s1 000  tons of output.
stockyard s2 000  tons of output.
stockyard s3 400  tons of output.
stockyard s4 100  tons of output.
stockyard s5 200  tons of output.

Mine m3 has to deliver to:
stockyard s1 400  tons of output.
stockyard s2 000  tons of output.
stockyard s3 000  tons of output.
stockyard s4 300  tons of output.
stockyard s5 000  tons of output.

Overall minimum cost of production and delivery: 223100
```

5.7.4 Delivering mining output 2

The slowness of the `5_18_mines_1.ecl` is partially due to the extent of domains declared in lines `/*4*/`, `/*5*/` and `/*6*/`. To accelerate the computations we could express the domains in hundreds of tons, swapping the lines `/*4*/`,..., `/*15*/` by the following:

```
 % Mine m1 is delivering A1 hundred tons to stockyard s1,
 % A2 hundred tons to stockyard s2,...:
/*4*/       [A1,A2,A3,A4,A5] :: 0..6,
 % Mine m2 is delivering B1 hundred tons to stockyard s1,
 % B2 hundred tons to stockyard s2,...:
/*5*/       [B1,B2,B3,B4,B5] :: 0..7,
 % Mine m3 is delivering C1 hundred tons to stockyard s1,
 % C2 hundred tons to stockyard s2,...:
/*6*/       [C1,C2,C3,C4,C5] :: 0..7,

/*7*/       Cost :: 0..2500,

/*8*/       A1+A2+A3+A4+A5 #=6,    % Output of mine m1
/*9*/       B1+B2+B3+B4+B5 #=7,    % Output of mine m2
/*10*/      C1+C2+C3+C4+C5 #=7,    % Output of mine m3

/*11*/      A1+B1+C1 #=4,         % Capacity of stockyard s1
/*12*/      A2+B2+C2 #=4,         % Capacity of stockyard s2
```

```
/*13*/      A3+B3+C3 #=4,         % Capacity of stockyard s3
/*14*/      A4+B4+C4 #=4,         % Capacity of stockyard s4
/*15*/      A5+B5+C5 #=4,         % Capacity of stockyard s5
```

The cost domain is also expressed for hundreds of tons and decreased in view of the results obtained by program `5_18_mines_1.ecl`. Introducing obvious changes to lines `/*18*/`,...,`/*36*/`, the program `5_19_mines_2.ecl`[13] is obtained, which solves the problem in a jiffy generating the message:

```
Found a solution with cost 2328
Found a solution with cost 2310
Found a solution with cost 2292
...
Found a solution with cost 2241
Found a solution with cost 2236
Found a solution with cost 2231
Found no solution with cost 0.0 .. 2230.0

Mine m1 has to deliver to:
stockyard s1 000 tons of output.
stockyard s2 400 tons of output.
stockyard s3 000 tons of output.
stockyard s4 000 tons of output.
stockyard s5 200 tons of output.

Mine m2 has to deliver to:
stockyard s1 000 tons of output.
stockyard s2 000 tons of output.
stockyard s3 400 tons of output.
stockyard s4 100 tons of output.
stockyard s5 200 tons of output.

Mine m3 has to deliver to:
stockyard s1 400 tons of output.
stockyard s2 000 tons of output.
stockyard s3 000 tons of output.
stockyard s4 300 tons of output.
stockyard s5 000 tons of output.

Overall minimum cost of production and delivery: 223100
```

[13] This is an OS-type problem.

5.7.5 Delivering mining output 3

Examples discussed in Sections 5.7.3 and 5.7.4 are *integer programming* examples: the objective function is linear in integer decision variables, and the constraints are equations or inequalities linear in integer decision variables as well. For such problems ECL^iPS^e CPS makes available an efficient *solver* named *eplex*. In *eplex* symbols of arithmetic operations and relations have to be prefixed by `$`. Its application will be illustrated by the already discussed mine production and transportation problem using program `5_20_mines_3.ecl`[14]:

```
/*1*/   :- lib(eplex).
 /*2*/   top :-
/*3*/       solve(_,_).
/*4*/   solve(Cost,Variables):-
/*5*/       Variables = [A1,A2,A3,A4,A5,B1,B2,B3,B4,B5,C1,C2,C3,C4,C5],
/*6*/        Variables $:: 0.0..1.0Inf,
 % A default domain for all variables of problems
 % solved with the \emph{eplex} solver is -1.0Inf..1.0Inf.
 %    An integer solution is to be determined:
/*7*/        integers(Variables),

  % Output of mine m1:
/*8*/        A1+A2+A3+A4+A5 $=600,
  % Output of mine m2:
/*9*/        B1+B2+B3+B4+B5 $=700,
  % Output of mine m3:
/*10*/       C1+C2+C3+C4+C5 $=700,

  % Stockyard capacities:
/*11*/       A1+B1+C1 $=400,
/*12*/       A2+B2+C2 $=400,
/*13*/       A3+B3+C3 $=400,
/*14*/       A4+B4+C4 $=400,
/*15*/       A5+B5+C5 $=400,

/*16*/       Cost $= 14*A1+5*A2+9*A3+24*A4+15*A5+
      30*B1+24*B2+11*B3+8*B4+19*B5+9*C1+22*C2+15*C3+7*C4+18*C5+
      108*A1+108*A2+108*A3+108*A4+108*A5+96*B1+96*B2+96*B3+96*B4+
      96*B5+102*C1+102*C2+102*C3+102*C4+102*C5,

/*17*/       eplex_solver_setup(min(Cost),
/*18*/       eplex_solve(Cost),

/*19*/      write("Mine m1 has to deliver to:"),nl,
/*29*/      write("stockyard s1 = "),write(A1),write(" tons of output."),nl,
```

[14] This is an OS-type problem.

```
/*21*/      write("stockyard s2 = "),write(A2),write(" tons of output."),nl,
/*22*/      write("stockyard s3 = "),write(A3),write(" tons of output."),nl,
/*23*/      write("stockyard s4 = "),write(A4),write(" tons of output."),nl,
/*24*/      write("stockyard s5 = "),write(A5),write(" tons of output."),nl,nl,

/*25*/      write("Mine m2 has to deliver to:"),nl,
/*26*/      write("stockyard s1 = "),write(B1),write(" tons of output."),nl,
/*27*/      write("stockyard s2 = "),write(B2),write(" tons of output."),nl,
/*28*/      write("stockyard s3 = "),write(B3),write(" tons of output."),nl,
/*29*/      write("stockyard s4 = "),write(B4),write(" tons of output."),nl,
/*30*/      write("stockyard s5 = "),write(B5),write(" tons of output."),nl,nl,

/*31*/      write("Mine m3 has to deliver to:"),nl,
/*32*/      write("stockyard s1 = "),write(C1),write(" tons of output."),nl,
/*33*/      write("stockyard s2 = "),write(C2),write(" tons of output."),nl,
/*34*/      write("stockyard s3 = "),write(C3),write(" tons of output."),nl,
/*35*/      write("stockyard s4 = "),write(C4),write(" tons of output."),nl,
/*36*/      write("stockyard s5 = "),write(C5),write(" tons of output."),nl,nl,
/*37*/      write("Overall minimum cost of production and delivery: "),write(Cost).
```

The solution obtained after 0.12 s contains triples:

`Lower_bound..Upper_bound..@ Value of Variable`,

the last one only being of interest:

```
Mine m1 has to deliver to:
stockyard s1 = _6066{0.0 .. 1.79769313486232e+308 @ 0.0} tons of output.
stockyard s2 = _6050{0.0 .. 1.79769313486232e+308 @ 400.0} tons of output.
stockyard s3 = _6034{0.0 .. 1.79769313486232e+308 @ 0.0} tons of output.
stockyard s4 = _6018{0.0 .. 1.79769313486232e+308 @ 0.0} tons of output.
stockyard s5 = _6002{0.0 .. 1.79769313486232e+308 @ 200.0}tons of output.

Mine m2 has to deliver to:
stockyard s1 = _5986{0.0 .. 1.79769313486232e+308 @ 0.0} tons of output.
stockyard s2 = _5970{0.0 .. 1.79769313486232e+308 @ 0.0} tons of output.
stockyard s3 = _5954{0.0 .. 1.79769313486232e+308 @ 400.0} tons of output.
stockyard s4 = _5938{0.0 .. 1.79769313486232e+308 @ 300.0} tons of output.
stockyard s5 = _5922{0.0 .. 1.79769313486232e+308 @ 0.0} tons of output.

Mine m3 has to deliver to:
stockyard s1 = _5906{0.0 .. 1.79769313486232e+308 @ 400.0} tons of output.
stockyard s2 = _5890{0.0 .. 1.79769313486232e+308 @ 0.0} tons of output.
stockyard s3 = _5874{0.0 .. 1.79769313486232e+308 @ 0.0} tons of output.
stockyard s4 = _5858{0.0 .. 1.79769313486232e+308 @ 100.0} tons of output.
stockyard s5 = _5842{0.0 .. 1.79769313486232e+308 @ 200.0}00 tons of output.
```

```
Overall minimum cost of production and delivery:
223100.000
```

5.7.6 Delivering mining output 4

The final massage from programs `5_20_mines_3.ecl` was rather awkward. It could be made better as shown by program `5_21_mines_4.ecl`:

```
/*1*/ :- lib(eplex).
/*2*/ top :-
/*3*/       Variables = [A1,A2,A3,A4,A5,B1,B2,B3,B4,B5,C1,C2,C3,C4,C5],
/*4*/       Variables $:: 0.0..1.0Inf,
/*5*/       integers(Variables),

 % Output of mine m1:
/*6*/       A1+A2+A3+A4+A5 $=600,

 % Output of mine m2:
/*7*/       B1+B2+B3+B4+B5 $=700,

 % Output of mine m3:
/*8*/       C1+C2+C3+C4+C5 $=700,

/*9*/       A1+B1+C1 $=400,
/*10*/      A2+B2+C2 $=400,
/*11*/      A3+B3+C3 $=400,
/*12*/      A4+B4+C4 $=400,
/*13*/      A5+B5+C5 $=400,

/*14*/      Cost $=
                  14*A1+5*A2+9*A3+24*A4+15*A5+
                  30*B1+24*B2+11*B3+8*B4+19*B5+
                  9*C1+22*C2+15*C3+7*C4+18*C5+
                  108*A1+108*A2+108*A3+108*A4+108*A5+
                  96*B1+96*B2+96*B3+96*B4+96*B5+
                  102*C1+102*C2+102*C3+102*C4+102*C5,

/*15*/      eplex_solver_setup(min(Cost)),
/*16*/      eplex_solve(Cost),
/*17*/      eplex_get(vars,Vars),
/*18*/      eplex_get(typed_solution,Vals),
/*19*/      Vars = Vals,nl,

/*20*/     write("Mine m1 has to deliver to:"),nl,
      write("stockyard s1 = "),write(A1),write(" tons of output."),nl,
      write("stockyard s2 = "),write(A2),write(" tons of output."),nl,
```

```
      write("stockyard s3 = "),write(A3),write(" tons of output."),nl,
      write("stockyard s4 = "),write(A4),write(" tons of output."),nl,
      write("stockyard s5 = "),write(A5),write(" tons of output."),nl,nl,

/*21*/     write("Mine m2 has to deliver to:"),nl,
      write("stockyard s1 = "),write(B1),write(" tons of output."),nl,
      write("stockyard s2 = "),write(B2),write(" tons of output."),nl,
      write("stockyard s3 = "),write(B3),write(" tons of output."),nl,
      write("stockyard s4 = "),write(B4),write(" tons of output."),nl,
      write("stockyard s5 = "),write(B5),write(" tons of output."),nl,nl,

/*22*/     write("Mine m3 has to deliver to:"),nl,
      write("stockyard s1 = "),write(C1),write(" tons of output."),nl,
      write("stockyard s2 = "),write(C2),write(" tons of output."),nl,
      write("stockyard s3 = "),write(C3),write(" tons of output."),nl,
      write("stockyard s4 = "),write(C4),write(" tons of output."),nl,
      write("stockyard s5 = "),write(C5),write(" tons of output."),nl,nl,

/*23*/     write("Overall minimum cost of production and delivery: "),write(Cost).
```

The message is:

```
Mine m1 has to deliver to:
      stockyard s1 = 0 tons of output.
      stockyard s2 = 400 tons of output.
      stockyard s3 = 0 tons of output.
      stockyard s4 = 0 tons of output.
      stockyard s5 = 200 tons of output.

Mine m2 has to deliver to:
      stockyard s1 = 0 tons of output.
      stockyard s2 = 0 tons of output.
      stockyard s3 = 400 tons of output.
      stockyard s4 = 300 tons of output.
      stockyard s5 = 0 tons of output.

Mine m3 has to deliver to:
      stockyard s1 = 400 tons of output.
      stockyard s2 = 0 tons of output.
      stockyard s3 = 0 tons of output.
      stockyard s4 = 100 tons of output.
      stockyard s5 = 200 tons of output.


   Overall minimum cost of production and delivery: 223100.0
```

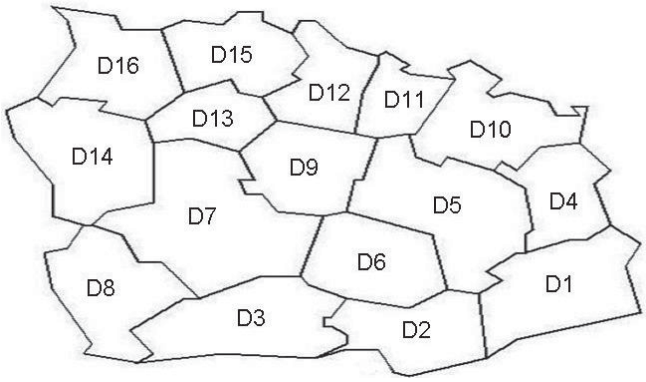


Figure 5.10: The administrative map of Absurdoland

5.7.7 Map coloring

Let's try to test the Graph Coloring Theorem (see Sections 2.4.7 and 3.7.4) for coloring a map.

This has to be done for the administrative map of Absurdoland showing the country's division into districts, see Figure 5.10 where districts are denoted by alphanumeric `Di` symbols, so that a minimum number of colors is used and adjacent districts have different colors.

This is done by program `5_22_map_coloring.ecl`[15]:

```
  /*1*/   :- lib(ic).
/*2*/   :- lib(ic_edge_finder3).
/*3*/   :- lib(branch_and_bound).

/*4*/   top:-
/*5*/       Districts = [D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,D13,D14,D15,D16],
/*6*/       Districts :: 1..16,
/*7*/       L :: 1..16,
/*8*/       color([D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,D13,D14,D15,D16]),
/*9*/       maxlist([D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,D13,D14,D15,D16],L),
/*10*/      minimize(labeling([D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,D13,D14,
                        D15,D16]),L),nl,nl,
/*11*/      write("Minimum number of colors = "),write(L),nl,
```

[15]This is an OS-type problem.

```
/*12*/      write("Districts = "),write("D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,
                        D11,D12,D13,D14,D15,D16"),nl,
/*13*/      write("Colors   = "), write(Districts).

/*14*/  color([D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,D13,D14,D15,D16]):-
/*15*/      D1 #\= D4,     /*16*/      D1 #\= D5,
/*17*/      D1 #\= D2,     /*18*/      D2 #\= D5,
/*19*/      D2 #\= D6,     /*20*/      D2 #\= D3,
/*21*/      D3 #\= D6,     /*22*/      D3 #\= D7,
/*23*/      D3 #\= D8,     /*24*/      D4 #\= D10,
/*25*/      D4 #\= D5,     /*26*/      D5 #\= D10,
/*27*/      D5 #\= D11,    /*28*/      D5 #\= D9,
/*29*/      D5 #\= D6,     /*30*/      D6 #\= D9,
/*31*/      D6 #\= D7,     /*32*/      D7 #\= D9,
/*33*/      D7 #\= D13,    /*34*/      D7 #\= D14,
/*35*/      D7 #\= D8,     /*36*/      D8 #\= D14,
/*37*/      D9 #\= D11,    /*38*/      D9 #\= D12,
/*39*/      D9 #\= D13,    /*40*/      D10 #\= D11,
/*41*/      D11 #\= D12,   /*42*/      D12 #\= D15,
/*43*/      D12 #\= D13,   /*44*/      D13 #\= D15,
/*45*/      D13 #\= D16,   /*46*/      D13 #\= D14,
/*47*/      D14 #\= D16,   /*48*/      D15 #\= D16,
/*49*/      D14 #\= D8.
```

The solution is given by:

```
Found a solution with cost 4
Found no solution with cost 1.0 .. 3.0

Minimum number of colors = 4
 Districts =  D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,D13,D14,D15,D16
 Colours   = [1, 2, 1, 2, 3, 4, 2, 3, 1, 1,  2,  3,  4,  1,  1,  2]
```

The outcome is difficult to understand. It means e.g. that the district `D1` (which corresponds on the list of districts to the first integer `1`) is of different color than the district `D2` (which corresponds on the list of districts to the first integer `2`). If the following coloring code is assumed:

```
 1 = pink      2 = yellow
 3 = white     4 = blue-green,
```

then the map looks as shown in Figure 5.11.

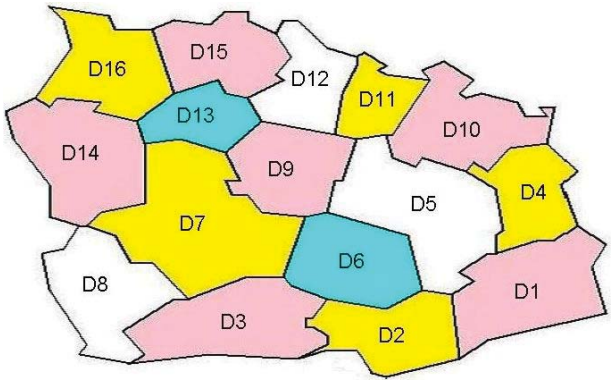


Figure 5.11: Coloring the administrative map of Absurdoland

5.7.8 Fighting for rainfall justice

Another problem of set representations consists in finding such set of elements from other sets that minimizes the cost of elements included. The problem simplest formulation is an OR formulation, corresponding in fact to the canonical form of integer programming problems. This is best illustrated by the following example:

The *World Organization for Total Justice* considers the struggle with unequal rainfalls as one of its basic missions. Rainfall diversity is - as viewed by the Organization - a basic injustice towards Mother Earth and its inhabitants, because to it may eventually be traced all other forms of injustice. However, the Organizations accomplishments on this particular field are - considering 15 years of activity - rather modest. The reason for this was rightly attributed to the lack of funds, the scarcity of which enabled only the most primitive forms of rainfall justice restoration, like rain pipelines transporting rainwater from regions of its abundance to those of its scarcity. Only after the World Government introduced a Common Rain Tax paid by all countries, those with heavy rainfalls as well as those with no rainfalls at all, a fundamental restructuring of the struggle with rainfall diversity could be accomplished. In particular it was deemed necessary to establish the following *Rain Agencies*: 1)*International Rain Fund*, collecting taxes and donations and financing projects, 2)*Airborne Rain Flotilla*, consisting of *Rainfall Causing Airplanes* and *Rainfall Stopping Airplanes*, 3)*Rainfall Satellite Monitoring*, 4)*Local Air Ionizers* to provoke intensive rainfalls, 5)*Rain*

Education Agency, to coordinate rain education at all levels, starting with junior classes on *Rainfall Justice*, through senior classes on *Rain Management*, up to chains of *Educational and Correctional Institutions* (to convince and win over the most ardent opponents of rainfall justice), 6) *Rain Lobbying Agency* to encourage the leaders of nations to contribute additionally to rain funds as well as supporting groups of *Rainpeace* activists, 7) *World Rain Institute*, to manage and finance rain research, to provide *Young Researcher Rain Grants* and to organize *Scientific Rain Summits* on selected football stadiums.

Obviously, to successfully implement such broad range of complicated actions, highly qualified experts are needed. Luckily, three world-reputable rain activists, Professor Hoaxman, Professor Luftmensch and Colonel Baron Fraud of Bluffbury - have been blessed with a progeny that from their earliest days, while listening to discussions at the Family Tables, had acquired such deep knowledge and understanding of rainfall theory and practice, which would be impossible to get at the best universities. Luckily as well, this progeny is - for different reasons - busily looking for new jobs:

So the two daughters of the Colonel Baron had to vacate the posts of vice-chairwoman of the Silly Initiative Monetary Fund. The elderly - it turned out - did not quite understood the meaning of percentages, thereby causing huge financial losses[16], the younger one had problems with grasping the difference between European and Anglo-American billions, causing a number of quite embarrassing and costly blunders[17].

Professors Hoaxman son, after being dismissed from a Sport Academy, was employed as *caddy* by an exclusive Golf Club. There, listening for some time to the palaver of playing bank officials, it occurred to him rightly that he would surely be successful in this profession. Hence he started dreaming about asserting himself in some banking business.

The young Luftmensch in his wildest dreams envisaged himself in uniforms, of course some elegant ones, dark blue or white, with golden braids and multicolored ribbons, and of course with the inseparable personal power and adoring girls all around. In such uniforms he could well manage the *Rain Flotilla*, e.g. using the white uniform to command the *Rainfall Stopping Planes*, and the blue uniform to command the *Rainfall Causing Planes*. Strongly believing that

[16] The dear one should not be blamed: she could not take - because of acute drug-and-booze poisoning - *Home Math* classes on percentages at her beloved *Quick Results College*.

[17] This should really be excused because the poor girl - while studying at the renowned *Quick Results College* - could not attend *Home Math* classes on large numbers; this was due to the urgent need to get rid of the fruit of some exciting night spend with somebody she can't remember.

hard work never hurt anybody, he aspired to simultaneously commanding the *Educational and Correctional Institutions*. Unfortunately, because of advanced emotional instability, his application to the Famous Military Academy has been rejected.

Now, all four of them saw their chance. Following their daddies advice, they submitted applications to organize and run rain agencies. Their applications were quite laconic, containing just the names of agencies and the expected salary in billions of MU; after all the names speak for themselves. All applicants, on the wave of drug-and-booze generated enthusiasm, declared the praiseworthy willingness to organize and run more than one agency. However, the daddies did not harmonize their applications, so there was some overlap: the same agencies were considered worthwhile for more than one applicant, as can be seen from Table 5.9.

Agenda	Applicants			
	Hoaxman Jr	Luftmensch Jr	Older Ms Bluffbury	Younger Ms Bluffbury
Int. Rain Fund	×			×
Rain Flotilla		×	×	
Satellite Monitoring			×	×
Air Ionizers		×		×
Education		×	×	
Lobbing	×			×
Rain Research	×	×		
Salaries (MM MU)	6	5	5	12

Table 5.4: Proposals to organize and run Rain Agencies

The Illuminati Management of the Organization did not mind this overlap, because - like any other management - it liked nothing so much as resolving competence conflicts among their subordinates; if there are no conflicts, the pleasure to resolve them is obviously lost. However, to keep appearances of competitiveness, it was decided that the least expensive applications will be accepted. For this end the program `5_23_rainfall_justice_OR.ecl`[18] may be useful:

[18] This is an OS-type problem.

```
/*1*/  :-lib(ic).
/*2*/  :-lib(branch_and_bound).
/*3*/  top :-
 % Xj = 1 - the application of candidate j has been accepted.
 % Xj = 0 - the application of candidate j has been rejected.
/*4*/  Variables=[X1,X2,X3,X4],
/*5*/  Variables :: 0..1,

/*6*/  X1 + X4 #>= 1,
/*7*/  X2 + X3 #>= 1,
/*8*/  X3 + X4 #>= 1,
/*9*/  X2 + X4 #>= 1,
/*10*/ X3 + X4 #>= 1,
/*11*/ X1 + X4 #>= 1,
/*12*/ X1 + X2 #>= 1,

/*13*/ Cost #= 6*X1 + 5*X2 + 5*X3 + 12*X4,

/*14*/ minimize(search(Variables,0,first_fail,indomain,
          complete,[]),Cost),
/*15*/ writeln("Variables":Variables ),
/*16*/ writeln("Cost":Cost).
```

The message is:

```
Found a solution with cost 17
Found a solution with cost 16
Found no solution with cost 0.0 .. 15.0
Variables : [1, 1, 1, 0]
Cost : 16
```

Luckily only one application (of younger Ms Bluffbury) has been rejected.

Notice the discrepancy between the length of the story and the shortness of the program. Well, there is no *iunctim* between the length of a story (i.e. between the complex circumstances giving raise to the problem) and the length of its program: sometimes to explain the background knowledge of some simple integer programming programs, a lot of things needs to be presented.

5.7.9 Send Most Money

For the popular puzzle *Send More Money* (see Section 4.4.1) an optimization version known as *Send Most Money* may be found, see Kjellerstrand's website [Kjellerstrand-13], which aims at maximizing the value of *Money*. It is given by

program `5_24_smm.ecl`[19] :

```
/*1*/  :-lib(ic).
/*2*/  :-lib(branch_and_bound).
/*3*/  top :-
   % 1) Finding a single solution that maximizes MONEY:
   %    a)A list LD with 8 variables is created. The variables
   %      correspond to the eight letters in "Send Most Money":
/*4*/    length(LD, 8),
   %    b)The domain of LD must include all single-position digits,
   %    because it is not known, which of them will be finally needed:
/*5*/    LD :: 0..9,
   % c)This is the main constraint:
/*6*/    send_most_money(LD, MONEY),
   %    Maximization is needed, but only the built-in  minimize/2
   %    is available, so negative MONEY is to be minimized:
/*7*/    MONEY_NEGATIVE #= -MONEY,

/*8*/    writeln("Determining a single solution for maximum value of MONEY:"),
/*9*/    minimize(search(LD,0,first_fail,indomain,complete,[]),MONEY_NEGATIVE),
/*10*/   writeln([MONEY, LD]),

   % 2) Determining all solutions for maximum value of MONEY:
/*11*/   length(LD2, 8),
/*12*/   LD2 :: 0..9,
/*13*/   findall(LD2, (send_most_money(LD2, MONEY),
              labeling(LD2)), Everything),
/*14*/   length(Everything, Length),
/*15*/   printf("%d solutions for the maximum value of MONEY = %d:\n",
              [Length, MONEY]),
/*16*/   writeln("[S, E, N, D, M, O, T, Y]"),
/*17*/   write_list(Everything).
 /*18*/ send_most_money([S,E,N,D,M,O,T,Y], MONEY) :-
/*19*/   MONEY #= 10000 * M + 1000 * O + 100 * N + 10 * E + Y,
/*20*/   alldifferent([S,E,N,D,M,O,T,Y]),
/*21*/   M #\= 0,
/*22*/   S #\= 0,
/*23*/   1000 * S + 100 * E + 10 * N + D +
                              1000 * M + 100 * O + 10 * S + T #= MONEY.

/*24*/ write_list(Everything):-
/*25*/   member(L,Everything),
/*26*/   writeln(L),
/*27*/   fail.
/*28*/ write_list([]).
```

[19] This is an OS-type problem.

The message is:

```
Determining a single solution for maximum value of MONEY:
Found a solution with cost -10437
Found a solution with cost -10438
Found a solution with cost -10548
Found a solution with cost -10657
Found a solution with cost -10765
Found a solution with cost -10768
Found a solution with cost -10875
Found a solution with cost -10876
Found no solution with cost -10878.0 .. -10877.0
[10876, [9, 7, 8, 2, 1, 0, 4, 6]]

2 solutions for the maximum value of MONEY = 10876:
[S, E, N, D, M, O, T, Y]
[9, 7, 8, 2, 1, 0, 4, 6]
[9, 7, 8, 4, 1, 0, 2, 6]
```

The nesting of `labeling(LD2)` into the `findall` built-in in line `/*13*/` for the purpose of finding all optimum solutions is worth noticing. It may also be applied to other optimum-seeking problems.

5.8 Advanced optimum assignment problems

5.8.1 Warehouse location problem - OR

The basic classical *warehouse location problem* (*WLP*) can be formulated as follows: given a number of customers and a number of warehouse locations, which warehouses should be build in order to minimize the costs of building the warehouses and delivering the demanded goods to the customers[20]. Finding the optimum location for warehouses is of crucial importance from investors' point of view. Therefore many variants of this problem have been solved in OR. Let's start with a rather simple *WLP*: There are 3 potential locations for warehouses serving 5 customers. The building costs and delivery costs are presented by Table 5.5. The solution is given by program `5_25_warehouses_OR.ecl`[21]:

[20] A CLP approach to this problem has been first presented in [van Hentenryck-89].
[21] This is an OS-type problem.

Customer	Warehouse		
	1	2	3
1	5	7	20
2	4	20	1
3	20	2	5
4	20	20	4
5	3	20	8
Building cost	18	20	28

Table 5.5: Delivery and building costs for 3 warehouses and 5 customers

```
/*1*/  :- lib(ic).
/*2*/  :- lib(branch_and_bound).
/*3*/  top:-
/*4*/       warehouses(_,_).

/*5*/  warehouses(Z,Cost):-
  %  Wj = 1 - warehouse j is built
  %  Wj = 0 - warehouse j is not built
  %  Tij = 1 - customer i is serviced by warehouse j
  %  Tij = 0 - customer i is not serviced by warehouse j
/*6*/       Z=[W1,W2,W3,T11,T12,T13,T21,T22,T23,T31,T32,T33,T41,T42,T43,T51,T52,T53],
/*7*/       Z::0..1,

/*8*/       T11 + T12 + T13 #= 1,  % customer 1 is serviced just by one warehouse
/*9*/       T21 + T22 + T23 #= 1,  % customer 2 is serviced just by one warehouse
/*10*/      T31 + T32 + T33 #= 1,  % customer 3 is serviced just by one warehouse
/*11*/      T41 + T42 + T43 #= 1,  % customer 4 is serviced just by one warehouse
/*12*/      T51 + T52 + T53 #= 1,  % customer 5 is serviced just by one warehouse

/*13*/      T11 #=< W1,   % if warehouse 1 is built, it may service customer 1
/*14*/      T21 #=< W1,   % if warehouse 1 is built, it may service customer 2
/*15*/      T31 #=< W1,   % if warehouse 1 is built, it may service customer 3
/*16*/      T41 #=< W1,   % if warehouse 1 is built, it may service customer 4
/*17*/      T51 #=< W1,   % if warehouse 1 is built, it may service customer 5

/*18*/      T12 #=< W2,   % if warehouse 2 is built, it may service customer 1
/*19*/      T22 #=< W2,   % if warehouse 2 is built, it may service customer 2
/*20*/      T32 #=< W2,   % if warehouse 2 is built, it may service customer 3
/*21*/      T42 #=< W2,   % if warehouse 2 is built, it may service customer 4
/*22*/      T52 #=< W2,   % if warehouse 2 is built, it may service customer 5

/*23*/      T13 #=< W3,   % if warehouse 3 is built, it may service customer 1
/*24*/      T23 #=< W3,   % if warehouse 3 is built, it may service customer 2
```

```
/*25*/     T33 #=< W3,   % if warehouse 3 is built, it may service customer 3
/*26*/     T43 #=< W3,   % if warehouse 3 is built, it may service customer 4
/*27*/     T53 #=< W3,   % if warehouse 3 is built, it may service customer 5

/*28*/     Cost #= 18*W1+10*W2+28*W3+5*T11+7*T12+100*T13+4*T21+100*T22+1*T23+
               100*T31+2*T32+5*T33+100*T41+100*T42 +4*T43+3*T51+100*T52+8*T53 ,

/*29*/     bb_min(labeling([W1,W2,W3,T11,T12,T13,T21,T22,T23,T31,T32,T33,
               T41,T42,T43,T51,T52,T53]),Cost, bb_options{strategy:restart}),  nl,

/*30*/     write("List of warehouses: "),writeln([W1,W2,W3]),nl,
/*31*/     write("List of customers and warehouses:"),nl,
/*32*/     writeln("[T11,T12,T13,T21,T22,T23,T31,T32,T33,T41,T42,T43,T51,T52,T53]"),
/*32*/     writeln([T11,T12,T13,T21,T22,T23,T31,T32,T33,T41,T42,T43,T51,T52,T53]),
/*33*/     write("Cost: "),writeln(Cost),nl.
```

The message is:

```
Found a solution with cost 66
Found a solution with cost 64
Found no solution with cost 0.0 .. 63.0

List of warehouses: [1, 0, 1]

List of customers and warehouses:
[T11,T12,T13,T21,T22,T23,T31,T32,T33,T41,T42,T43,T51,T52,T53]
[1,  0,  0,  0,  0,  1,  0,  0,  1,  0,  0,  1,  1,  0,  0]

Cost: 64
```

The meaning of the list of customers and warehouses is as follows:

```
T11 = 1, i.e. customer 1 is served from warehouse 1;
T23 = 1, i.e. customer 2 is served from warehouse 3;
T33 = 1, i.e. customer 3 is served from warehouse 3;
T43 = 1, i.e. customer 4 is served from warehouse 3;
T51 = 1, i.e. customer 5 is served from warehouse 1;
```

5.8.2 Warehouse location problem 1 CLP

The warehouse location problem may be solved using a number of different *CLP* approaches. A CLP version using data from Table 5.5 is given by program

5_26_warehouses_CLP_1.ecl[22]:

```
/*1*/  :- lib(ic).
/*2*/  :- lib(branch_and_bound).
 %op(Precedence, +Associativity, ++Name)+
/*3*/  :- op(960, fx, if).
/*4*/  :- op(950,xfx, then).

/*5*/  top:-
/*6*/          warehouses(_,_,_).

/*7*/  warehouses(Ws,Cs,Cost):-
/*8*/          Ws=[W1,W2,W3],
      % if Wi=0,  warehouse "i" is not build
      % if Wi=1,  warehouse "i" is  build
/*9*/          Ws::0..1,

/*10*/         Cs=[C1,C2,C3,C4,C5],
/*11*/         Cs::1..3,
      %   Cj - number of warehouse  serving the "j"-th customer

/*12*/         element(C1,[5,7,20],Cost_1),
/*13*/         element(C2,[4,20,1],Cost_2),
/*14*/         element(C3,[20,2,5],Cost_3),
/*15*/         element(C4,[20,20,4],Cost_4),
/*16*/         element(C5,[3, 20,8],Cost_5),

         % if warehouse "i" is not established, it won't appear in list Cs:
/*17*/         if (W1 #= 0) then outof(Cs,1),
/*18*/         if (W2 #= 0) then outof(Cs,2),
/*19*/         if (W3 #= 0) then outof(Cs,3),

/*20*/         Cost #= 18*W1+20*W2+28*W3+Cost_1+Cost_2+Cost_3+Cost_4+Cost_5,

/*21*/         bb_min((labeling(Ws),labeling(Cs)),Cost,
                       bb_options{strategy:restart}),

/*22*/         write("   List of warehouses: "),
                  writeln([W1,W2,W3]),
/*23*/         write("   List of customers and warehouses: "),
                  writeln([C1,C2,C3,C4,C5]),
/*24*/         write("   Cost: "),
                  writeln(Cost).

/*25*/ outof([],_).
/*26*/ outof([K|Ks],N):-
```

[22]This is an OS-type problem.

```
/*27*/         K #\= N,
/*28*/         outof(Ks,N).

/*29*/ if Cond then Goal :-
/*30*/         Cond =.. CList,
/*31*/         append(CList, [Bool], RList),
/*32*/         Reified =.. RList,
/*33*/         call(Reified),
/*34*/         call_if(Goal, Bool).

/*35*/ delay call_if(_Goal, Bool) if var(Bool).
/*36*/ call_if(_Goal, 0).
/*37*/ call_if(Goal, 1) :-
/*38*/         call(Goal).
```

The message is:

```
Found a solution with cost 66
Found a solution with cost 64
Found no solution with cost 15.0 .. 63.0

List of warehouses: [1, 0, 1]
List of customers and warehouses: [1, 3, 3, 3, 1]
Cost: 64
```

The meaning of the list of customers and warehouses (`[1, 3, 3, 3, 1]`) is as follows:
- customer `1` is served from warehouse `1`,
- customers verb"2", `3` and `4` are served from warehouse `3`,
- customer `5` is served from warehouse `1`. The meaning of the list of warehouses (`[1, 0, 1]`) is as follows:
only warehouses `1` and `3` will be established.

The use of following built-ins deserves some comments:

- `?Term =.. ?List` succeeds if `List` is the list, which has the name of predicate `Term` as its first element and the predicates arguments, if any, as its successive elements. E.g.:

  ```
  Term =.. [likes,"John",play].
  ```

  gives

  ```
  Term = likes("John",play),
  ```

  and:

```
s([1,4,5,6]) =.. List.
```

gives

```
List = [s,[1,4,5,6]].
```

- `call(+Goal)` succeeds if `Goal` succeeds: it calls the goal `Goal`. This built-in is used to call goals that are grounded only at the time they are called. For lines `/*35*/,..,/*38*/` wait until `Bool` is grounded, then call `Goal`, or simply succeed.

- `op(960, fx, if)` and `op(950,xfx, then)` mean that the "`then`" part from lines `*/17*/`, `*/18*/` and `*/19*/` is evaluated after the "`if`" part was, see Section 2.1.4.

As before, the number of variables needed to model the warehouse location problem *OR-wise* is decisively larger than the number needed to model it *CLP-wise*.

5.8.3 Warehouse location problem 2 CLP

The program `5_26_warehouses_CLP_1.ecl` discussed so far has weak propagation properties[23], which is due to multiple callings of the `element/3` built-in. The next program `5_27_warehouses_CLP_2.ecl`, which is a slightly modified version of the *Warehouse location* program authored by J. Schimpfa (see [Schimpf-10]), has better propagation properties . It uses a heuristic which orders - for each client - warehouses according the the rising delivery cost. This is illustrated for a more complicated problem given by table 5.6:

As before we would like to know, which warehouses should be built, and for which customers, in order to minimize the overall delivery and building cost.

The program `5_26_warehouses_CLP_1` discussed before has rather poor propagation properties, mainly due to the multiple use of the `element/3` built-in. As result, to solve more complicated problems takes long times. The next program `5_27_warehouses__CLP_2.ecl`[24], which is a slightly modified version of the *Warehouse location* program by Schimpf (see [Schimpf-10]), is much better. It is based on a following heuristic: for each customer the warehouses have to be ordered according to rising delivery costs. The program is as follows:

[23] This mean slow convergence for larger problems.
[24] This is an OS-type problem.

Customers	Warehouses			
	1	2	3	4
1	5	7	1	20
2	14	8	100	300
3	2	20	50	12
4	110	2	200	5
5	300	300	8	200
6	3	100	8	5
7	30	40	20	80
8	230	50	70	8
9	20	350	70	98
10	30	450	370	250
Building cost	18	10	28	20

Table 5.6: Delivery and building costs for 4 warehouses and 10 customers

```
/*1*/  :- lib(ic).
/*2*/  :- lib(ic_sets).
/*3*/  :- lib(branch_and_bound).
 /*4*/  top:-
      % declare the data:
/*5*/     building_cost_array(BuildingCostArray),
/*6*/     delivery_cost_array(DeliveryCostArray),
/*7*/     dim(DeliveryCostArray,[NumberOfClients,NumberOfHouses]),
/*8*/     dim(BuildingCostArray,[NumberOfHouses]),

      % declare constraints:
/*9*/     intset(ListOfBuildHouses,1,NumberOfHouses),
/*10*/    (
/*11*/    for(ClientsId, 1, NumberOfClients),
/*12*/    foreach(NumberOfHouseForClient,HousesForClients),
/*13*/    foreach(DeliveryCostForClient,ListOfDeliveryCostsForClients),
/*14*/    param(ListOfBuildHouses,DeliveryCostArray,NumberOfHouses)
/*15*/    do
/*16*/    ListOfDeliveryCosts is
              DeliveryCostArray[ClientsId,1..NumberOfHouses],
/*17*/    element(NumberOfHouseForClient,ListOfDeliveryCosts,DeliveryCostForClient),
/*18*/    NumberOfHouseForClient in ListOfBuildHouses
/*19*/    ),
/*20*/    weight(ListOfBuildHouses,BuildingCostArray,BuildingCost),

          % objective function:
/*21*/    OverallCost #= BuildingCost + sum(ListOfDeliveryCostsForClients),
```

```
     % search and propagation:
/*22*/sort_houses(DeliveryCostArray,SortedListsOfHousesForClients),
/*23*/    minimize(
/*24*/    (
/*25*/    insetdomain(ListOfBuildHouses, increasing, _, _),
/*26*/    labeling(HousesForClients,SortedListsOfHousesForClients),

     % displaying intermediate results:
/*27*/    write("List of  constructed warehouses = "),
               writeln(ListOfBuildHouses),
/*28*/    write("Warehouses associated to clients = "),
               writeln(HousesForClients),
/*29*/    write("List of delivery costs for clients = "),
               writeln(ListOfDeliveryCostsForClients)
/*30*/    ),
/*31*/    OverallCost),nl,

     % displaying final results:
/*32*/    write("List of built warehouses = "),writeln(ListOfBuildHouses),
/*33*/    write("Warehouses associated with clients = "),
               writeln(HousesForClients),
/*34*/    write("Sorted lists of warehouses for clients = "),
               writeln(SortedListsOfHousesForClients),
/*35*/    write("List of delivery costs for clients = "),
               writeln(ListOfDeliveryCostsForClients),
/*36*/    write("Overall cost: "),writeln(OverallCost).

  % heuristics: sort warehouses for all clients in order of increasing delivery cost:
/*37*/  sort_houses(DeliveryCostArray,SortedListsOfHousesForClients) :-
/*38*/   dim(DeliveryCostArray,[NumberOfClients,NumberOfHouses]),
/*39*/    ( for(I,1,NumberOfHouses),
/*40*/    foreach(I,ListOfHouseId)
/*41*/    do
/*42*/    true
/*43*/    ),
/*44*/    (
/*45*/    for(ClientsId, 1, NumberOfClients),
/*46*/    foreach(SortedListOfHousesForClient,SortedListsOfHousesForClients),
/*47*/    param(DeliveryCostArray,NumberOfHouses,ListOfHouseId)
/*48*/    do
/*49*/    DeliveryCosts is DeliveryCostArray[ClientsId,1..NumberOfHouses],
/*50*/    sorting(DeliveryCosts,ListOfHouseId,SortedListOfHousesForClient)
/*51*/    ).

     % bounding variables "HousesForClients"

        % according to the heuristic
/*52*/   labeling(HousesForClients,SortedListsOfHousesForClients) :-
```

```
/*53*/    (
/*54*/    foreach(NumberOfHouseForClient,HousesForClients),
/*55*/    foreach(SortedListOfHousesForClient,SortedListsOfHousesForClients)
/*56*/    do
/*57*/    member(NumberOfHouseForClient,SortedListOfHousesForClient)
/*58*/    ).

      % intermediate constraint: sorting heuristic
/*59*/  sorting(Keys, Values, SortedValues):-
/*60*/    (foreach(K,Keys),
/*61*/    foreach(W,Values),
/*62*/    foreach(K-W,KeyValues)
/*63*/    do
/*64*/    true),
/*65*/    keysort(KeyValues, SortedKeyValues),
/*66*/    (foreach(W,SortedValues),
/*67*/    foreach(_K-W,SortedKeyValues)
/*68*/    do
/*69*/    true).

/*70*/    delivery_cost_array([](
/*71*/    [](5,7,1,20),
/*72*/    [](14,8,100,300),
/*73*/    [](2,20,50,12),
/*74*/    [](110,2,200,5),
/*75*/    [](300,300,8,200),
/*76*/    [](3,100,8,5),
/*77*/    [](30,40,20,80),
/*78*/    [](230,50,70,8),
/*79*/    [](20,350,70,98),
/*80*/    [](30,450,370,250)
/*81*/    )).

/*81*/    building_cost_array([](18,10,28,20)).
```

The message is:

```
List of  built warehouses = [1]
Warehouses associated to clients = [1,1,1,1,1,1,1,1,1,1]
List of delivery costs for clients = [5,14,2,110,300,3,30,230,20,30]
Found a solution with cost 762
List of  built warehouses = [1, 2]
Warehouses associated to clients = [1,2,1,2,1,1,1,2,1,1]
List of delivery costs for clients = [5,8,2,2,300,3,30,50,20,30]
Found a solution with cost 478
List of  built warehouses = [1, 3]
```

```
Warehouses associated to clients = [3,1,1,1,3,1,3,3,1,1]
List of delivery costs for clients = [1,14,2,110,8,3,20,70,20,30]
Found a solution with cost 324
List of  built warehouses = [1,2,3]
Warehouses associated to clients = [3,2,1,2,3,1,3,2,1,1]
List of delivery costs for clients = [1,8,2,2,8,3,20,50,20,30]
Found a solution with cost 200
List of  built warehouses = [1,3,4]
Warehouses associated to clients = [3,1,1,4,3,1,3,4,1,1]
List of delivery costs for clients = [1,14,2,5,8,3,20,8,20,30]
Found a solution with cost 177
Found no solution with cost 102.0 .. 176.0
List of  built warehouses = [1, 3, 4]
Warehouses associated to clients = [3,1,1,4,3,1,3,4,1,1]
Sorted lists of warehouses for clients =
    [[3,1,2,4],[2,1,3,4],[1,4,2,3],[2,4,1,3],[3,4,1,2],
    [1,4,3,2],[3,1,2,4],[4,2,3,1],[1,3,4,2],[1,4,3,2]]
List of delivery costs for clients = [1,14,2,5,8,3,20,8,20,30]
Overall cost: 177
```

5.8.4 Warehouse location problem 3 CLP

An efficient program comparable to the one from Section 5.8.3 may also be designed by not using sets but using the built-in `fromto/4`. This is demonstrated by example `5_28_warehouses_CLP_3.ecl`[25], where the following variables have been used:

- `ListOfClientHouses` - list of variables corresponding to warehouse numbers associated with consecutive customers, e.g. `ListOfClientHouses = [3, 1, 1, 4, 3, 1, 3, 4, 1, 1]` means that customer `5` will be served by warehouse `3`.
- `ListOfHousesBuild` - list of variables denoting warehouses that will be build. E.g. `ListOfHousesBuild = [1,0,1, 1]` means that warehouse `2` is not going to be build.
- `DeliveryCostArray` - one-dimensional array of delivery costs for consecutive clients.

[25] This program has been proposed by Łukasz Domagała.

- `BuildingCostList` - list of building costs for consecutive warehouses.
- `OverallCost` - the sum of building costs and delivery costs.

Program `5_28_warehouses_3.ecl`[26] is:

```
/*1*/  :-lib(ic).
/*2*/  :-lib(ic_global).
/*3*/  :-lib(branch_and_bound).
/*4*/  top:-
/*5*/     declare_data(DeliveryCostArray,BuildingCostList),
/*6*/     constrain(ListOfClientHouses,
                  ListOfHousesBuild,DeliveryCostArray,
                       BuildingCostList,OverallCost),
/*7*/     find_optimum_solution(ListOfClientHouses,
                  OverallCost),
/*8*/     display_results(ListOfClientHouses,
                  ListOfHousesBuild,OverallCost).

/*9*/  declare_data(DeliveryCostArray,BuildingCostList):-
/*10*/    DeliveryCostArray=[](
          /*        H1  H2  H3  H4  */
/*11*/    /* K1 */ [5  ,7  ,1  ,20 ],
/*12*/    /* K2 */ [14 ,8  ,100,300],
/*13*/    /* K3 */ [2  ,20 ,50 ,12 ],
/*14*/    /* K4 */ [110,2  ,200,5  ],
/*15*/    /* K5 */ [300,300,8  ,200],
/*16*/    /* K6 */ [3  ,100,8  ,5  ],
/*17*/    /* K7 */ [30 ,40 ,20 ,80],
/*18*/    /* K8 */ [230,50 ,70 ,8  ],
/*19*/    /* K9 */ [20 ,350,70 ,98 ],
/*20*/    /* K10*/ [30 ,450,370,250]
/*21*/    ),

/*22*/    BuildingCostList=[18,10,28,20].

/*23*/ constrain(ListOfClientHouses,
                     ListOfHousesBuild,DeliveryCostArray,
                         BuildingCostList,OverallCost):-
/*24*/    dim(DeliveryCostArray,[NumberOfClients]),
/*25*/    length(BuildingCostList,MaxNumberOfHouses),
    % Knowing  "NumberOfClients" the unbounded
    % "ListOfClientHouses" is created:
/*26*/    length(ListOfClientHouses,NumberOfClients),
    % Its domain includes all warehouses under consideration
/*27*/    ListOfClientHouses#::[1..MaxNumberOfHouses],
```

[26] This is an OS-type problem.

```
    % Warehous building costs:
/*28*/    (foreach(HouseBuildingCost,BuildingCostList),
/*29*/    foreach(HouseBuild,ListOfHousesBuild),
/*30*/    fromto(ListOfCosts,[
                HouseBuildingCostOr0|ListOfCostsOut],
                      ListOfCostsOut,ListOfCostsForClient),
/*31*/    count(HouseId,1,MaxNumberOfHouses),
/*32*/    param(ListOfClientHouses)
/*33*/    do
     % Number of clients for warehouse number HouseId:
/*34*/    occurrences(HouseId, ListOfClientHouses,
                      NumberOfClientsForHouseiNr),
/*35*/    #>(NumberOfClientsForHouseiNr,0,HouseBuild),
/*36*/    HouseBuildingCostOr0#=
                HouseBuild * HouseBuildingCost
/*37*/    ),

    % Warehous delivery costs:
/*38*/    (foreach(ClientsHouse,ListOfClientHouses),
/*39*/    foreacharg(ListOfDeliveryCosts,DeliveryCostArray),
/*40*/    foreach(CostForClient,ListOfCostsForClient)
/*41*/    do
/*42*/    element(ClientsHouse,ListOfDeliveryCosts,CostForClient)
/*43*/    ),

    % Overall cost determination:
/*44*/    sumlist(ListOfCosts, OverallCost).

/*45*/ find_optimum_solution(ListOfClientHouses,OverallCost):-
/*46*/    BBOptions=bb_options{strategy:continue, from:0},
/*47*/    bb_min(labeling(ListOfClientHouses),OverallCost,BBOptions).

/*48*/ display_results(ListOfClientHouses,ListOfHousesBuild,OverallCost):-
/*49*/    write("Overall Cost = "),write(OverallCost),nl,
/*50*/    write("List of built warehouses = "),write(ListOfHousesBuild),nl,
/*51*/    write("Warehouses associated with clients = "),write(ListOfClientHouses),nl.
```

The message is:

```
Found a solution with cost 762
Found a solution with cost 592
Found a solution with cost 560
Found a solution with cost 498
Found a solution with cost 328
Found a solution with cost 296
```

```
Found a solution with cost 286
Found a solution with cost 220
Found a solution with cost 198
Found a solution with cost 188
Found a solution with cost 181
Found a solution with cost 177
Found no solution with cost 102.0 .. 176.0
Overall Cost=177
List warehouses to be built:[1,0,1,1]
Warehouses associated with clients:[3,1,1,4,3,1,3,4,1,1]
```

5.8.5 Real-valued objective functions

For real-valued objective functions, even if the decision variables are integers, *branch-and-bound* is not delivering: we have to resort to *eplex*. This is illustrated by the following example:
In order to promote tolerance and fight discrimination, the Absurdoland's Ministry of National Brainwashing, after analyzing a number of public surveys, has ordered that the enrollment to any High School in Absurdoland must be at least 10% gay or lesbian. As a result of this, the Happy Town School Authorities are facing a following problem: there are five High School Districts with numbers of straight and gay/lesbian students as shown by Table 5.7, and two High Schools (HS), with mean distances from the districts shown by the same Table.

District	Straight	Gay/lesbian	Distance to HS 1	Distance to HS 2
1	80	15	3	6
2	70	13	1	1.5
3	90	8	2	0.8
4	50	20	2.6	1.8
5	60	15	3	1.2

Table 5.7: Happy Town student population and traveling distances

The School Board policy requires all students from a given district attend the same High School. Assuming that each High School must have an enrollment of at least 130 students, write a program that will minimize the mean total distance student must travel to High Schools while respecting the enrollment restrictions.

Introducing the following notation:
`Di_HS1 = 1` students from district i travel to HS1
`Di_HS2 = 1` students from district i travel to HS2 `Di_HS1 = 0` students from district i do not travel to HS1 `Di_HS2 = 0` students from district i do not travel to HS2 it is possible to formulate the balances:
1) Balance of all students enrolled in `HS1`:

```
D1_HS1 * 80 + D2_HS1 * 70 + D3_HS1 * 90 + D4_HS1 * 50+
         D5_HS1  * 60 >= 130
```

2) Balance of all students enrolled in `HS2`:

```
D1_HS2 * 80 + D2_HS2 * 70 + D3_HS2 * 90 + D4_HS2 * 50+
         D5_HS2  * 60 >= 130
```

3) Balance of gay/lesbian students enrolled in HS1:

```
D1_HS1 * 15 + D2_HS1 * 13 + D3_HS1 * 8 + D4_HS1 * 20 +
D5_HS1  * 15 >= 0.1 *(D1_HS1 * 80 + D2_HS1 * 70 + D3_HS1 * 90 +
D4_HS1 * 50+ D5_HS1  * 60)
```

4) Balance of gay/lesbian students enrolled in HS2:

```
D1_HS2 * 15 + D2_HS2 * 13 + D3_HS2 * 8 + D4_HS2 * 20 +
D5_HS2  * 15 >= 0.1 *(D1_HS2 * 80 + D2_HS2 * 70 + D3_HS2 * 90 +
D4_HS2 * 50+ D5_HS2  * 60)
```

The balances 3) and 4) may be put into a more simple form:
3a) `D1_HS1*7 + D2_HS1*6 - D3_HS1 + D4_HS1*15 + D5_HS1 *9 >= 0`
4a) `D1_HS2*7 + D2_HS2*6 - D3_HS2 + D4_HS2*15 + D5_HS2 *9 >= 0`

Now we can formulate program `5_29_school_enrollment.ecl` for solving this problem:

```
  /*1*/  :- lib(eplex).
/*2*/ top:-+

/*3*/      Variables=[D1_HS1,D1_HS2,D2_HS1,D2_HS2,D3_HS1,D3_HS2,
                                                 D4_HS1,D4_HS2,D5_HS1,D5_HS2],
/*4*/      Variables $:: 0.0..1.0Inf,
/*5*/      integers(Variables),

/*6*/      D1_HS1 + D1_HS2 $= 1,
/*7*/      D2_HS1 + D2_HS2 $= 1,
/*8*/      D3_HS1 + D3_HS2 $= 1,
/*9*/      D4_HS1 + D4_HS2 $= 1,
/*10*/     D5_HS1 + D5_HS2 $= 1,

/*11*/     D1_HS1*80 + D2_HS1*70 + D3_HS1*90 + D4_HS1*50+ D5_HS1 *60 $>= 130,
/*12*/     D1_HS2*80 + D2_HS2*70 + D3_HS2*90 + D4_HS2*50 + D5_HS2 *60 $>= 130,
```

```
/*13*/     D1_HS1*7 + D2_HS1*6 - D3_HS1 + D4_HS1*15 + D5_HS1 *9 $>= 0,
/*14*/     D1_HS2*7 + D2_HS2*6 +  - D3_HS2  +    D4_HS2*15 + D5_HS2 *9 $>= 0,


/*15*/     Distance $= D1_HS1 * 3   + D1_HS2 * 6 +
                                      D2_HS1 * 1   + D2_HS2 * 1.5 +
                                      D3_HS1 * 2   + D3_HS2 * 0.8 +
                                      D4_HS1 * 2.6 + D4_HS2 * 1.8 +
                                      D5_HS1 * 3   + D5_HS2 * 1.2,


/*16*/     eplex_solver_setup(min(Distance)),
/*17*/     eplex_solve(Distance),
/*18*/     eplex_get(vars,Vars),
/*19*/     eplex_get(typed_solution,Vals),
/*20*/     Vars = Vals,nl,
/*21*/     write(Variables),nl,


/*22*/     (foreach(A,["D1_HS1","D1_HS2","D2_HS1","D2_HS2","D3_HS1","D3_HS2",
                          "D4_HS1","D4_HS2","D5_HS1","D5_HS2"]),
/*23*/     foreach(X,[D1_HS1,D1_HS2,D2_HS1,D2_HS2,D3_HS1,D3_HS2,
                           D4_HS1,D4_HS2,D5_HS1,D5_HS2])
/*24*/     do
/*25*/     write(A),write(" = "),write(X),nl).
```

The solution is:

```
[1, 0, 1, 0, 0, 1, 0, 1, 0, 1]
D1_HS1 = 1
D1_HS2 = 0
D2_HS1 = 1
D2_HS2 = 0
D3_HS1 = 0
D3_HS2 = 1
D4_HS1 = 0
D4_HS2 = 1
D5_HS1 = 0
D5_HS2 = 1
```

5.9 Optimum timetabling problems

Timetabling is the process of deciding who should act (or what should happen) in a well-defined time span in order to satisfy a number of constraints and minimize some performance index. In the most elementary case it is the process of defining on the Cartesian product of two sets (the set of actors or actions and the set of time intervals) a subset satisfying constraints and minimizing some objective function, and known as *timetable*.

5.9.1 Fast food bar crew roster

A roster is a list showing the order in which people are to perform a set of duty. A crew roster problem aims at determining an allocation of the duties into rosters satisfying constraints of job regulations and minimizing the number of people involved.

A large fast food bar operate seven days each week and faces the problem of deciding how many employees to use on what day. The bar has a reliable forecast of the number of employees needed for each day of the week, which shows that for Monday 20 employees are needed, for Tuesday – 16, Wednesday - 13, for Thursday – 16, for Friday - 19, Saturday – 14 and for Sunday - 12. The bar hires employees to work at five consecutive days with two consecutive days off. How many employees need to start work each day of the week to minimize the total number of employees hired? The solution is presented by program `5_30_crew_rostering.ecl`[27]:

```
/*1*/  :- lib(ic).
/*2*/  :- lib(branch_and_bound).
/*3*/   top:-
  % Demand for employees working on consecutive days starting with Monday:
/*4*/         Demand = [20,16,13,16,19,14,12],
  % Domains for variables:
  % Mon - number of employees starting work on Monday, etc.
/*5*/         [Mon,Tue,Wed,Thu,Fri,Sat,Sun] :: 0..50,

  % On Mondays are working employees who started on Monday,
  % or on Thursday, or on Friday, or on Saturday, or on Sunday.
  % Monday is a day off for those who started on Tuesday and Wednesday.
/*6*/         Monday #= Mon + Thu + Fri + Sat + Sun,
  % The number of employees working on Monday should meet the demand:
/*7*/         element(1,Demand,D1),
/*8*/         Monday #>= D1,
```

[27] This is an OST-type problem.

```
  % Similar constraints are defined for the remaining days:
/*9*/         Tuesday #= Tue + Fri + Sat + Sun + Mon,
/*10*/        element(2,Demand,D2),
/*11*/        Tuesday #>= D2,
/*12*/        Wednesday #= Wed + Sat + Sun + Mon + Tue,
/*13*/        element(3,Demand,D3),
/*14*/        Wednesday #>= D3,
/*15*/        Thursday #= Thu + Sun + Mon + Tue + Wed,
/*16*/        element(4,Demand,D4),
/*17*/        Thursday #>= D4,
/*18*/        Friday #= Fri + Mon + Tue + Wed + Thu,
/*19*/        element(5,Demand,D5),
/*20*/        Friday #>= D5,
/*21*/        Saturday #= Sat + Tue + Wed + Thu + Fri,
/*22*/        element(6,Demand,D6),
/*23*/        Saturday #>= D6,
/*24*/        Sunday #= Sun + Wed + Thu + Fri + Sat,
/*25*/        element(7,Demand,D7),
/*26*/        Sunday #>= D7,

  % The "bb_min(_)" predicate is used to minimize the number of
  % employees needed to meet the weekly schedule. This is done by
  % simply labeling the variables Mon,Tue,Wed,Thu,Fri,Sat,Sun:
/*27*/        NumberOfEmployees #= Mon+Tue+Wed+Thu+Fri+Sat+Sun,
/*28*/        bb_min(labeling([Mon,Tue,Wed,Thu,Fri,Sat,Sun]),NumberOfEmployees,
                       bb_options with [strategy:step]),

  write("On Mondays "),write(Mon),write(" employees start working."),nl,
  write("On Tuesdays "),write(Tue),write(" employees start working."),nl,
  write("On Wednesday "),write(Wed),write(" employees start working."),nl,

  write("On Thursday  "),write(Thu),write(" employees start working."),nl,
  write("On Friday  "),write(Fri),write(" employees start working."),nl,
  write("On Saturday  "),write(Sat),write(" employees start working."),nl,
  write("On Sunday "),write(Sun),write(" employees start working."),nl,
  write("All together "),write(NumberOfEmployees),write(" employees are needed.")
```

The message is:

```
  Found a solution with cost 35
  Found a solution with cost 34
  ...
  Found a solution with cost 22
  Found no solution with cost 0.0 ..21.0
  On Mondays 8 employees start working.
```

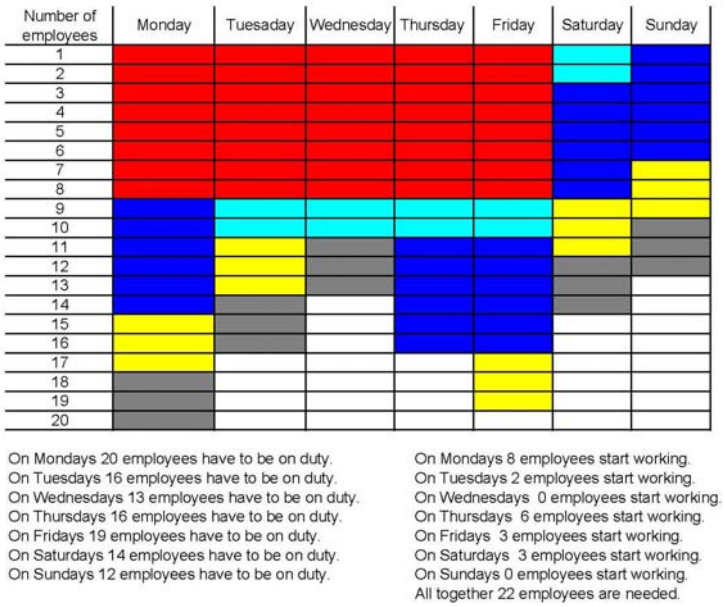


Figure 5.12: Crew roster for fast food bar

```
On Tuesdays 2 employees start working.
On Wednesday 0 employees start working.
On Thursday  6 employees start working.
On Friday  3 employees start working.
On Saturday  3 employees start working.
On Sunday 0 employees start working.
All together 22 employees are needed.
```

The solution has been depicted by Figure 5.12.

The Cartesian product from the timetable definition is the product of the set of days (Monday,Tuesday, Wednesday, Thursday. Friday, Saturday, Sunday) and the employee set of employees (employee_1, employee_2, ..., employee_22). The product corresponds to all "boxes" from Figure 5.15. The solution is given by subsets of the Cartesian product, marked by colours:

- red (employees starting work on Monday and working up to Friday);
- green (employees starting work on Tuesday and working up to Saturday));
- blue (employees starting work on Thursday and working up to Monday);
- yellow (employees starting work on Friday and working up to Tuesday);
- grey (employees starting work on Saturday and working up to Wednesday).

5.9.2 The power and misery of optimization

Optimality (in the strict sense used in this book) means just that the solution optimizes some objective function. The practical value of such optimum solution may (in some cases) be at odds with the theoretical result. To bridge the gap between both notions of optimality, a reformulation of the problem or a change of objective function may often be needed. This is illustrated by the following crew roster problem for toll collectors.

5.9.3 Toll collectors roster

A tollway has a toll plaza with the following staffing demands for each 24-hour period:

```
 from 24 to 6 - 2 collectors
from  6 to 10  - 8 collectors
from 10 to 12 - 4 collectors
from 12 to 16  - 3 collectors
from 16 to 18 - 6 collectors
from 18 to 22  - 5 collectors
from 22 to 24 - 3 collectors
```
[28]

Each collector works four hours, is off one hour, and then works another four hours. A collector may start the work at any hour. How many collectors should start work at each hour in order to minimize the number of collectors hired? The following variables are needed:

```
X1 - number of collectors that start work at 1
X2 - number of collectors that start work at 2
X3 - number of collectors that start work at 3
```

[28] Well, the 24-hour clock system, although not popular in English-speaking countries, is decidedly more CLP-friendly and less error-prone for around the clock time-tabling tasks.

```
X4 - number of collectors that start work at 4
...
X24 -number of collectors that start work at 24.
```

The solution is given by `5_31_toll_collectors.ecl`[29]:

```
/*1*/  :- lib(eplex).
/*2*/  top:-
/*3*/         Variables = [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,
                          X13,X14,X15,X16,X17,X18,X19,X20,X21,X22,X23,X24],
/*4*/         Variables $:: 0.0..1.0Inf,
/*5*/          integers(Variables),

  %  Number of collectors on duty from 24 to 1:
/*6*/         X24+X23+X22+X21+X19+X18+X17+X16 $>= 2,

  %  Number of collectors on duty from 1 to 2:
/*7*/          X1+X24+X23+X22+X20+X19+X18+X17 $>= 2,

  %  Number of collectors on duty from 2 to 3:
/*8*/         X2+X1+X24+X23+X21+X20+X19+X18 $>= 2,

  %  Number of collectors on duty from 3 to 4:
/*9*/          X3+X2+X1+X24+X22+X21+X20+X19 $>= 2,

  %  Number of collectors on duty from 4 to 5:
/*10*/         X4+X3+X2+X1+X23+X22+X21+X20 $>= ),

  %  Number of collectors on duty from 5 to 6:
/*11*/         X5+X4+X3+X2+X24+X23+X22+X21 $>= 2,

  %  Number of collectors on duty from 6 to 7:
/*12*/         X6+X5+X4+X3+X1+X24+X23+X22 $>= 8,

  %  Number of collectors on duty from 7 to 8:
/*13*/         X7+X6+X5+X4+X2+X1+X24+X23 $>= 8,

  %  Number of collectors on duty from 8 to 9:
/*14*/         X8+X7+X6+X5+X3+X2+X1+X24 $>= 8,

  %  Number of collectors on duty from 9 to 10:
/*15*/         X9+X8+X7+X6+X4+X3+X2+X1 $>= 8),

  %  Number of collectors on duty from 10 to 11:
/*16*/         X10+X9+X8+X7+X5+X4+X3+X2 $>= 4,
```

[29] This is an OST-type problem.

```
  %  Number of collectors on duty from 11 to 12:
/*17*/         X11+X10+X9+X8+X6+X5+X4+X3 $>= 4,

  %  Number of collectors on duty from 12 to 13:
/*18*/         X12+X11+X10+X9+X7+X6+X5+X4 $>= 3,

  %  Number of collectors on duty from 13 to 14:
/*19*/  X13+X12+X11+X10+X8+X7+X6+X5 $>= 3,

  %  Number of collectors on duty from 14 to 15:
/*20*/         X14+X13+X12+X11+X9+X8+X7+X6 $>= 3,

  %  Number of collectors on duty from 15 to 16:
/*21*/         X15+X14+X13+X12+X10+X9+X8+X7 $>= 3,

  %  Number of collectors on duty from 16 to 17:
/*22*/         X16+X15+X14+X13+X11+X10+X9+X8 $>= 6,

  %  Number of collectors on duty from 17 to 18:
/*23*/         X17+X16+X15+X14+X12+X11+X10+X9 $>= 6,

  %  Number of collectors on duty from 18 to 19:
/*24*/  X18+X17+X16+X15+X13+X12+X11+X10 $>= 5,

  %  Number of collectors on duty from 19 to 20:
/*25*/         X19+X18+X17+X16+X14+X13+X12+X11 $>= 5,

  %  Number of collectors on duty from 20 to 21:
/*26*/         X20+X19+X18+X17+X15+X14+X13+X12 $>= 5,

  %  Number of collectors on duty from 21 to 22:
/*27*/  X21+X20+X19+X18+X16+X15+X14+X13+X12 $>= 5,

  %  Number of collectors on duty from 22 to 23:
/*28*/         X22+X21+X20+X19+X17+X16+X15+X14 $>= 3,

  %  Number of collectors on duty from 23 to 24:
/*29*/         X23+X22+X21+X20+X18+X17+X16+X15 $>= 3,

/*30*/         NumberOfCollectors $= X1+X2+X3+X4+X5+X6+X7+X8+X9+X10+
                X11+X12+X13+X14+X15+X16+X17+X18+X19+X20+X21+X22+X23+X24,

/*31*/       eplex_solver_setup(min(NumberOfCollectors)),
/*32*/        eplex_solve(NumberOfCollectors),
/*33*/        eplex_get(vars,Vars),
/*34*/        eplex_get(typed_solution,Vals),
/*35*/        Vars = Vals,nl,
```

```
/*36*/       Number is X1+X2+X3+X4+X5+X6+X7+X8+X9+X10+X11+X12+
                X13+X14+X15+X16+X17+X18+X19+X20+X21+X22+X23+X24,
/*37*/        write("Overall number of collectors = "),write(Number),nl,nl,

/*38*/  (foreach(A,["1","2","3","4","5","6","7","8","9","10","11","12",
           "13","14","15","16","17","18","19","20","21","22","23","24"]),
/*39*/  foreach(X,[X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,
          X13,X14,X15,X16,X17,X18,X19,X20,X21,X22,X23,X24])
/*40*/  do
/*41*/  write("Number of collectors starting work at "),
                  write(A),write(" o'clock = "),write(X),nl).
```

As can be seen, the collector balances (lines `/*6*/` - `/*29*/`) are formulated so as to fulfill the main constraint: each collector works 4 hours, has an hour break, and works for another 4 hours.

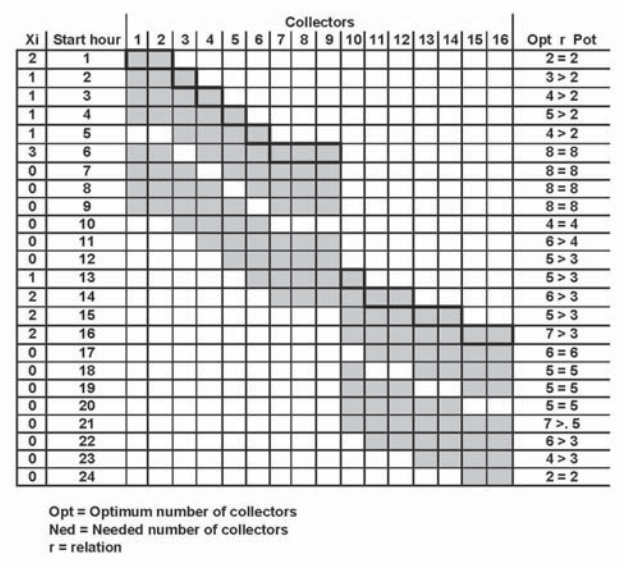
The solution obtained is as follows:

```
      Overall number of collectors = 16

   Number of collectors staring to work at 1 o'clock    = 2
   Number of collectors staring to work at 2 o'clock  = 1
   Number of collectors staring to work at 3 o'clock  = 1
   Number of collectors staring to work at 4 o'clock  = 1
   Number of collectors staring to work at 5 o'clock  = 1
   Number of collectors staring to work at 6 o'clock  = 3
   Number of collectors staring to work at 7 o'clock  = 0
   Number of collectors staring to work at 8 o'clock  = 0
   Number of collectors staring to work at 9 o'clock  = 0
   Number of collectors staring to work at 10 o'clock  = 0
   Number of collectors staring to work at 11 o'clock  = 0
   Number of collectors staring to work at 12 o'clock  = 0
   Number of collectors staring to work at 13 o'clock  = 1
   Number of collectors staring to work at 14 o'clock  = 2
   Number of collectors staring to work at 15 o'clock  = 2
   Number of collectors staring to work at 16 o'clock  = 2
   Number of collectors staring to work at 17 o'clock  = 0
   Number of collectors staring to work at 18 o'clock  = 0
   Number of collectors staring to work at 19 o'clock  = 0
   Number of collectors staring to work at 20 o'clock  = 0
   Number of collectors staring to work at 21 o'clock  = 0
   Number of collectors staring to work at 22 o'clock  = 0
```

```
Number of collectors staring to work at 23 o'clock  = 0
Number of collectors staring to work at 24 o'clock  = 0
```

The solution is depicted by Figure 5.13. Although it is optimum, it is not parsimonious: for many hours the number of collectors on duty is larger then the needed number. This shows that some rethinking and reformulation of the problem is needed.



Xi	Start hour	Collectors 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Opt r Pot
2	1																	2 = 2
1	2																	3 > 2
1	3																	4 > 2
1	4																	5 > 2
1	5																	4 > 2
3	6																	8 = 8
0	7																	8 = 8
0	8																	8 = 8
0	9																	8 = 8
0	10																	4 = 4
0	11																	6 > 4
0	12																	5 > 3
1	13																	5 > 3
2	14																	6 > 3
2	15																	5 > 3
2	16																	7 > 3
0	17																	6 = 6
0	18																	5 = 5
0	19																	5 = 5
0	20																	5 = 5
0	21																	7 >. 5
0	22																	6 > 3
0	23																	4 > 3
0	24																	2 = 2

Opt = Optimum number of collectors
Ned = Needed number of collectors
r = relation

Figure 5.13: Crew roster for toll collectors

5.9.4 Dog Service

The "Dog Service Company" is well-known as breeder, trainer and provider of dogs being experts in discovering smuggled alcohol, tobacco, drugs, explosives and ammunition. The "Great Southern Boarder Crossing" is an important customer of the company. Dogs made available to the crossings authorities may be working any day and night no longer than 6 hours, and after any hour enjoy an hourly rest. A thorough data mining discovered the following pattern of

dog-demand throughout any day and night:

```
from 12 a.m. to 4 a.m. - 2 dogs
from 4 a.m. to 8 a.m. - 4 dogs
from 8 a.m. to 10 p.m. - 6 dogs
from 10 p.m. to 12 p.m. - 8 dogs
from 12 p.m. to 4 a.m. - 6 dogs
from 4 a.m. to 6 a.m. - 4 dogs
from 6 a.m. to 10 a.m. - 2 dogs
from 10 a.m. to 12 a.m. - 3 dogs
```

How many dogs should start working at any hour throughout day and night in order to minimize the overall number of dogs working around the clock?

The following variables are needed;

```
X1 - number of dogs that start working at 1
X2 - number of dogs that start working at 2
...
X24 -number of dogs that start working at 24.
```

The dog balances for each hour are formulated so as to fulfill the main constraint: each each dog works one hour with an hour long break afterwards. The program `5_32_dogs.ecl` looks like this:

```
/*1*/ :- lib(eplex).
/*2*/ top:-
/*3*/  Dogs = [X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,
          X13,X14,X15,X16,X17,X18,X19,X20,X21,X22,X23,X24],

/*4*/  Dogs $:: 0.0..1.0Inf,
/*5*/  integers(Dogs),

  % Number of dogs working from 24 to 1:
/*6*/  X24+X22+X20+X18+X16+X14 $>= 2,

  % Number of dogs working from 1 to 2:
/*7*/  X1+X23+X21+X19+X17+X15 $>= 2,

  % Number of dogs working from 2 to 3:
/*8*/  X2+X24+X22+X20+X18+X16 $>= 2,
```

```
  % Number of dogs working from 3 to 4:
/*9*/  X3+X1+X23+X21+X19+X17 $>= 2,

  % Number of dogs working from 4 to 5:
/*10*/  X4+X2+X24+X22+X20+X18 $>= 4,

  % Number of dogs working from 5 to 6:
/*11*/  X5+X3+X1+X23+X21+X19 $>= 4,

  % Number of dogs working from 6 to 7:
/*12*/  X6+X4+X2+X24+X22+X20 $>= 4,

  % Number of dogs working from 7 to 8:
/*13*/  X7+X5+X3+X1+X23+X21 $>= 4,

  % Number of dogs working from 8 to 9:
/*14*/  X8+X6+X4+X2+X24+X22 $>= 6,

  % Number of dogs working from 9 to 10:
/*15*/  X9+X7+X5+X3+X1+X23 $>= 6,

  % Number of dogs working from 10 to 11:
/*16*/  X10+X8+X6+X4+X2+X24 $>= 8,

  % Number of dogs working from 11 to 12:
/*17*/  X11+X9+X7+X5+X3+X1 $>= 8,

  % Number of dogs working from 12 to 13:
/*18*/  X12+X10+X8+X6+X4+X2 $>= 6,

  % Number of dogs working from 13 to 14:
/*19*/  X13+X11+X9+X7+X5+X3 $>= 6,

  % Number of dogs working from 14 to 15:
/*20*/  X14+X12+X10+X8+X6+X4 $>= 6,

  % Number of dogs working from 15 to 16:
/*21*/  X15+X13+X11+X9+X7+X5 $>= 6,

  % Number of dogs working from 16 to 17:
/*22*/  X16+X14+X12+X10+X8+X6 $>= 4,

  % Number of dogs working from 17 to 18:
/*23*/  X17+X15+X13+X11+X9+X7 $>= 4,

  % Number of dogs working from 18 to 19:
/*24*/  X18+X16+X14+X12+X10+X8 $>= 2,
```

```
  % Number of dogs working from 19 to 20:
/*25*/  X19+X17+X15+X13+X11+X9 $>= 2,

  % Number of dogs working from 20 to 21:
/*26*/  X20+X18+X16+X14+X12+X10 $>= 2,

  % Number of dogs working from 21 to 22:
/*27*/  X21+X19+X17+X15+X13+X11 $>= 2,

  % Number of dogs working from 22 to 23:
/*28*/  X22+X20+X18+X16+X14+X12 $>= 3,

  % Number of dogs working from 23 to 24:
/*29*/  X23+X21+X19+X17+X15+X13 $>= 3,

/*30*/  Number_of_dogs $= X1+X2+X3+X4+X5+X6+X7+X8+X9+X10+X11+X12+
        X13+X14+X15+X16+X17+X18+X19+X20+X21+X22+X23+X24,

/*31*/  eplex_solver_setup(min(Number_of_dogs)),
/*32*/  eplex_solve(Number_of_dogs),
/*33*/  eplex_get(vars,Vars),
/*34*/  eplex_get(typed_solution,Vals),
/*35*/  Vars = Vals,nl,

/*36*/ Number is X1+X2+X3+X4+X5+X6+X7+X8+X9+X10+X11+X12+
          X13+X14+X15+X16+X17+X18+X19+X20+X21+X22+X23+X24,
/*37*/ write("Minimum number of dogs needed = "),write(Number),nl,nl,

/*38*/  (foreach(A,["1","2","3","4","5","6","7","8","9","10","11","12",
           "13","14","15","16","17","18","19","20","21","22","23","24"]),
/*39*/ foreach(X,[X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,
          X13,X14,X15,X16,X17,X18,X19,X20,X21,X22,X23,X24])
/*40*/ do
/*41*/ write("Number of dogs starting work at "),write(A),
                      write(" o'clock  is  "),write(X),nl).
```

The solution is as follows:

```
  Minimum number of dogs needed = 22

  Number of dogs starting work at 1 o'clock is  0
  Number of dogs starting work at 2 o'clock is  5
  Number of dogs starting work at 3 o'clock is  0
  Number of dogs starting work at 4 o'clock is  2
  Number of dogs starting work at 5 o'clock is  4
  Number of dogs starting work at 6 o'clock is  1
  Number of dogs starting work at 7 o'clock is  4
  Number of dogs starting work at 8 o'clock is  0
  Number of dogs starting work at 9 o'clock is  0
```

```
Number of dogs starting work at 10 o'clock is  0
Number of dogs starting work at 11 o'clock is  0
Number of dogs starting work at 12 o'clock is  0
Number of dogs starting work at 13 o'clock is  0
Number of dogs starting work at 14 o'clock is  3
Number of dogs starting work at 15 o'clock is  0
Number of dogs starting work at 16 o'clock is  0
Number of dogs starting work at 17 o'clock is  3
Number of dogs starting work at 18 o'clock is  0
Number of dogs starting work at 19 o'clock is  0
Number of dogs starting work at 20 o'clock is  0
Number of dogs starting work at 21 o'clock is  0
Number of dogs starting work at 22 o'clock is  0
Number of dogs starting work at 23 o'clock is  0
Number of dogs starting work at 24 o'clock is  0
```

It is depicted by the roster from Figure 5.14.

5.9.5 Police officers

The number of optimum solutions with the same value of objective function could - for some problems - be large indeed. Consider the following example:
The City Police Station[30] needs at least, for successive 4-hour intervals around-the-clock, the number of police officers on duty as given by Table 5.8:

Time (hours)	Interval	Number of officers required
2 - 6	1	22
6 - 10	2	55
10 - 14	3	88
14 - 18	4	110
18 - 22	5	44
22 - 2	6	33

Table 5.8: Minimum number of required police officers

Each officer is on duty for 8 consecutive hours, starting at the beginning of some 4-hour interval. The minimum number of police officers needed to meet the schedule is determined by program `5_33_police_officers.ecl`:

```
/*1*/  :-lib(ic).
/*2*/  :-lib(branch_and_bound).
```

[30] This example is from [Wagner-75].

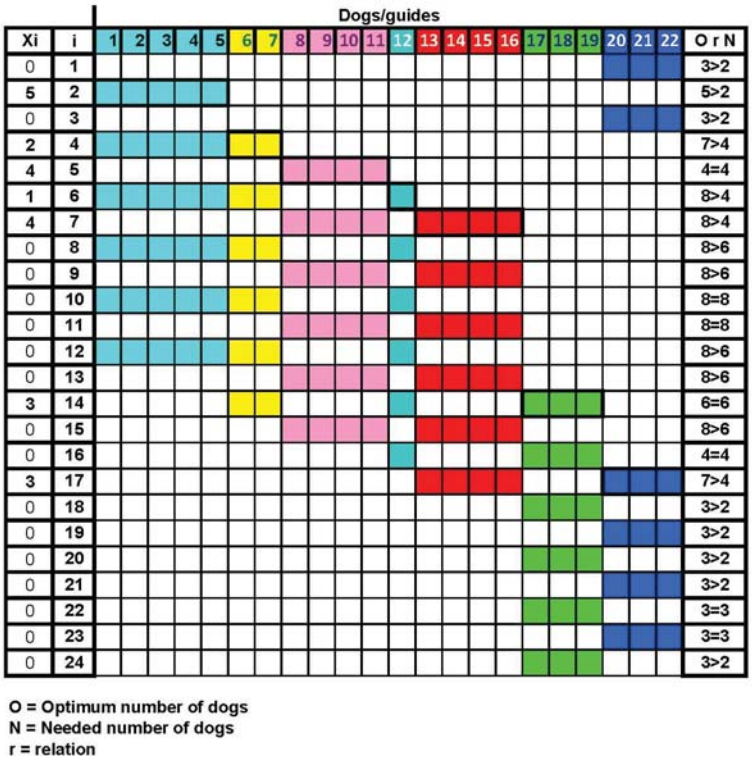


Figure 5.14: Dog roster for Great Southern Boarder Crossing

```
/*3*/  top :-
   % Officers_i - the number of officers starting their service
   % at the beginning of interval i
/*4*/     [Officers_1,Officers_2,Officers_3,Officers_4,Officers_5,Officers_6]
                    ::0..100,

/*5*/     Officers_on_duty_at_interval_1 #= Officers_6 + Officers_1,
/*6*/     Officers_on_duty_at_interval_1 #>= 22,

/*7*/     Officers_on_duty_at_interval_2 #= Officers_1 + Officers_2,
/*8*/     Officers_on_duty_at_interval_2 #>= 55,

/*8*/     Officers_on_duty_at_interval_3 #= Officers_2 + Officers_3,
```

```
/*9*/     Officers_on_duty_at_interval_3 #>= 88,

/*10*/    Officers_on_duty_at_interval_4 #= Officers_3 + Officers_4,
/*11*/    Officers_on_duty_at_interval_4 #>= 110,

/*12*/    Officers_on_duty_at_interval_5 #= Officers_4 + Officers_5,
/*13*/    Officers_on_duty_at_interval_5 #>= 44,

/*14*/    Officers_on_duty_at_interval_6 #= Officers_5 + Officers_6,
/*15*/    Officers_on_duty_at_interval_6 #>= 33,

/*16*/    Sum  #= Officers_1+Officers_2+Officers_3+Officers_4+Officers_5+Officers_6,

/*17*/    bb_min(labeling([Officers_1,Officers_2,Officers_3,
                    Officers_4,Officers_5,Officers_6]),
                    Sum,bb_options with [strategy:step]), nl,

/*18*/    write("Number of officers starting service at the beginning of :"),nl,
/*19*/    write("interval_ 1: "),write(Officers_1),nl,
/*20*/    write("interval_ 2: "),write(Officers_2),nl,
/*21*/    write("interval_ 3: "),write(Officers_3),nl,
/*22*/    write("interval_ 4: "),write(Officers_4),nl,
/*23*/    write("interval_ 5: "),write(Officers_5),nl,
/*24*/    write("interval_ 6: "),write(Officers_6),nl,nl,
/*25*/    write("Minimum number of police officers needed: "),write(Sum),nl,nl,
/*26*/    fail.

/*28*/ top:-
/*28*/    write("That's all!"),nl,nl.
```

The message generated is:

```
Found a solution with cost 198
Found no solution with cost 20.0 .. 197.0
Number of officers starting their service at the beginning of :
interval_ 1: 0
interval_ 2: 55
interval_ 3: 33
interval_ 4: 77
interval_ 5: 0
interval_ 6: 33
Minimum number of police officers needed: 198
That's all!
```

As before, "fail" is impotent for "branch-and-bound". However, a strong suspicion is nurtured about the existence of many more optimum solutions. To dispel any doubt, the approach already presented in Section 5.6.1 is applied once more: the known minimum number of police officers is used to constrict the domain of variable `Sum` for the program `5_34_all_police_officers.ecl` that just determines all feasible solutions for the optimum number 198 of police officers:

```
/*1*/   :-lib(ic).
/*2*/   top :-
/*3*/     assert(counter(0)),
   % Officers_i - the number of officers starting their service
                 at the beginning of interval i
/*4*/     [Officers_1,Officers_2,Officers_3,
                 Officers_4,Officers_5,Officers_6]::0..100,

/*5*/     Officers_on_duty_at_interval_1 #= Officers_6 + Officers_1,
/*6*/     Officers_on_duty_at_interval_1 #>= 22,

/*7*/     Officers_on_duty_at_interval_2 #= Officers_1 + Officers_2,
/*8*/     Officers_on_duty_at_interval_2 #>= 55,

/*8*/     Officers_on_duty_at_interval_3 #= Officers_2 + Officers_3,
/*9*/     Officers_on_duty_at_interval_3 #>= 88,

/*10*/    Officers_on_duty_at_interval_4 #= Officers_3 + Officers_4,
/*11*/    Officers_on_duty_at_interval_4 #>= 110,

/*12*/    Officers_on_duty_at_interval_5 #= Officers_4 + Officers_5,
/*13*/    Officers_on_duty_at_interval_5 #>= 44,

/*14*/    Officers_on_duty_at_interval_6 #= Officers_5 + Officers_6,
/*15*/    Officers_on_duty_at_interval_6 #>= 33,

/*16*/    Officers_1+Officers_2+Officers_3+Officers_4+Officers_5+
            Officers_6  #= 198,

/*17*/    labeling([Officers_1,Officers_2,Officers_3,
                 Officers_4,Officers_5,Officers_6]),

/*18*/    count(Number),
/*19*/    write("Number of solution: "),write(Number),nl,
/*20*/    write("Number of officers starting service at the beginning of:"),nl,
/*21*/    write("interval_ 1: "),write(Officers_1),nl,
/*22*/    write("interval_ 2: "),write(Officers_2),nl,
/*23*/    write("interval_ 3: "),write(Officers_3),nl,
/*24*/    write("interval_ 4: "),write(Officers_4),nl,
```

```
/*25*/    write("interval_ 5: "),write(Officers_5),nl,
/*26*/    write("interval_ 6: "),write(Officers_6),nl,nl,
/*27*/    write("Minimum number of police officers needed: "),write(Sum),nl,nl,
/*28*/    fail.

/*29*/ top:-
/*30*/    write("That's all!"),nl,nl.

/*31*/ count:-
/*32*/    retract(counter(Old)),
/*33*/    New is Old  1, +
/*34*/    assert(counter(New)).
```

This time the program generates 32154 solutions; only the first and last is shown below:

```
Number of solution: 1
Number of officers starting their service at the beginning of
interval_ 1: 0
interval_ 2: 55
interval_ 3: 33
interval_ 4: 77
interval_ 5: 0
interval_ 6: 33
Minimum number of police officers needed: 198

......

Number of solution:  32154:
Number of officers starting their service at the beginning of:
interval_ 1: 55
interval_ 2: 0
interval_ 3: 99
interval_ 4: 11
interval_ 5: 33
interval_ 6: 0
Minimum number of police officers needed: 198
```

The solutions are depicted in Figure 5.15.

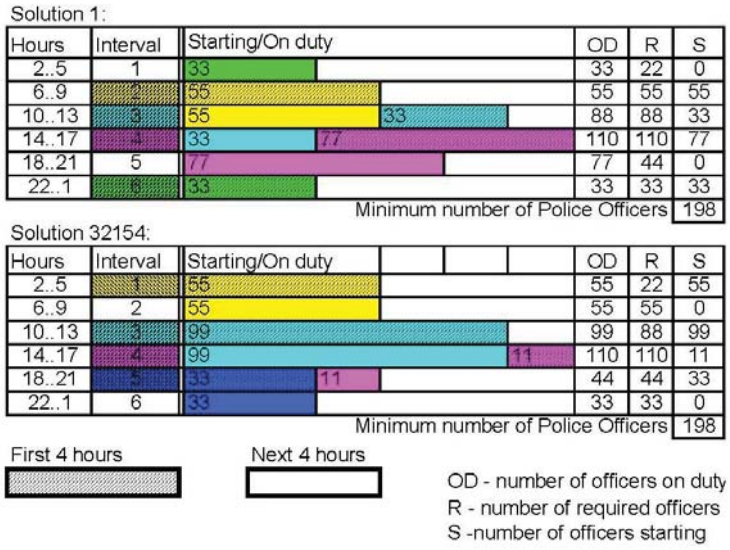


Figure 5.15: Optimum time-tables for police officers

5.10 Optimum sequencing problems

One of the more important applications of ECL^iPS^e is sequencing. *Sequencing* means determining the order of elements from some set so as to fulfill *precedence constraints* and *disjunctive constraints* for those elements while minimizing some objective function.

The meaning of those constraints is as follows:

- *precedence constraints in the time-domain*, which state that some tasks may begin only after some other task of known duration has been completed:

```
Start_of_task_i #>= Start_of_task_j + Duration_of_task_j.
```

- *precedence constraints on the order-line*, which state that some tasks may

may have higher position on some order-line than other tasks:

```
Position_of_task_i #> Position_of_task_j
```

- *disjunctive constraints*, which state that two or more tasks using the same resource must be performed sequentially. For the simple case of two tasks (say task `i` and task `j`), either task `i` must start after task `j` has been completed, or task `j` must start after task `i` has been completed, which can be expressed as:

```
disjunctive(Start_of_task_i,Duration_of_task_i,
               Start_of_task_j, Duration_of_task_j):-
   Start_of_task_i #>= Start_of_task_j + Duration_of_task_j.

disjunctive(Start_of_task_i,Duration_of_task_i,
               Start_of_task_j, Duration_of_task_j):-
   Start_of_task_j #>= Start_of_task_i + Duration_of_task_i.
```

5.10.1 Precedence constraints - building a house

Precedence constraints are typical for a variety of projects, where something must be done before something else may begin. As example may serve building a house. The table of precedence constraints and duration of activities are presented in Table 5.9

The network of precedence constraints for all activities and the activity durations are presented in Figure 5.16. This is an *Activities on Arc* (AoA) network: it uses directed arcs to represent activities.

Assuming that the project begins at time `0`, the problem is to find the shortest duration for the project, i.e, the earliest time of completion. It would be also desirable to determine the *critical path* of the project, i.e. the shortest sequence of activities starting from the initial activity and ending with the final activity. Knowing the critical path of a project is important because the only way to shorten the projects duration is to shorten the duration of critical path activities[31]. The modest house building example is solved by program `5_35_house.ecl`, which offers three options:

[31] The goals mentioned are pursued in OR under the heading *PERT* (*Program Evaluation and Review Technique*) or *CPM* (*Critical Path Method*), developed to assist managers in tracking the progress of large projects. Its first application was to the Polaris submarine

Activity name	Activity duration	Precedence
Foundation	5	Nothing
Walls	6	Foundation
Sanitary installation	3	Foundation
Roof	5	Walls
Electrical installation	3	Walls
Painting	2	Electrical installation Sanitary installation Roof
End	0	Painting

Table 5.9: House building data

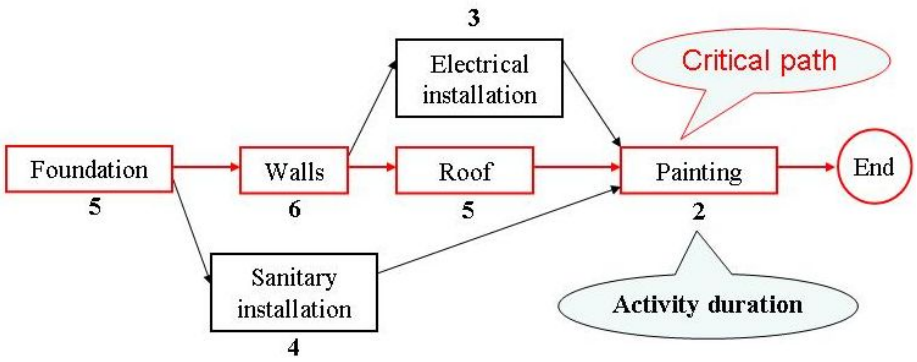


Figure 5.16: AoA network of precedence constraints for house building

1. To find a single optimum solution. For this option lines singled out by `/*?a*/` have to be decommented and lines singled out by `/*?b*/` have to be commented, as shown in the program.

2. To find for a known single optimum solution all other optimum solutions. For this option lines singled out by `/*?b*/` have to be decommented and lines singled out by `/*?a*/` have to be commented.

ballistic missile project; thanks to PERT the project is believed to be completed 18 month ahead of schedule.

3. To determine the *critical path*, for the last change additionally lines `/*x*/` have to be decommented and lines `/*17b`, `/*18*/`, `/*19*/`,...,`/*24*/` have to be commented.

The program `5_35_house.ecl`[32] is as follows:

```
/*1*/  :-lib(ic).
      % for a single minimum-time solution:
/*2a*/ :-lib(branch_and_bound).
 /*3*/  top:-

      % for a single minimum-time solution:
/*4a*/    house(_).

      % for all minimum-time solutions:
/*4b*/%   findall(Operations, house(Operations),_).

/*5*/  house(Operations):-
/*6*/    Operations = [Foundation,Walls,SanitaryInstallation,Roof,
                       ElectricalInstallation,Painting,End],

      % for a single minimum-time solution:
/*7a*/  Operations :: 0..25,

      % for all minimum-time solutions:
/*7b*/% Operations :: 0..18,

/*8*/    Walls #>= Foundation + 5,
/*9*/    SanitaryInstallation #>= Foundation + 5,
/*10*/   Roof #>= Walls + 6,
/*11*/   ElectricalInstallation #>= Walls + 6,
/*12*/   Painting #>=  ElectricalInstallation + 3,
/*13*/   Painting #>= SanitaryInstallation + 4,
/*14*/   Painting #>=  Roof + 5,
/*15*/   End #>= Painting + 2,

   %    for a single minimum-time solution:
/*16a*/  End #=< 25,

   %    for all minimum-time solutions:
/*16b*/%   End #= 18,

   %    for a single minimum-time solution:
/*17a*/   minimize(labeling(Operations),End),nl,
```

[32] This is an OST-type problem.

```
/*x*/%   get_domain(Foundation,DFoundation),
/*x*/%   get_domain(Walls,DWalls),
/*x*/%   get_domain(SanitaryInstallation,DSanitaryInstallation),
/*x*/%   get_domain(Roof,DRoof),
/*x*/%   get_domain(ElectricalInstallation,DElectricalInstallation),
/*x*/%   get_domain(Painting,DPainting),
/*x*/%   get_domain(End,DEnd),

/*x*/%   write("Domain of foundation: "),write(DFoundation),nl,
/*x*/%   write("Domain of walls: "),write(DWalls),nl,
/*x*/%   write("Domain of sanitary installation: "),
/*x*/%                     write(DSanitaryInstallation),nl,
/*x*/%   write("Domain of roof: "),write(DRoof),nl,
/*x*/%   write("Domain of electrical installation: "),
/*x*/%                     write(DElectricalInstallation),nl,
/*x*/%   write("Domain of painting: "),write(DPainting),nl,
/*x*/%   write("Domain of end: "),write(DEnd),nl.

    %     for all minimum-time solutions:
/*17b*/%  labeling(Operations),

/*18*/  write("Starting time for foundation: "),write(Foundation),
/*19*/  nl,write("Starting time for walls: "),write(Walls),nl,
/*20*/  write("Starting time for sanitary installation: "),
                       write(SanitaryInstallation),nl,
/*21*/  write("Starting time for roof: "),write(Roof),nl,
/*22*/  write("Starting time for electrical installation: "),
                      write(ElectricalInstallation),
/*23*/  nl,write("Starting time for painting: "),write(Painting),nl,
/*24*/  write("End: "),write(End),nl.
```

For the case of single optimum solution the message is:

```
Found a solution with cost 18
Starting time for foundation: 0
Starting time for walls: 5
Starting time for sanitary installation: 5
Starting time for roof: 11
Starting time for electrical installation: 11
Starting time for painting: 16
End: 18
```

In case all optimum solutions are to be determined, all lines numbered by

/*?b*/ have to be decommented, lines numbered by /*?a*/ have to be commented, and the shortest duration (End: 18) is used as the upper bound of the Operations domain in line /*7b*/. The modified program (referred to as 5_36_house_all.ecl) then generates 24 optimum solutions, from which the following three are presented:

```
Starting time for foundation: 0
Starting time for walls: 5
Starting time for sanitary installation: 5
Starting time for roof: 11
Starting time for electrical installation: 11
Starting time for painting: 16
End: 18

Starting time for foundation: 0
Starting time for walls: 5
Starting time for sanitary installation: 5
Starting time for roof: 11
Starting time for electrical installation: 12
Starting time for painting: 16
End: 18

Starting time for foundation: 0
Starting time for walls: 5
Starting time for sanitary installation: 5
Starting time for roof: 11
Starting time for electrical installation: 13
Starting time for painting: 16
End: 18
```

Now, if for the last change the x lines are additionally decommented, and the lines /*17b*/, /*18*/, /*19*/,...,/*24*/ are commented, then the modified program (referred to as 5_37_house_crit_path.ecl) generates the following result:

```
Domain of foundation: 0
Domain of walls: 5
Domain of sanitary installation: 5 .. 12
Domain of Roof: 11
Domain of electrical installation: 11 .. 13
Domain of painting: 16
Domain of End: 18
```

Single value domains indicate that the corresponding activities (for founda-

tion, walls, roof and painting) determine the critical path: in order to decrease the projects duration, durations of critical path activities must be decreased.

5.10.2 Disjunctive constraints - limited resources

All resources are limited[33]. Simple scheduling with disjunctive constraints is presented by the program `5_38_disjunctive_sequencing.ecl`[34] for four tasks with variable start times `Z1`,..`Z4`. The tasks with start times `Z2` and `Z3` are disjunctive because they use a common resource, which is just large enough for servicing one of the tasks only. The program is as follows:

```
/*1*/  :-lib(ic).
/*2*/  :-lib(branch_and_bound).
/*3*/  top :-
/*4*/         schedule(_).

/*5*/  schedule([Z1,Z2,Z3,Z4,End]):-
/*6*/         [Z1,Z2,Z3,Z4,End] :: 0..15,
/*7*/         Z1 + 3 #=< Z2,
/*8*/         Z1 + 3 #=< Z3,
/*9*/         Z2 + 4 #=< Z4,
/*10*/        Z3 + 2 #=< Z4,
/*11*/        Z4 + 1 #= End,
/*12*/        disjunctive([Z2,4,Z3,2]),
/*13*/        minimize(labeling([Z1,Z2,Z3,Z4,End]),End),
/*14*/        writeln("Z1 ":Z1),
/*15*/        writeln("Z2 ":Z2),
/*16*/        writeln("Z3 ":Z3),
/*17*/        writeln("Z4 ":Z4),
/*18*/        writeln("End ":End).

/*19*/ disjunctive([Z1,D1,Z2,_]):-
/*20*/        Z1 + D1 #=< Z2.

/*21*/ disjunctive([Z1,_,Z2,D2]):-
/*22*/        Z2 + D2 #=< Z1.
```

The message is:

```
Found a solution with cost 10
Z1 : 0
```

[33] Well, resources of some banks seem to be an exception.
[34] This is an OST-type problem.

```
Z2 : 3
Z3 : 7
Z4 : 9
End  : 10
```

This solution is best presented by a Gantt chart[35] from Figure 5.17a).

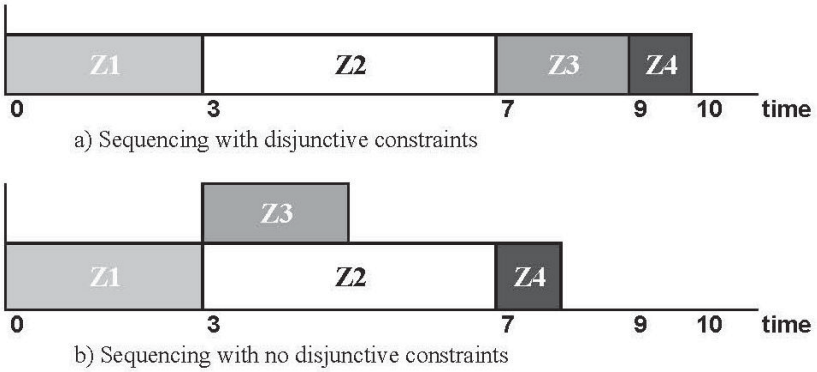


Figure 5.17: Gantt charts for simple sequencing problem

If the disjunctive constraint is removed (i.e. if line `/*12*/` is removed, which means the common resource is large enough to service simultaneously tasks `Z2` and `Z3`), the message is:

[35] A Gantt chart is a graphical representation of resource allocation over time for concurrently performed tasks. It is named after Henry Gantt (1861-1919), a mechanical engineer and management consultant who, in the second decade of the 20th century, developed this visual tool to show the progress of concurrent activities in time, see e.g. http://www.ganttchart.com/History.html). Gantt charts were first used on large construction projects like the Hoover Dam, which started in 1931, and the interstate highway network, which started in 1956.

It should be noted that the Patron of the Economic University in Katowice (Poland), Karol Adamiecki (1866-1933), presented in 1903 a similar technique of describing scheduling programs and applied it to steel mill scheduling at the Iron Works he was employed as Chief Technical Officer.

```
Found a solution with cost 8
Z1 : 0
Z2 : 3
Z3  : 3
Z4 : 7
End  : 8
```

This corresponds to the Gantt chart from Figure 5.17b).

It should be noticed that the number of disjunctions growth rapidly with the number of tasks: for the discussed example with 2 disjunctive tasks there are 2 disjunctions, for 3 tasks there will be $3 * 2 = 6$ disjunctions, for 4 tasks - $4 * 6 = 24$ disjunctions, and for n tasks there will be x_n disjunctions with $x_n = n * x_{n-1}$. This is the reason that solving problems with disjunctions using the approach just presented is numerically inefficient. Therefore, although the model used is easily readable and strongly declarative, in the next chapter a more efficient approach to modeling disjunctions with global constraints `cumulative/4`, `cumulative/5`, and `disjunctive/2` will be presented

5.10.3 Sequencing with conflicting constraints - a photo

Conflicting constraints are constraints that cannot be fulfilled simultaneously. They are common in most real-world applications. Any academic teacher is well-acquainted with having the preferred lecture room at the preferred day and preferred time slice already reserved by a colleague. A simple case of conflicting constraints, inspired by an example from the `Mozart/Oz` system website ([Mozart/Oz-10]) may be stated as follows:

Anna, Ben, Charles, Derek, Eva, Fred, Gary line up for a commemorative photo. Some of them have preferences next to whom they want to stand: :

1. `Anna wants to stand next to Eva and Fred.`
2. `Ben wants to stand next to Anna and Eva.`
3. `Derek wants to stand next to Fred and Charles.`
4. `Garry wants to stand next to Derek and Charles,`

see Figure 5.18.

It is easy to demonstrate that the preferences are contradictory. This is done by the program `5_39_photo_1.ecl`[36] :

[36] This is an FS-type problem.

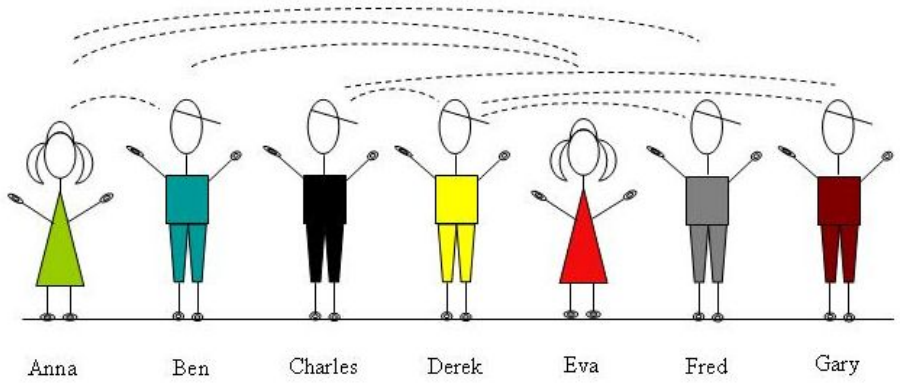


Figure 5.18: Candidates for a commemorative photo and their preferences

```
/*1*/  :-lib(ic).
/*2*/  top :-
/*3*/       Persons = [Anna, Ben, Charles, Derek, Eva, Fred, Gary],
  % The meaning of variables is as follows: Anna is the position number
  % (counting from left) occupied by person "Anna", etc.
/*4*/       Persons :: 1..7,
/*5*/       alldifferent(Persons),

/*6*/       next_to(Anna,Eva),
/*7*/       next_to(Anna,Fred),
/*8*/       next_to(Ben,Anna),
/*9*/       next_to(Ben,Eva),
/*10*/      next_to(Derek,Fred),
/*11*/      next_to(Derek,Charles),
/*12*/      next_to(Gary,Derek),
/*13*/      next_to(Gary,Charles),

/*14*/      search(Persons,0,first_fail,indomain,complete,[]),
/*15*/      writeln("Persons ":Persons).

/*16*/ next_to(X,Y):-
/*17*/      X #= Y + 1.
/*18*/ next_to(X,Y):-
/*19*/      Y #= X + 1.
```

The message generated is `No`. However, if lines `/*6*/` and `/*11*/` are removed

(no attention is payed to one preference by Anna and one by Derek), the solution obtained is:

```
Persons  : [5, 6, 1, 3, 7, 4, 2]
Persons  : [3, 2, 7, 5, 1, 4, 6]
```

and this corresponds to two alignments (see Figure 5.19:

```
Charles - Gary  - Derek  - Fred -  Anna - Ben - Eva
Eva - Ben - Anna - Fred - Derek  - Gary - Charles
```

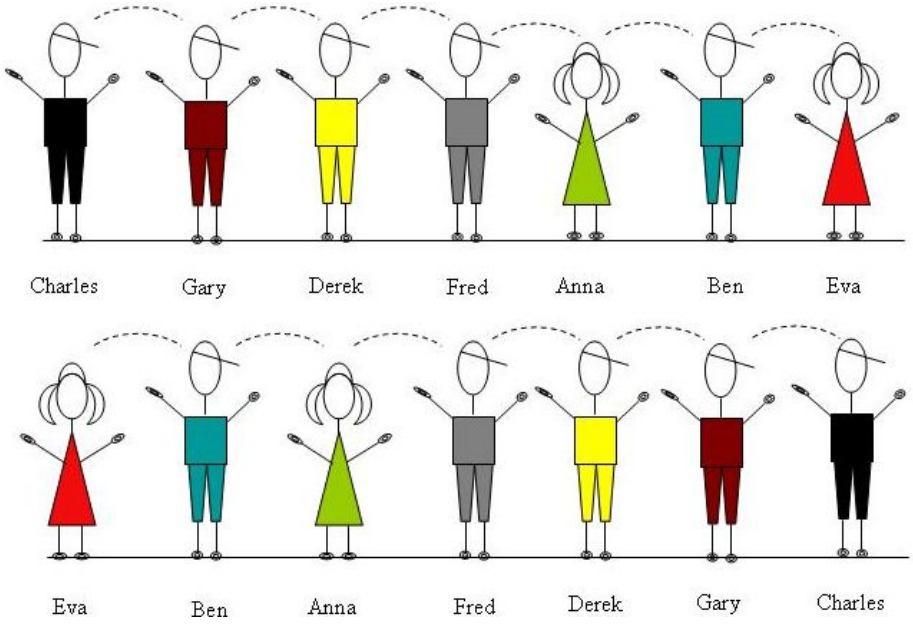


Figure 5.19: Alignment with no constraints 6 and 11.

The question may well be asked how many of the declared preferences may be satisfied at most, if all of them cannot. The maximum number of satisfied

preferences is determined by program `5_40_photo_2.ecl`[37] :

```
/*1*/  :-lib(ic).
/*2*/  :-lib(branch_and_bound).
/*3*/  top :-
/*4*/  preferences(Preferences),
/*5*/  dim(Preferences,[NumberOfPreferences,2]),
/*5*/  dim(Positions, [7]),
/*6*/  Positions :: 1..7,
/*7*/  alldifferent(Positions),

/*8*/  length(Differences, NumberOfPreferences),
/*9*/  (for(I,1,NumberOfPreferences),
/*10*/        fromto(Differences,Out,In,[]), % collect reifications
/*11*/         param(Preferences,Positions)
/*12*/        do
/*13*/        P1 #= Positions[Preferences[I,1]],
/*14*/        P2 #= Positions[Preferences[I,2]],
/*15*/        Difference #= P1-P2,
    % reifying the condition that the modulus of variable Difference be equal 1:
/*16*/        Reif #= (Difference #= 1 or Difference #= -1),
/*17*/        Out = [Reif|In]
/*18*/ ),
/*19*/ flatten_array(Preferences, FlattenedPreferences),
/*20*/ NumberOfFlattenedPreferences #= sum(FlattenedPreferences),
/*21*/ Z :: 0..NumberOfFlattenedPreferences,
    % Z - number of preferences fulfilled:
/*22*/ Z #= sum(Differences),
/*23*/ flatten_array(Positions, Variables),
/*24*/ ZNeg #= -Z,

/*25*/ minimize(search(Variables,0,first_fail,
              indomain,complete,[]),ZNeg),
/*26*/ writeln("Names: Anna, Ben, Charles, Derek, Eva, Fred, Gary"),
/*27*/ writeln("Positions":Positions),
/*28*/ writeln("Number of preferences claimed":
              NumberOfPreferences),
/*29*/ writeln("Number of preferences fulfilled":Z).

 % Remainder about preferences:
 %  [Anna, Ben, Charles, Derek, Eva, Fred, Gary]
 % 1. Anna wishes to stand next to Eva and Fred:
 %     1 next to 5, 1 next to 6
 % 2. Ben wishes to stand next to  Anna and Eva:
 %     2 next to 1, 2 next to 5
 % 3. Derek wishes to stand next to Fred and Charles:
```

[37] This is an OS-type problem.

```
%     4 next to 6, 4 next to 3
% 4. Gary wishes to stand next to Derek and Charles:
%     7 next to 4, 7 next to 3


/*30*/ preferences([](
                []](1,5),
                []](1,6),
                []](2,1),
                []](2,5),
                []](4,6),
                []](4,3),
                []](7,4),
                []](7,3))).
```

The message is:

```
Found a solution with cost -2
Found a solution with cost -3
Found a solution with cost -4
Found a solution with cost -5
Found a solution with cost -6
Found no solution with cost -8.0 .. -7.0
Names: Anna, Ben, Charles, Derek, Eva, Fred, Gary
Positions : [](3, 1, 6, 5, 2, 4, 7)
Number of preferences claimed : 8
Number of preferences fulfilled : 6
```

The solution corresponds to the alignment:

```
Ben,Eva,Anna,Fred,Derek,Charles,Gary
```

The alignment is shown in Figure 5.20. There are two preferences not fulfilled. Obviously, this is not a unique optimum solution: program `5_39_photo_1.ecl`, with lines `/*6*/` and `/*11*/` removed, generated two solutions with different `6` preferences fulfilled.

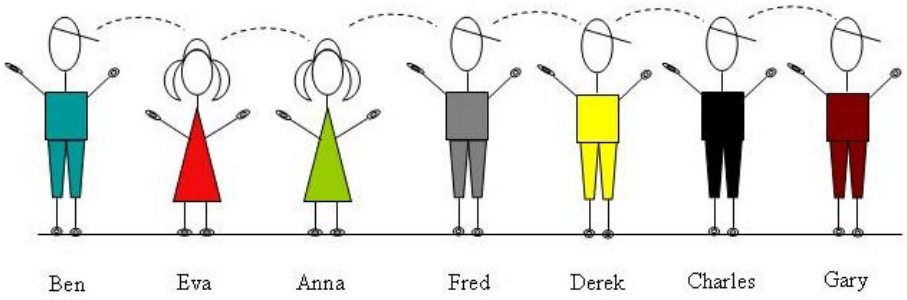


Figure 5.20: Alignments minimizing the number of violated constraints

5.11 Exercises

Five textbooks

Find the optimum solution to the following problem: Bookco Publishers is considering publishing five textbooks. The maximum number of copies of each textbook that can be sold, the variable cost of producing each textbook, the sales price of each textbook, and the fixed cost of a production run for each book are given in Table 5.10. Thus for example, producing 2000 copies of book l brings in a revenue of 2000*50 = 100000 but costs 80000 + 25*2000 = 130000 MU. Bookco can produce at most 10 thousand books. How can they maximize profit?

Textbooks	1	2	3	4	5
Maximum demand	5000	4000	3000	4000	3000
Variable cost (MU)	25	20	15	18	22
Sales price (MU)	50	40	38	32	40
Fixed cost (thousands MU)	80	50	60	30	40

Table 5.10: Textbooks data

Increasing the pension fund while going green at the same time

The increasing number of childless young couples parenting dogs i.e. devoting their attention and love to them, prompted the Absurdoland's Parliament (worried about future declining tax revenues that could accelerate the collapse of the non-sustainable financial pyramid of state-guaranteed

pensions) to look for a mechanism that could make the dogs to generate some income towards the retiree benefits of their masters. A hastily created *Think Tank* considered a number of proposals, but its *Final Scrutiny Report* presented in detail just one recommendation referred to by the acronym `TET`, meaning *Tail Energy Taps*. The idea was to convert the dogs natural (and pretty useless) tail-wagging into useful energy by means of a small computer-controlled and tail-driven electrical power generator loading a small battery. Such `TET`'s would be attached to the dogs behind and interfaced with their tails. All dog masters would be obliged by law to download the energy stored in the battery once per week at the local *Dog Energy Sink* (DES), were it would be used to drive pumps rising water to a cascade of elevated reservoirs, thus converting dogs energy into stored gravitational potential energy for future uses. For lazy dogs an enhanced `TET` model was envisaged, allowing to control the dogs tail-wagging through Internet, and prompt the dogs declining activity by a series of gentle randomly changing mechanical and acoustical signals.

The submitted proposal was enthusiastically supported by the overwhelming majority of parliamentarians. The environmentalists could not praise enough the brilliant idea of tapping a hitherto untouched source of green energy, its sustainability and renewability. Some of them even envisaged `TET`'s to tail-wag the way to energy independence. Parliamentarian do-gooders of all stripes were just enthralled by the bright prospect of creating a number of green technical jobs in newly created branches of `TET` production, `TET` maintenance, `DES` building, and `DES` maintenance, as well as green management jobs in the newly created *District Dog Energy Coordination Outlets* (`DDECO`), the *National Department of Dog Energy* (`NDDE`, in the *Ministry of Energy*), and the *Dog Energy Police Task Force* (`DEPTF`, in the *Ministry of Interior*) for chasing dogs with no `TET`'s attached to their behinds. The objections raised by a small group of `TET`-sceptics, who argued that without massive taxpayer funded subsidies dog-energy is unsustainable, were brushed aside, and a *Dog Energy Bill* was passed quickly. This was welcomed by a number of companies thinking about downloading on the market some of their outdated mechanical and electronic hardware that still gathered dust on the shelves, and give something to do to their underemployed and overpaid unionized manpower. The Bill had plenty of gaps, which could be profitably exploited by shrewd companies. The most important gap was the lack of *canigraphic*[38] data; nobody

[38] A doggie equivalent of "demographic".

in Absurdoland knew how many large dogs and how many small dogs live there, but the Bill distinguished those groups of dogs, prescribing for each group a unique `TET` contraption. The reasonable approach by all companies engaged was to stop worrying about the dog *canilation*[39] and start maximizing profit, pretty sure that their output will be bought by state-controlled `NDDE` outlets at state-established prices.

So did the renowned *Junk Techno Company*, which saw the opportunity to get rid of two of their dust gathering mechanical appliances, `Type_A` and `Type_B`, rejected by both Army and Navy. Both appliances could practically at small cost be converted into correspondingly small dog `TET`'s (`Type_B` appliance only) and large dog `TET`'s (`Type_A` plus `Type_B` appliances). What's more, they constitute the main cost factor of the `TET`'s, the needed accompanying electronics being practically freely available at various electronics graveyards. The daily conversion of `Type_A` appliances could not be larger than `60` items, the daily conversion of `Type_B` appliances could not be larger than `50` items. The furnishing of all produced appliances with electronics could be done at the pace of `120` appliances daily.

On the basis of NDDE-approved purchasing prices for small and large dog `TET`'s, the Company estimated its daily profit from getting rid of a single `Type_A` appliance as being no less than `300` MU, and from getting rid of a single `Type_B` appliance - `500` MU. Write a program that determines the number of appliances produced to maximize the daily profit of the company.

Glue

Glueco produces three types of glue on two different production lines. Each line can be utilized by up to seven workers at a time. Workers are paid 500 MU per week on production line l, and 900 MU per week on production line 2. For a week of production it costs 1000 MU to set up production line l and 2000 MU to set up production line 2. During a week on a production line, each worker produces the number of units of glue shown in Table 5.11. Each week, at least 120 units of glue l, at least 150 units of glue 2, and at least 200 units of glue 3 must be produced. Write a program to minimize the total cost of meeting weekly demands.

Allocating machines

A product can be produced on four different machines. Each machine

[39] A doggie equivalent of "population".

Production lines	Glue 1	Glue 2	Glue 3
Production line 1	20	30	40
Production line 2	50	35	45

Table 5.11: Glue production data

has a fixed setup cost, variable production costs per-unit-processed, and a production capacity given in Table 5.12. A total of 2000 units of the product must be produced. Write a program that determines machine loads that minimize total costs.

Machine	Fixed cost (MU)	Variable cost per unit (MU)	Capacity
1	1000	20	900
2	920	24	1000
3	800	16	1200
4	700	28	1600

Table 5.12: Machines data

Paper rolls

A paper factory manufactures large rolls of paper that have a width of 105cm. However, retailers demand rolls of smaller width, which have to be cut from the large ones. For instance, a standard width roll could be cut into two rolls of 35cm each and one roll of 30cm. The factory received orders shown in Table 5.13.

Width cm	Number of rolls
25	100
30	125
35	80

Table 5.13: Orders data

Write a program minimizing the number of produced large rolls needed to satisfy the order.

Five projects

Five projects are being evaluated over a 3-year planning horizon. Table 5.14[40] gives the expected returns and the associated yearly expenditure for each project.

Project	Expenditure (million MU per year)			Returns (million MU)
	year 1	year 2	year 3	
1	5	1	8	20
2	4	7	10	40
3	3	9	2	20
4	7	4	1	15
5	8	6	10	30
Available funds (milion MU)	25	25	25	

Table 5.14: Projects data

Write a program to determine, which project should be selected over the 3-year horizon to maximize the overall returns. Modify the program to take into account the following constraint: project 5 must be selected if either project 1 or project 3 is selected. Modify the program to take into account the following constraint: project 2 and project 3 are mutually exclusive.

Allocating benefits to Napoleonides

Napoleonism means, in respect of an individual, the individual's deeply felt internal and personal experience of being *Napoleon Bonaparte*, which may not exactly correspond to the role assigned to the individual by the oppressive society.

This sad discrepancy is a source of enormous sufferings of all those persons considering themselves to be Napoleon Bonaparte. It is therefore not surprising that the *World Institute for Wellness* has finally taken seriously into consideration the plight of those persons, referred to officially as *Napoleonides*. As a result a series of directives was issued urging Local Governments to consider *Napoleonism* not as a mental disorder, but as a

[40] This exercise is from [Taha-08].

normal state of health that contributes substantially to the diversification of society (*Diversity is our strength!*), and deserves not only widespread respect and some intelligent publicity (lets say establishing a. o. *The World Napoleonides Day* as Public Holiday, with "Napoleonides Parades" and TV campaigns), but also parity in employment, wages and membership in representative organs as well as special financial support from the public purse. The directives also incorporated napoleonophobia into the constantly increasing spectrum of heavily punished hate speeches, phobias and descriminatory practices, forbidding therapy to turn people away from thinking they are Napoleon Bonaparte, and introducing sensitivity training for napoleonism into syllabuses of elementary education.
The Parliament of Absurdoland, always in the vanguard of legislative organs eager to quickly implement any whim of the *World Institute for Wellness*, ordered its *Commission on Discrimination and Exclusion* to solve the problem as soon as possible. The Commission started with inviting a number of Napoleonide *activists* to present their grievances and was deeply shocked by what they heard. The activists complained bitterly about them being addressed simply as '*Mr. Smith*', and not by "*Your Imperial Highness*", about the necessity to go to work by tram or buss instead of riding on horses or in a horse-drawn carriage, with the assistance of some generals and adjutants as well as a small bunch of *Chevaux Légeres*, all in proper uniforms. They complained about haters calling them *bonacrazies* or *bonacranks*. Their main complaints were about the cost of making them as similar as possible to their archetype: the cost of face matching surgery - although quite substantial - happened to be negligible compared to the costs of height matching surgery. A number of activists presented arguments in favour of providing them with small-sized servants-staffed *manor houses* to enable them to lead a truly Napoleon-like everyday existence. However, the straw that broke the camel's neck was given by tales of tragic institutionalizations, incarcerations and persecutions of Napoleonides in *Closed Mental Institutions*. The Commission immediately changed all laws handicapping or force-medicating Napoleonides and started to work on optimizing benefits to ameliorate their fortune. The activists provided a list of `150` well-known declared Absurdoland's Napoleonides (their number increased substantially on the aftermath of the *World Institute for Wellness* directives). The Government of Absurdoland quickly changed its budget by taking `10` MM MU out of the *Social Security Fund* to provide deserving and decent living conditions for Napoleonides. In the meantime the activists presented *Happiness Val-*

ues for their most wanted benefits, and suggested that the Commission should do its job by maximizing *Happines* over all entitled beneficiaries by selecting the number of different benefits granted. Next the Commission established prices for those benefits, the Ministry of Medical Technology informed, that - because of technological constraints - no more than 30 height-matching and 15 face-matching operations could be performed yearly, and the Ministry of Cultural Heritage declared that the number of small-sized servants-staffed manor houses available for manorless Napoleonides is (unfortunately, at least for the time being) limited to 6. It was also considered reasonable that (in view of the sorry state of economy) Napoleonides equipped with horses for riding should not claim horse-driven carriages. The basic problem data is presented by Table 5.15.

Number of beneficiaries	Type of benefit	Happiness Value	Cost of benefit per beneficiary
N1	Napoleon-like outfit	6	40
N2	Horse for riding	25	300
N3	Horse-drawn carriage	50	1500
N4	Face matching	180	2000
N5	Height matching	200	6500
N6	Manor house	500	13000

Table 5.15: Data for allocating benefits to napoleonides

Determine the numbers of various beneficiaries to maximize the happiness value for the napoleonide population.

Producing cars

Clunker Motors Co has four car manufacturing plants. Each is capable of producing any of the company's three flagships (Clunker SUV, Clunker Electric and Clunker Green), but only one of them. The main economic data for the production are shown in Table 5.16.

They include the fixed cost of running each plant for a year and the variable costs of producing a single car. The constraints are:
1. Each plant can produce only one type of car. 2. The total production of each type must be located at a single plant. 3. If plants 3 and 4 are producing cars, then plant 1 must also produce cars.

Clunker Motors Co must produce 600000 cars of each type per year. Write

Plant	Fixed cost	Variable cost		
		SUV	Electric	Green
1	7 billion MU	15000 MU	19000 MU	15000 MU
2	6 billion MU	12000 MU	18000 MU	11000 MU
3	4 billion MU	17000 MU	16000 MU	12000 MU
4	2 billion MU	19000 MU	22000 MU	9000 MU

Table 5.16: Car manufacturing data

a program that determines how to minimize the annual overall cost of producing cars while meeting production quotas.

Fast food outlets

The well-known chain of popular fast food outlets "Tasty Poison" is conquering the Absurdoland's fast food market after the collapse of the communist state-owned-and-run "Eating Joints". A set of specialist analyzed possible locations in the Ancient Capital, proposing 6 locations were "Tasty Poison" outlets could be placed. Those outlets served a number of Ancient Capital districts, the "Tasty Poison" policy being to serve any district by at least one outlet. The specialists provided - after some hard work - trusted estimates of cost for building any of the six fast food outlets, shown in Table 5.17.

District	Fast food outlet					
	1	2	3	4	5	6
1	×	×		×		
2		×		×		
3	×		×			
4	×			×		
5			×		×	
6			×		×	×
7		×		×		
8		×			×	×
Cost of building (MM MU)	10	15	12	8	12	13

Table 5.17: Fast food project data

Write a program that determines what outlets to build in order to mini-

mize cost and serve all districts.

Hot buttered toasts [41]

There is an old toaster with two hinged doors on each side. It can take two pieces of toast at a time, and it only toasts one side of a piece at a time. The times for the activities are: 1)It takes 30 seconds to toast one side of a piece of bred (the toaster can take up two pieces at a time). 2)It takes 3 seconds to put a piece of bread in the toaster. 3)It takes 3 seconds to take out a piece of bread from the toaster. 4) It takes 3 seconds to reverse a piece of bread without removing it from the toaster. 5) It takes 12 seconds to butter a side of toast. In addition, the activities of inserting, reversing, removing and buttering a slice require both hands, so the cannot be performed at the same time. Each piece is only buttered on one side and the butter can only be applied after that side has been toasted. When we begin, the three pieces of bread are out of the toaster, and we have to complete the toasting with the three pieces also being out of the toaster. Develop such schedule that the three pieces of bread are toasted and buttered in the shortest time.

Crossing a bridge [42]

Four travelers (Mr. A, B, C and D) have to cross a bridge over a deep ravine. It is a very dark night and the travelers only have one oil lamp. The lamp is essential for successfully crossing the ravine because the bridge is very old and has plenty of holes and loose boards. What is worse, its construction is quite weak and it can only support two men at any time. It turns out that each traveler needs a different amount of time to cross the bridge. Mr. A is young and fast, and only needs a minute to cross the bridge. Mr. D, on the other hand, is an old man who recently had a hip replacement and will need 10 minutes to get across the bridge. Mr B and Mr C need two and five minutes, respectively. And since each traveler needs the light to cross, it is the slower man in a pair who determines the total time required to make the crossing. Write a program to determine a crossing schedule that minimizes the overall crossing time.

[41] This exercise is from [Michalewicz-07].
[42] This exercise is from [Michalewicz-07].

Students grievances [43]

A University is in the process of forming a committee to handle students grievances. The administration wants the committee to include at least one female, one male, one students, one administrator and one faculty member. Ten individuals a,b,c,...,j have been nominated to serve on the committee, see Table 5.18.

Category	Individuals
Females	a,b,c,d,e
Males	f,g,h,i,j
Students	a,b,c,j
Administrators	e,f
Faculty	d,g,h,i

Table 5.18: Committee candidates

The administration wants to formulate the smallest committee with representation of each of the five categories of persons. Write a program to solve this problem.

Constructing a pizzeria

Given data from Table 5.19 write a program to find the shortest duration for constructing the pizzeria and to determine the critical path activities[44].

[43] This exercise is from [Taha-08].

[44] This exercise is from http://faculty.ksu.edu.sa/ialharkan/default.aspx

No	Activity	Predecessor	Days
1	Design layout		1
2	Select contractor	1	5
3	Cleaning area	2	5
4	Plumbing	3	5
5	Install electricity	3	12
6	Install AC	3	7
7	Tile floors	4	7
8	Install walk-in cooler	7	1
9	Make partition	8	6
10	Tile walls and partitions	9	7
11	Ceiling, lighting, AC	5, 6, 7	4
12	Equipment installation	10, 11	2
13	Paste	12	3
14	Design store front	3	5
15	Install store front	14	5
16	Paint 1 and 2	13, 15	2
17	Install counters	16	4
18	Install electricity sockets	13	1
19	Install frames	16	1
20	Paint 3	17, 19	1
21	Final decoration	20	1
22	Prepare sign board	3	2
23	Install sign board	22	2
24	Print new store opening		2
25	Final test	18, 21, 23	1

Table 5.19: Pizzeria construction activities

Chapter 6

CLP with global constraints for optimal solutions

6.1 Introduction

Some global predicates were already introduced and applied in Chapter 4 for finding feasible solutions. Some of them (like `cumulative/4`, `cumulative/5`, `disjunctive/2`) and some new ones (like `disjoint/1` and `cycle/3`) are indispensable for solving a group of new optimization problems:
1. *Optimum scheduling problem*s. Scheduling is an extension of sequencing: scheduling is sequencing with the additional constraint on available resources and the aim of minimizing an objective function, given most often by the time to complete the schedule, known as makespan. Resource constraints may be defined as follows:

```
Demand_A_for_shared_resource + ... +
        Demand_Z_for_shared_resource <=
            Maximum_amount_of_shared_resource_available.
```

This constraint is fundamental and universal, almost like the Law of Gravitation (*toutes proportion gard´ee*): no rational decision-making is possible while abstracting from the finiteness of resources.
2. *Optimum bin packing problems*. The aim of those problems is most often to pack the highest-value number of objects of different values into a bin of fixed

volume. An elementary bin-packing problem is the knapsack problem, discussed in sections 5.6.3 and 5.6.3.
3. *Optimum vehicle routing problems*. The aim of those problems is to service a number of spatially distributed customers with a fleet of vehicles in the most parsimonious manner. The basic vehicle routing problem is the celebrated *travelling salesman problem*, discussed in Section 6.19.
The mentioned predicates have found application also for solving some of the already discussed problems, like optimum sequencing problems.

Global predicates are useful for combinatorial optimization problems because they:

- are rich enough to capture substantial parts of the optimization problem structure;
- provide major abstractions common to a broad range of combinatorial optimization problems;
- provide exceptional custom-tailored computing power for some optimization-relevant constraints;
- enhance the program declarativity and readability by introducing names highly relevant to the functionality of the constraints.

The discussed and applied global constraints (`cumulative/4`, `cumulative/5`, `disjunctive/2`, `circuit/1`, `cycle/3`) are just a tiny subset of all global constraints available for optimization purposes.

6.2 The 'cumulative/4' built-in

The cumulative constraint:

```
cumulative(+StartTimes, +Durations,+Resources,++Limit)
```

expresses the fact that the total of a shared resource used by many tasks may not, at any time instant, exceed a given limit. Its arguments have the following meaning (see Figure 6.1):

- `StartTimes = [S1,...,Sn]` is a list of domain variables representing start times for `n` tasks;

- `Durations = [D1,...,Dn]` is a list of domain variables representing task durations;

- `Resources = [R1,...,Rn]` is a list of domain variables representing amounts of shared resource needed by tasks;

- `End Times Ej` for all tasks `1<=j<=n` are given by `Sj+Dj=Ej`;

- `Limit` is the maximum amount of the available shared resource at any time instant `i` such that for `a<=i<=b`:
  `max(Sum Rj) <= Limit`
  for all such `j` that `Sj <= i <= Sj+Dj-1`, where:
  `a = min(min(S1),...,min(Sn))`
  `is the earliest beginning of all tasks, and`
  `b = max((max(S1)+max(D1)),...,(max(Sn)+max(Dn)))`
  `is the latest end of all tasks, with:`
  `min(X)` being the minimum value of `X` in its domain, and
  `max(X)` being the minimum value of `X` in its domain.

- The end time of all tasks is equal `End = max(Sj+Dj).`

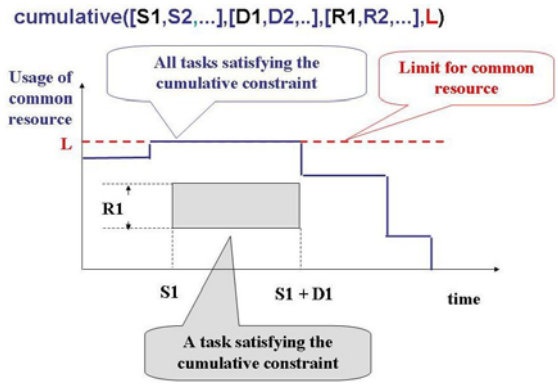


Figure 6.1: Tasks satisfying a *cumulative/4* constraint

6.3 Cumulative scheduling 1

Let's apply the `cumulative/4` built-in to data from Section 5.10.2. This can be done as shown in program `6_1_cumu_schedule_1.ecl`[1]:

```
/*1*/  :-lib(ic).
/*2*/  :- lib(ic_edge_finder3).
/*3*/  :-lib(branch_and_bound).

/*4*/  top :-
/*5*/  schedule(_).

/*6*/  schedule([Z1,Z2,Z3,Z4,End]):-
/*7*/  [Z1,Z2,Z3,Z4,End] :: 0..15,
/*8*/  Z1 + 3 #=< Z2,
/*9*/  Z1 + 3 #=< Z3,
/*10*/ Z2 + 4 #=< Z4,
/*11*/ Z3 + 2 #=< Z4,
/*12*/ Z4 + 1 #= End,

/*13*/ cumulative([Z2,Z3],[4,2],[1,1],1),
/*14*/ minimize(labeling([Z1,Z2,Z3,Z4,End]),End),
/*15*/ writeln("Z1 ":Z1),
/*16*/ writeln("Z2 ":Z2),
/*17*/ writeln("Z3 ":Z3),
/*18*/ writeln("Z4 ":Z4),
/*19*/ writeln("End ":End).
```

The message is:

```
Found a solution with cost 10
Found no solution with cost 8.0 .. 9.0
Z1 : 0
Z2 : 3
Z3 : 7
Z4 : 9
End  : 10.
```

It corresponds to the already presented Gantt chart from Figure 5.17a).
If line `/*13*/` is removed and line `/*13a*/` activated, then the message gener-

[1] This is an OST-type problem.

ated is:

```
Found a solution with cost 8
Z1  : 0
Z2  : 3
Z3  : 3
Z4  : 7
End  : 8.
```

It corresponds to the already presented Gantt chart from Figure 5.17b).

6.4 Cumulative scheduling 2

Consider a slightly more complicated cumulative scheduling, see [Baldiceanu-94]:
There are seven tasks, each of them characterized by its duration and the amount of shared resource needed, see Table 6.1:

Task	Duration	Resource
1	16	2
2	6	9
3	13	3
4	7	7
5	5	10
6	18	1
7	4	11

Table 6.1: Data for simple cumulative scheduling

A schedule is to be found that minimizes the overall end of all tasks while not exceeding the resource capacity equal 13. This can be done as shown in program `6_2_cumu_schedule_2.ecl`[2]:

```
/*1*/  :- lib(ic).
/*2*/  :- lib(ic_edge_finder3).
/*3*/  :- lib(branch_and_bound).
```

[2]This is an OST-type problem.

```
/*4*/top:-
/*5*/    LS = [S1,S2,S3,S4,S5,S6,S7],   %list of task start times
/*6*/    LD = [16, 6,13, 7, 5,18, 4],   %list of task durations
/*7*/    LE = [E1,E2,E3,E4,E5,E6,E7],   %list of task end times
/*8*/    LR = [2,9,3,7,10,1,11],   %list of task resource requirements
/*9*/    LS :: 1..100,
/*10*/   End :: 1..100,
/*11*/   LE :: 1..100,
/*12*/   Limit :: 1..13,

/*13*/   cumulative(LS,LD,LR,Limit),

/*14*/   E1 #= S1 + 16,
/*15*/   E2 #= S2 + 6,
/*16*/   E3 #= S3 + 13,
/*17*/   E4 #= S4 + 7,
/*18*/   E5 #= S5 + 5,
/*19*/   E6 #= S6 + 18,
/*20*/   E7 #= S7 + 4,

/*21*/   maxlist([E1,E2,E3,E4,E5,E6,E7],End),
/*22*/   minimize(labeling([S1,S2,S3,S4,S5,S6,S7,
           E1,E2,E3,E4,E5,E6,E7]),End),

/*23*/   write("LS = "), writeln(LS),
/*24*/   write("LE = "), writeln(LE),
/*25*/   write("Limit = "), writeln(Limit),
/*26*/   write("End = "), writeln(End).
```

The message is:

```
LS=[ 1,17,10,10, 5, 5,1]
LE=[17,23,23,17,10,23,5]
Limit=13
End=23
```

A Gantt chart illustrating this schedule is given by Figure 6.2. The numbers inside rectangles are task numbers. This figure may also be interpreted as a solution for a bin-packing problem, namely the problem of cutting a rectangle with dimension 13×23 into smaller rectangles given by the tasks.

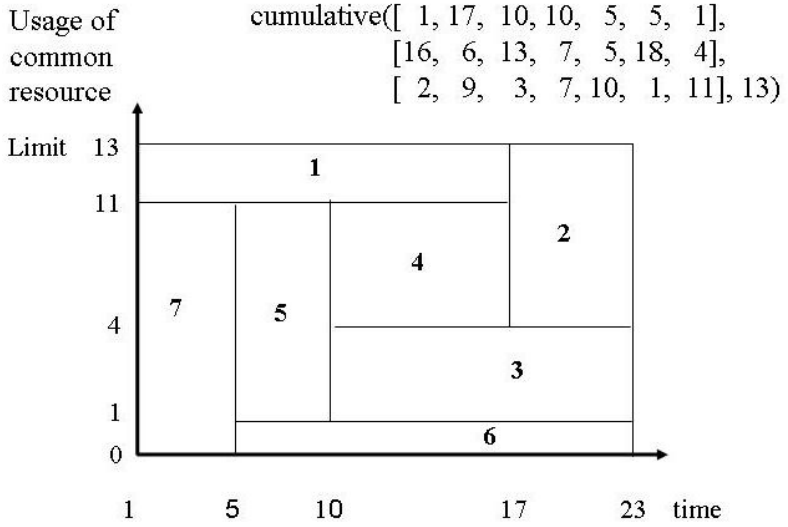


Figure 6.2: Gantt chart for cumulative scheduling

6.5 Cumulative sequencing

The `cumulative/4` predicate may be used also for optimum sequencing problems. This is illustrated by the following assembly line example: a sequence of tasks should be determined that fulfills precedence and time constraints as well as minimizes the overall assembly time. The following set of tasks and their duration is given:

```
 jobs:         A   B   C   D   E   F   G   H  I    J   K
duration:     45  11   9  50  15  12  12  12  12   8   9 .
```

The precedence constraints are:

```
first_next(predecessor, successor)
first_next(A,B).
first_next(B,C).
first_next(C,F).
first_next(C,G).
first_next(F,J).
```

```
first_next(G,J).
first_next(J,K).
first_next(D,E).
first_next(E,H).
first_next(E,I).
first_next(H,J).
first_next(I,J).
```

For any pair (predecessor-successor), predecessor cannot start before successor has ended. Program 6_3_sequencing_opti_cum.ecl uses the cumulative/4 global built-in to determine a sequence of jobs that minimizes the overall assembly time:

```
/*1*/ :- lib(ic).
/*2*/ :- lib(ic_edge_finder3).
/*3*/ :- lib(branch_and_bound).

/*4*/     top:-
  %  task start times:
/*5*/        LS=[As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks],
/*6*/        LS :: 0..250,
/*7*/        End :: 0..250,

  % precedence and time constraints:
/*8*/       As + 45 #=< Bs,
/*9*/       Bs + 15 #=< Cs,
/*10*/      Cs + 9 #=< Fs,
/*11*/      Cs + 9 #=< Gs,
/*12*/      Fs + 12 #=< Js,
/*13*/      Gs + 12 #=< Js,
/*14*/      Js + 8 #=< Ks,
/*15*/      Ds + 50 #=< Es,
/*16*/      Es + 15 #=< Hs,
/*17*/      Es + 15 #=< Is,
/*18*/      Hs + 12 #=< Js,
/*19*/      Is + 12 #=< Js,
/*20*/      Ks + 9 #= End,

/*21*/     cumulative([As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks],
               [45,15,9,50,15,12,12,12,12,8,9],
                                  [ 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1],1),

/*22*/     minimize(labeling([As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks]),End),

/*23*/     writeln([As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks,End]).
```

A single solution generated by ECL^iPS^e) is as follows:

```
Found a solution with cost 199
Found no solution with cost 98.0 .. 198.0
[0, 45, 60, 69, 119, 134, 146, 158, 170, 182, 190, 199]
```

Our intuition suggest however that there may be more solutions. This is to be verified using program `6_4_sequencing_opti_cum_all.ecl`, with variable `End` grounded on optimum value `199`:

```
/*1*/ :- lib(ic).
/*2*/ :- lib(ic_edge_finder3).

/*3*/     top:-
/*4*/         assert(counter(0)),
  %  task start times:
/*5*/         LS=[As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks],
/*6*/         LS :: 0..250,

  %  precedence and time constraints:
/*7*/         As + 45 #=< Bs,
/*8*/         Bs + 15 #=< Cs,
/*9*/         Cs + 9 #=< Fs,
/*10*/        Cs + 9 #=< Gs,
/*11*/        Fs + 12 #=< Js,
/*12*/        Gs + 12 #=< Js,
/*13*/        Js + 8 #=< Ks,
/*14*/        Ds + 50 #=< Es,
/*15*/        Es + 15 #=< Hs,
/*16*/        Es + 15 #=< Is,
/*17*/        Hs + 12 #=< Js,
/*18*/        Is + 12 #=< Js,
/*19*/        Ks + 9 #= End,
/*20*/        End is 199,

/*21*/        cumulative([As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks],
               [45,15,9,50,15,12,12,12,12,8,9],
                                  [ 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1],1),

/*22*/        labeling([As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks]),

/*23*/        my_count,
/*24*/        counter(Number),

/*25*/        write("Optimum solution "), write(Number),write(":"),nl,
/*26*/        writeln([As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks,End]),
```

```
/*27*/       fail.

/*28*/    top:-
/*29*/       write("Those are all optimum solutions.").

/*30*/    my_count:-
/*31*/      retract(counter(Old)),
/*32*/      New is Old  1, +
/*33*/      assert(counter(New)).
```

Our intuition was well-founded: there are 504 optimum solutions. Only some of them are shown below:

```
Optimum solution 1:
[0, 45, 60, 69, 119, 134, 146, 158, 170, 182, 190, 199]
.............................................................
Optimum solution 61:
[0, 45, 110, 60, 119, 134, 146, 158, 170, 182, 190, 199]
.............................................................
Optimum solution 141:
[0, 95, 110, 45, 119, 134, 146, 158, 170, 182, 190, 199]
.............................................................
Optimum solution  504:
[89, 134, 149, 0, 50, 170, 158, 77, 65, 182, 190, 199]
Those are all optimum solutions.
```

The Gantt chart for those solutions shown in Figure 6.3 presents a proper interpretation of the above numerical results.

6.6 The 'disjunctive/2' built-in

The disjunctive constraint:

```
disjunctive(+Start_Times, +Durations)
```

is fulfilled if there is no overlap of tasks with start times from the list `Start_Times` and corresponding durations from the list `Durations`, as shown in Figure 6.4. Both lists must have equal numbers of elements.

In contrast with `cumulative/3`, which constraints the task along the resource coordinate (on Gantt charts - vertically), `disjunctive/2` constraints

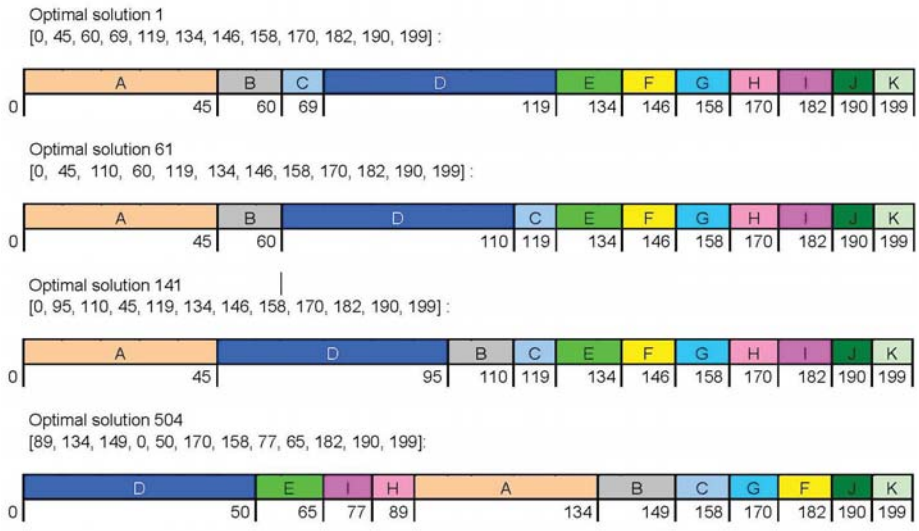


Figure 6.3: Gantt charts of some optimum assembly sequences

tasks along the time coordinate (on Gantt charts - horizontally). However - as shown by the following example - `disjunctive/2` may sometimes fulfill the role of `cumulative/3`.

6.7 Disjunctive sequencing

The `disjunctive/2` built-in is most often used for sequencing problems. This is illustrated by example `6_5_opti_dis.ecl` dealing with the already solved problem from Section 6.5:

```
/*1*/ :- lib(ic).
/*2*/ :- lib(ic_edge_finder3).
/*3*/ :- lib(branch_and_bound).

/*4*/     top:-
  %  task start times:
/*5*/        LS=[As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks],
/*6*/        LS :: 0..250,
```

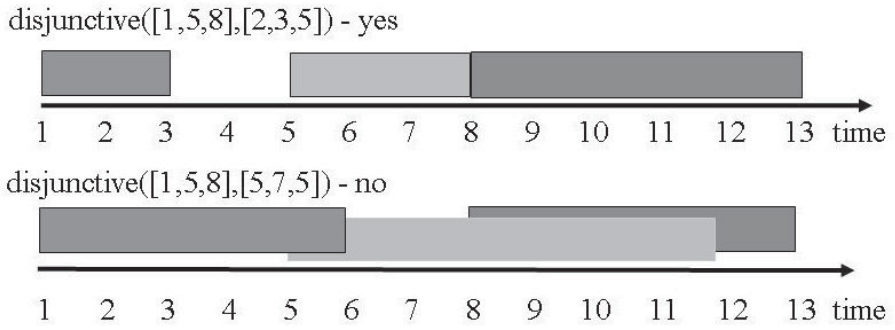


Figure 6.4: Properties of the *disjunctive/2* constraint

```
/*7*/        End :: 0..250,

  %  precedence and time constraints:
/*8*/       As + 45 #=< Bs,
/*9*/       Bs + 15 #=< Cs,
/*10*/      Cs + 9 #=< Fs,
/*11*/      Cs + 9 #=< Gs,
/*12*/      Fs + 12 #=< Js,
/*13*/      Gs + 12 #=< Js,
/*14*/      Js + 8 #=< Ks,
/*15*/      Ds + 50 #=< Es,
/*16*/      Es + 15 #=< Hs,
/*17*/      Es + 15 #=< Is,
/*18*/      Hs + 12 #=< Js,
/*19*/      Is + 12 #=< Js,
/*20*/      Ks + 9 #= End,

/*21*/     disjunctive([As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks],
                      [45,15,9,50,15,12,12,12,12,8,9]),

/*22*/     minimize(labeling([As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks]),End),

/*23*/     writeln([As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks,End]).
```

The solution is identical with that obtained for `6_3_sequencing_opti_cum.ecl`.

Multiple optimum solutions are given by `6_6_sequencing_opti_dis_all.ecl`:

```
  /*1*/ :- lib(ic).
/*2*/ :- lib(ic_edge_finder3).
```

```
/*3*/    top:-
/*4*/       assert(counter(0)),
  %  task start times:
/*5*/       LS=[As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks],
/*6*/       LS :: 0..250,

  %  precedence and time constraints:
/*7*/       As + 45 #=< Bs,
/*8*/       Bs + 15 #=< Cs,
/*9*/       Cs + 9 #=< Fs,
/*10*/      Cs + 9 #=< Gs,
/*11*/      Fs + 12 #=< Js,
/*12*/      Gs + 12 #=< Js,
/*13*/      Js + 8 #=< Ks,
/*14*/      Ds + 50 #=< Es,
/*15*/      Es + 15 #=< Hs,
/*16*/      Es + 15 #=< Is,
/*17*/      Hs + 12 #=< Js,
/*18*/      Is + 12 #=< Js,
/*19*/      Ks + 9 #= End,
/*20*/      End is 199,

/*21*/      disjunctive([As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks],
                 [45,15,9,50,15,12,12,12,12,8,9]),

/*22*/     labeling([As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks]),

/*23*/     my_count,
/*24*/     counter(Number),

/*25*/     write("Optimum solution "), write(Number),write(":"),nl,
/*26*/     writeln([As,Bs,Cs,Ds,Es,Fs,Gs,Hs,Is,Js,Ks,End]),
/*27*/     fail.

/*28*/  top:-
/*29*/     write("Those are all optimum solutions. ").

/*30*/  my_count:-
/*31*/     retract(counter(Old)),
/*32*/     New is Old  1, +
/*33*/     assert(counter(New)).
```

There are 504 optimum solutions, exactly the same as for program `6_4_sequencing_opti_cum_all.ecl`. See also Figure 6.3

6.8 Disjunctive scheduling

Let's solve the example from Section 5.10.2 using `disjunctive/2`. This is done by program `6_7_dis_schedule.ecl`[3] :

```
/*1*/  :- lib(ic).
/*2*/  :- lib(ic_edge_finder3).
/*3*/  :- lib(branch_and_bound).

/*4*/  top :-
/*5*/  schedule(_).

/*6*/  schedule([Z1,Z2,Z3,Z4,End]):-
/*7*/  [Z1,Z2,Z3,Z4,End] :: 0..15,
/*8*/  Z1 + 3 #=< Z2,
/*9*/  Z1 + 3 #=< Z3,
/*10*/ Z2 + 4 #=< Z4,
/*11*/ Z3 + 2 #=< Z4,
/*12*/ Z4 + 1 #= End,

/*13*/ disjunctive([Z2,Z3],[4,2]),
/*14*/ minimize(labeling([Z1,Z2,Z3,Z4,End]),End),
/*15*/ writeln("Z1 ":Z1),
/*16*/ writeln("Z2 ":Z2),
/*17*/ writeln("Z3 ":Z3),
/*18*/ writeln("Z4 ":Z4),
/*19*/ writeln("End ":End).
```

The message is:

```
Found a solution with cost 10
Found no solution with cost 8.0 .. 9.0
Z1 : 0
Z2 : 3
Z3 : 7
Z4 : 9
End  : 10,
```

depicted by the already generated Gantt chart from Figure 6.14a).

[3] This is an OST-type problem.

6.9 The 'disjoint2(Rectangles)' built-in

This is a generalization of the `disjunctive/2` predicate for the case of two dimensions. It constrains the position (and possibly size) of rectangles in `Rectangles` so that none overlaps. The rectangles are defined by structures:

```
rect{x:X,y:Y,w:W,h:H}
```

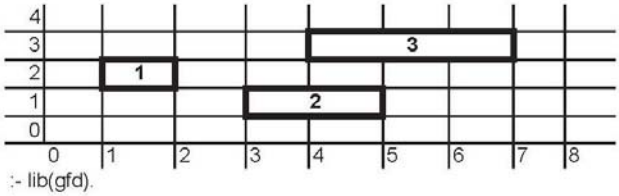
using the following fields:

- constant x: The x co-ordinate of the left side of the rectangle, equal to variable X;
- constant y: The y co-ordinate of the bottom side of the rectangle, equal to variable Y;
- constant w: The width of the rectangle equal to variable W;
- constant h: The height of the rectangle equal to variable H.

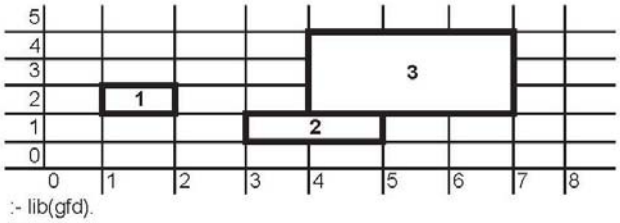
Its basic usage is illustrated by program `6_8_disjoint.ecl`:

```
:- lib(gfd).
top_1:-
      disjoint2([rect{x:1,y:2,w:1,h:1}, rect{x:3,y:1,w:2,h:1},rect{x:4,y:3,w:3,h:1}]).
top_2:-
      disjoint2([rect{x:1,y:2,w:1,h:1},rect{x:3,y:1,w:2,h:1},rect{x:4,y:2,w:3,h:3}]).
top_3:-
      disjoint2([rect{x:1,y:2,w:1,h:1},rect{x:3,y:1,w:2,h:3},rect{x:4,y:2,w:3,h:3}]).
```

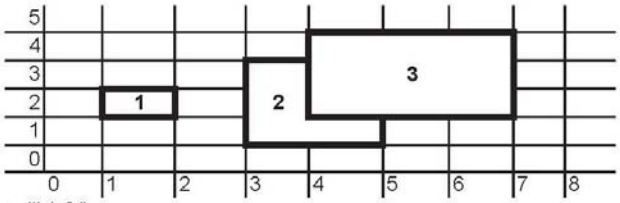
The solution to `top_1` and `top_2` is `yes`, the solution to `top_3` is no. Figure 6.5 depicts the rectangles involved in this program.




```
:- lib(gfd).
top:-
disjoint2([rect{x:1,y:2,w:1,h:1}, rect{x:3,y:1,w:2,h:1},rect{x:4,y:3,w:3,h:1}]).
yes,.
```




```
:- lib(gfd).
top:-
disjoint2([rect{x:1,y:2,w:1,h:1},rect{x:3,y:1,w:2,h:1},rect{x:4,y:2,w:3,h:3}]).
yes
```




```
:- lib(gfd).
top:-
disjoint2([rect{x:1,y:2,w:1,h:1},rect{x:3,y:1,w:2,h:3},rect{x:4,y:2,w:3,h:3}]).
no
```

Figure 6.5: Three examples of 'disjoint2(Rectangles)' application

6.10 Assembly line balancing

Assembly line balancing is the assignment of tasks to workstations so that, while fulfilling precedence constraints, each workstation has approximately the same amount of work to accomplish as measured by the time to complete it. The largest time to complete all tasks at some workstation is referred to as *cycle time*. Optimum line balancing aims at minimizing the cycle time.

Let us consider the `141` optimum solution to the cumulative sequencing problem from Chapter as shown in Figure 6.3. It is used for balancing a 4-workstations assembly line as shown by program `6_9_disjoint_balance.ecl`:

```
/*1*/ :-lib(gfd).
/*2*/  :- lib(branch_and_bound).

/*3*/ top:-
  %task start times:
/*4*/      LSZ=[ A, B, C, D, E, F, G, H, I, J, K],
  %workstations for tasks A,B,...
/*5*/      Lst=[Ast,Bst,Cst,Dst,Est,Fst,Gst,Hst,Ist,Jst,Kst],
  %task duration times:
/*6*/      LD=[Ad,Bd,Cd,Dd,Ed,Fd,Gd,Hd,Id,Jd,Kd],
  %task end times:
/*4*/      LE=[EA,EB,EC,ED,EE,EF,EG,EH,EI,EJ,EK],
  %task resource requirements:
/*8*/      LR=[ 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1],

  %domain declaration
/*9*/     LSZ #:: 0..100,
/*10*/    Lst #:: 1..4,
/*11*/    LE #:: 0..100,
/*12*/    Limit #:: 1..4,

  %task duration times declaration:
/*13*/     Ad is 45, Bd is 15, Cd is 9, Dd is 50, Ed is 15, Fd is 12,
                     Gd is 12, Hd is 12, Id is 12, Jd is 8, Kd is 9,
  %tasks assignment to workstations:
/*14*/     Ast is 1,  Bst is 3, Cst is 3, Dst is 2, Est is 3, Fst is 3,
                     Gst is 4, Hst is 4, Ist is 4, Jst is 4, Kst is 4,

  % constraints for tasks end times::
/*15*/     gfd_gac: (EA #>= A + Ad),
/*16*/     gfd_gac: (EB #>= B + Bd),
/*17*/     gfd_gac: (EC #>= C + Cd),
/*18*/     gfd_gac: (ED #>= D + Dd),
/*19*/     gfd_gac: (EE #>= E + Ed),
/*20*/     gfd_gac: (EF #>= F + Fd),
```

```
/*21*/     gfd_gac: (EG #>= G + Gd),
/*22*/     gfd_gac: (EH #>= H + Hd),
/*23*/     gfd_gac: (EI #>= I + Id),
/*24*/     gfd_gac: (EJ #>= J + Jd),
/*25*/     gfd_gac: (EK #>= K + Kd),

  %non-overlapping of tasks constraints:
/*26*/     disjoint2([
             rect{x:A,y:Ast,w:Ad,h:1},rect{x:B,y:Bst,w:Bd,h:1},
             rect{x:C,y:Cst,w:Cd,h:1},rect{x:D,y:Dst,w:Dd,h:1},
             rect{x:E,y:Est,w:Ed,h:1},rect{x:F,y:Fst,w:Fd,h:1},
             rect{x:G,y:Gst,w:Gd,h:1},rect{x:H,y:Hst,w:Hd,h:1},
             rect{x:I,y:Ist,w:Id,h:1}, rect{x:J,y:Jst,w:Jd,h:1},
             rect{x:K,y:Kst,w:Kd,h:1}]),

  %constraining the usage of resources
/*27*/     cumulative(LSZ,LD,LR,Limit),

/*28*/     append(LSZ,LE,LSZ_E),
/*29*/     append(LSZ_E, Lst,LSZ_E_Lst),

  %minimizing
/*30*/    gfd_gac: (max(LE, M)),
/*31*/    bb_min(labeling(LSZ_E_Lst), M, bb_options with
              [strategy:continue,from:0,to:100]),
/*32*/    write("End times of tasks = "),write(LE),nl,
/*33*/    write("Minimum cycle time = "),write(M),nl,nl,

/*34*/write("Workstation for A: "),write(Ast),write(", Start A = "),write(A),nl,
/*35*/write("Workstation for B: "),write(Bst),write(", Start B = "),write(B),nl,
/*36*/write("Workstation for C: "),write(Cst),write(", Start C = "),write(C),nl,
/*37*/write("Workstation for D: "),write(Dst),write(", Start D = "),write(D),nl,
/*38*/write("Workstation for E: "),write(Est),write(", Start E = "),write(E),nl,
/*39*/write("Workstation for F: "),write(Fst),write(", Start F = "),write(F),nl,
/*40*/write("Workstation for G: "),write(Gst),write(", Start G = "),write(G),nl,
/*41*/write("Workstation for H: "),write(Hst),write(", Start H = "),write(H),nl,
/*42*/write("Workstation for I: "),write(Ist),write(", Start I = "),write(I),nl,
/*43*/write("Workstation for J: "),write(Jst),write(", Start J = "),write(J),nl,
/*43*/write("Workstation for K: "),write(Kst),write(", Start K = "),write(K),nl.
```

The solution is:

```
Found a solution with cost 53
Found no solution with cost 50.0 .. 52.0
End times of tasks = [45, 15, 24, 50, 39, 51, 12, 24, 36, 44, 53]
```

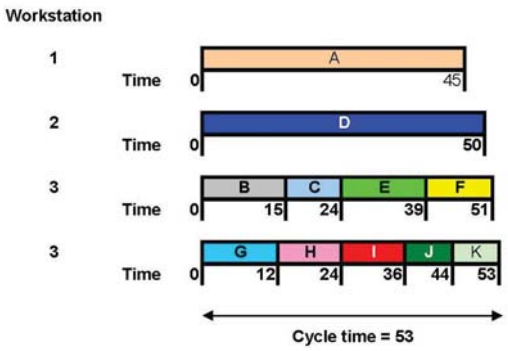


Figure 6.6: Solution of 'cumulative' for assembly line balancing

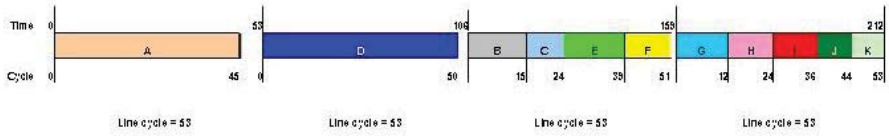


Figure 6.7: Gantt diagram for assembly line balancing

```
 Minimum cycle time = 53
 Workstation for A: 1 Start A = 0
 Workstation for B: 3 Start B = 0
 Workstation for C: 3 Start C = 15
 Workstation for D: 2 Start D = 0
 Workstation for E: 3 Start E = 24
 Workstation for F: 3 Start F = 39
 Workstation for G: 4 Start G = 0
 Workstation for H: 4 Start H = 12
 Workstation for I: 4 Start I = 24
 Workstation for J: 4 Start J = 36
 Workstation for K: 4 Start K = 44

 Delayed goals:
gfd : gfd_do_propagate(gfd_prob(nvars(35)))
 Yes (1258.02s cpu)
```

6.11 Reading newspapers 1

Let's have a look at a more complicated scheduling example. Its purpose is to show how to solve scheduling problems were cumulative and precedence constraints occur:

Four bright youngsters (Andy, Ben, Carl and Dusty) are studying *Community Organizing*, with a major in *Deceptions*, *Tensions and Scares*, at the best Absurdoland's university. They share a flat to which each morning the University Administration delivers four of the most influential Absurdoland's newspapers. They are: *Mainstream Drivel* (MD), *Daily Absurdities* (DA), *Morning Brainwasher* (MB) and *Gutter News* (GN). The students get up at different times and are ready to start reading at different times, as shown in Table 6.2. There also the reading orders and reading durations for all students may be found. The problem is to determine the earliest time they can all - after having read all newspapers - set off to the University[4].
Actually, the problem is one of finding the start times for reading each newspaper by each student in a non-conflicting way. The solution is given by program `6_4_newspapers_1.ecl`, where:
1) the non-overlapping constraint for reading newspapers is declared using `disjunctive/1`;
2) the availability of only a single copy of each newspaper is declared using `cumulative/4`;
3) time is divided into minutes with 08.00 taken as zero.

The program `6_4_newspapers_1.ecl`[5] reads as follows:

```
/*1*/ :- lib(ic).
/*2*/ :- lib(ic_edge_finder3).
/*3*/ :- lib(branch_and_bound).

/*4*/ top:-
/*5*/  A=[AMD,ADA,AMB,AGN], % reading start times for  Andy
/*6*/  B=[BDA,BMB,BMD,BGN], % reading start times for  Ben
/*7*/  C=[CMB,CDA,CMD,CGN], % reading start times for  Carl
```

[4]The subject has been inspired by the report of [Duncan-90], who quotes French ([French-82]) as the author who originally posed the problem.
[5]This is an OST-type problem.

Students	Task 1	Task 2	Task 3	Task 4
Andy starts at 8.30 with reading order: and duration:	To read MD 60 mins	To read DA 30 mins	To read MB 2 mins	To read GN 5 mins
Ben starts at 8.45 with reading order: and duration:	To read DA 75 mins	To read MB 3 mins	To read MD 25 mins	To read GN 10 mins
Carl starts at 8.45 with reading order: and duration:	To read MB 5 mins	To read DA 15 mins	To read MD 10 mins	To read GN 30 mins
Dusty starts at 9.30 with reading order and duration:	To read GN 90 mins	To read MD 5 mins	To read DA 5 mins	To read MB 5 mins

Table 6.2: Reading order duration for students and papers

```
/*8*/  D=[DGN,DMD,DDA,DMB], % reading start times for  Dusty
/*9*/  End=[A_end,B_end,C_end,D_end],
/*10*/              % end of reading times for students

/*11*/  A :: 30..360,
/*12*/  B :: 45..360,
/*13*/  C :: 45..360,
/*14*/  D :: 105..360,
/*15*/  End :: 90..360,
/*16*/  End_of_Ends :: 90..360,

/*17*/ AMD#>=30,
/*18*/ ADA#>=AMD+60,        % order constraints for Andy
/*19*/ AMB#>=ADA+30,
/*20*/ AGN#>=AMB+2,
/*21*/ A_end#>=AGN+5,

/*22*/ BDA#>=45,
/*23*/ BMB#>=BDA+75,         % order constraints for Ben
/*24*/ BMD#>=BMB+3,
/*25*/ BGN#>=BMD+25,
/*26*/ B_end#>=BGN+10,
```

```
/*27*/ CMB#>=45,
/*28*/ CDA#>=CMB+5,          % order constraints for Carl
/*29*/ CMD#>=CDA+15,
/*30*/ CGN#>=CMD+10,
/*31*/ C_end#>=CGN+30,

/*32*/ DGN#>=105,
/*33*/ DMD#>=DGN+90,         % order constraints for Dusty
/*34*/ DDA#>=DMD+5,
/*35*/ DMB#>=DDA+5,
/*36*/ D_end#>=DMB+5,

    % Constraining the number of students reading a paper.
    % Any paper may be read by one student only:
    % reading "Mainstream Drivel":
/*37*/ disjunctive([AMD,BMD,CMD,DMD],[60,25,10,5]),
    % reading "Daily Absurdities":
/*38*/ disjunctive([ADA,BDA,CDA,DDA],[30,75,15,5]),
    % reading "Morning Brainwasher":
/*39*/ disjunctive([AMB,BMB,CMB,DMB],[2,3,5,9]),
    % reading "Gutter Newsu":
/*40*/ disjunctive([AGN,BGN,CGN,DGN],[5,10,30,90]),

    % Constraining the number of papers read by student.
    % Any student may read only a single paper.
    % reads Andy:
/*41*/ cumulative([AMD,ADA,AMB,AGN],[60,30,2,5],[1,1,1,1],1),
    % reads Ben:
/*42*/ cumulative([BDA,BMB,BMD,BGN],[75,3,25,10],[1,1,1,1],1),
    % reads Carl:
/*43*/ cumulative([CMB,CDA,CMD,CGN],[5,15,10,30],[1,1,1,1],1),
    % reads Dusty:
/*44*/ cumulative([DGN,DMD,DDA,DMB],[90,5,5,5],[1,1,1,1],1),

/*45*/ maxlist(End,End_of_Ends),
/*46*/ minimize(labeling([AMD,ADA,AMB,AGN,BDA,BMB,BMD,BGN,
          CMB,CDA,CMD,CGN,DGN,DMD,DDA,DMB,A_end,B_end,
              C_end,D_end]), End_of_Ends),nl,

/*47*/ write("A = "),write(A),nl,
/*48*/ write("B = "),write(B),nl,
/*49*/ write("C = "),write(C),nl,
/*50*/ write("D = "),write(D),nl,

/*51*/ present_schedule([AMD,ADA,AMB,AGN,BDA,BMB,BMD,BGN,
         CMB,CDA,CMD,CGN,DGN,DMD,DDA,DMB],
         ["Andy","Mainstream Drivel",60,
         "Andy","Daily Absurdities",30,
```

```
        "Andy","Morning Brainwasher",2,
        "Andy","Gutter News",5,
        "Ben","Daily Absurdities",75,
        "Ben","Morning Brainwasher",3,
        "Ben","Mainstream Drivel",25,
        "Ben","Gutter News",10,
        "Carl","Morning Brainwasher",5,
        "Carl","Daily Absurdities",15,
        "Carl","Mainstream Drivel",10,
        "Carl","Gutter News",10,
        "Dusty","Gutter News",90,
        "Dusty","Mainstream Drivel",5,
        "Dusty","Daily Absurdities",5,
        "Dusty","Morning Brainwasher",5]).

/*52*/ present_schedule([],[]):-nl.
/*53*/ present_schedule([H1|T1],[H21,H22,H23|T2]) :-
/*54*/    convert_time(H1, FG, FM),
/*55*/    HH is H1+H23,
/*56*/    convert_time(HH, TG, TM),nl,
/*57*/    write(H21),write(" reads "),write(H22),write(" from "),
/*58*/    write(FG),write(":"),write(FM),
/*59*/    write(" to "),write(TG),write(":"),write(TM),
/*60*/ present_schedule(T1,T2).

/*61*/ convert_time(Time,Hours,Minutes) :-
/*62*/    div(Time, 60, G),
/*63*/    Hours is G + 8,
/*64*/    mod(Time,60,Minutes).
```

The message is:

```
Found a solution with cost 338
Found a solution with cost 263
Found a solution with cost 262
Found a solution with cost 257
Found a solution with cost 240
Found a solution with cost 238
Found a solution with cost 235
Found a solution with cost 210

A = [75, 140, 170, 195]
B = [65, 140, 143, 200]
C = [45, 50, 65, 75]
D = [105, 195, 200, 205]
```

```
Andy reads "Mainstream Drivel" from 9:15 to 10:15
Andy reads "Daily Absurdities" from 10:20 to 10:50
Andy reads "Morning Brainwasher" from 10:50 to 10:52
Andy reads "Gutter News" from 11:15 to 11:20
Ben reads "Daily Absurdities" from 9:5 to 10:20
Ben reads "Morning Brainwasher" from 10:20 to 10:23
Ben reads "Mainstream Drivel" from 10:23 to 10:48
Ben reads "Gutter News" from 11:20 to 11:30
Carl reads "Morning Brainwasher" from 8:45 to 8:50
Carl reads "Daily Absurdities" from 8:50 to 9:5
Carl reads "Mainstream Drivel" from 9:5 to 9:15
Carl reads "Gutter News" from 9:15 to 9:25
Dusty reads "Gutter News" from 9:45 to 11:15
Dusty reads "Mainstream Drivel" from 11:15 to 11:20
Dusty reads "Daily Absurdities" from 11:20 to 11:25
Dusty reads "Morning Brainwasher" from 11:25 to 11:30
```

As can be seen, the readings finish at 11:25 and then all students may set-off to the University.

The above message makes for hard reading. It is better presented as Gantt charts, one for presenting student activities (see Figure 6.8), the other one presenting the reading histories of papers (see Figure 6.9). The color codes for *boxes* of the Gantt chart for papers are necessarily different from those of the Gantt chart for students. Sticking to the same color codes would result in all boxes of the Gantt chart for papers to have the same color for the same paper, which would be rather uninformative.

It is obvious from those charts that the optimum reading order is not unique: e.g. reading of MB by Andy, MD by Ben and GN by Carl could start a little latter with no change to the minimum final time.

6.12 Reading newspapers 2

The problem could also be solved by a program that uses only the `cumulative/4` global constraint, as shown in `6_5_newspapers_2.ecl`[6]:

[6] This is an OST-type problem.

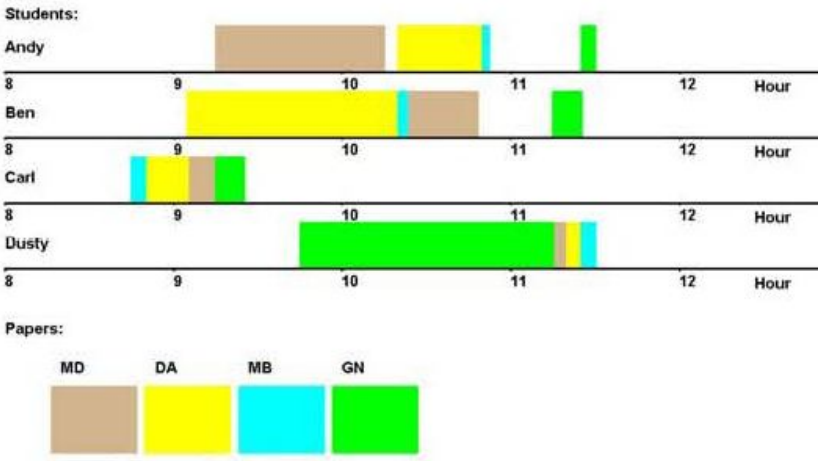


Figure 6.8: Gantt chart for students.

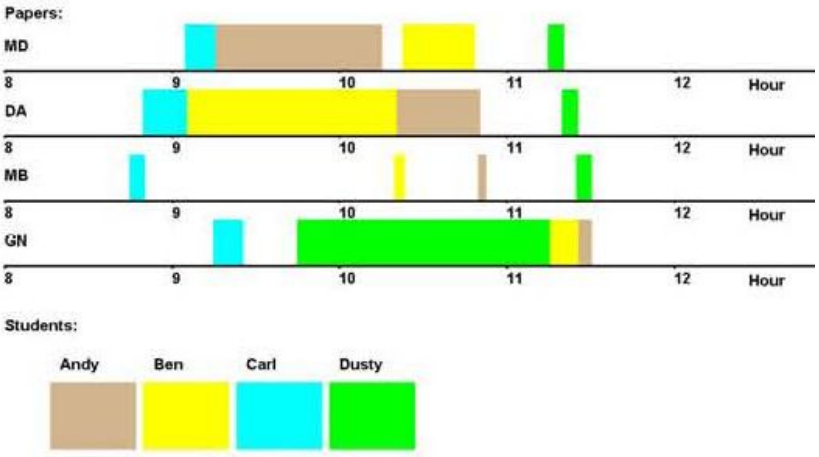


Figure 6.9: Gantt chart for papers.

```
/*1*/ :- lib(ic).
/*2*/ :- lib(ic_edge_finder3).
/*3*/ :- lib(branch_and_bound).

/*4*/ top:-
/*5*/  A=[AMD,ADA,AMB,AGN], % reading start times for Andy
/*6*/  B=[BDA,BMB,BMD,BGN], % reading start times for Ben
/*7*/  C=[CMB,CDA,CMD,CGN], % reading start times for Carl
/*8*/  D=[DGN,DMD,DDA,DMB], % reading start times for Dusty
/*9*/  End=[A_end,B_end,C_end,D_end],
              % end of reading times for students

/*10*/  A :: 30..360,
/*11*/  B :: 45..360,
/*12*/  C :: 45..360,
/*13*/  D :: 105..360,
/*14*/  End :: 90..360,
/*15*/  End_of_Ends :: 90..360,

/*16*/ AMD#>=30,
/*17*/ ADA#>=AMD+60,    % order constraints for Andy
/*18*/ AMB#>=ADA+30,
/*19*/ AGN#>=AMB+2,
/*20*/ A_end#>=AGN+5,

/*21*/ BDA#>=45,
/*22*/ BMB#>=BDA+75,    % order constraints for Ben
/*23*/ BMD#>=BMB+3,
/*24*/ BGN#>=BMD+25,
/*25*/ B_end#>=BGN+10,

/*26*/ CMB#>=45,
/*27*/ CDA#>=CMB+5,    % order constraints for Carl
/*28*/ CMD#>=CDA+15,
/*29*/ CGN#>=CMD+10,
/*30*/ C_end#>=CGN+30,

/*31*/ DGN#>=105,
/*32*/ DMD#>=DGN+90,    % order constraints for Dusty
/*33*/ DDA#>=DMD+5,
/*34*/ DMB#>=DDA+5,
/*35*/ D_end#>=DMB+5,

    % Constraining the number of students reading a paper.
    % Any paper may be read by one student only:
    % reading "Mainstream Drivel":
/*36*/ cumulative([AMD,BMD,CMD,DMD],[60,25,10,5],[1,1,1,1],1),
    % reading "Daily Absurdities":
/*37*/ cumulative([ADA,BDA,CDA,DDA],[30,75,15,5],[1,1,1,1],1),
```

```
    % reading "Morning Brainwasher":
/*38*/ cumulative([AMB,BMB,CMB,DMB],[2,3,5,5],[1,1,1,1],1),
    % reading "Gutter News":
/*39*/ cumulative([AGN,BGN,CGN,DGN],[5,10,30,90],[1,1,1,1],1),

    % Constraining the number of papers read by student.
    % Any student may read only a single paper.
    % reads Andy:
/*40*/ cumulative([AMD,ADA,AMB,AGN],[60,30,2,5],[1,1,1,1],1),
    % reads Ben:
/*41*/ cumulative([BDA,BMB,BMD,BGN],[75,3,25,10],[1,1,1,1],1),
    % reads Carl:
/*42*/ cumulative([CMB,CDA,CMD,CGN],[5,15,10,30],[1,1,1,1],1),
    % reads Dusty:
/*43*/ cumulative([DGN,DMD,DDA,DMB],[90,5,5,5],[1,1,1,1],1),

/*44*/ maxlist(End,End_of_Ends),
/*45*/ minimize(labeling([AMD,ADA,AMB,AGN,BDA,BMB,BMD,BGN,
          CMB,CDA,CMD,CGN,DGN,DMD,DDA,DMB,A_end,B_end,
              C_end,D_end]), End_of_Ends),nl,

/*46*/ write("A = "),write(A),nl,
/*47*/ write("B = "),write(B),nl,
/*48*/ write("C = "),write(C),nl,
/*49*/ write("D = "),write(D),nl,

/*50*/ present_schedule([AMD,ADA,AMB,AGN,BDA,BMB,BMD,BGN,
         CMB,CDA,CMD,CGN,DGN,DMD,DDA,DMB],
         ["Andy","Mainstream Drivel",60,
         "Andy","Daily Absurdities",30,
         "Andy","Morning Brainwasher",2,
         "Andy","Gutter News",5,
         "Ben","Daily Absurdities",75,
         "Ben","Morning Brainwasher",3,
         "Ben","Mainstream Drivel",25,
         "Ben","Gutter News",10,
         "Carl","Morning Brainwasher",5,
         "Carl","Daily Absurdities",15,
         "Carl","Mainstream Drivel",10,
         "Carl","Gutter News",10,
         "Dusty","Gutter News",90,
         "Dusty","Mainstream Drivel",5,
         "Dusty","Daily Absurdities",5,
         "Dusty","Morning Brainwasher",5]).

/*51*/ present_schedule([],[]):-nl.
/*52*/ present_schedule([H1|T1],[H21,H22,H23|T2]) :-
/*53*/    convert_time(H1, FG, FM),
/*54*/    HH is H1+H23,
```

```
/*55*/    convert_time(HH, TG, TM),nl,
/*56*/    write(H21),write(" reads "),write(H22),write(" from "),
/*57*/    write(FG),write(":"),write(FM),
/*58*/    write(" to "),write(TG),write(":"),write(TM),
/*59*/ present_schedule(T1,T2).

    %% Czasy przedstawiaja minuty po godzinie 08:00

/*60*/ convert_time(Time,Hours,Minutes) :-
/*61*/    div(Time, 60, G),
/*62*/    Hours is G + 8,
/*63*/    mod(Time,60,Minutes).
```

The message is:

```
Found a solution with cost 338
Found a solution with cost 263
Found a solution with cost 262
Found a solution with cost 257
Found a solution with cost 240
Found a solution with cost 238
Found a solution with cost 235
Found a solution with cost 210

A = [75, 140, 170, 195]
B = [65, 140, 143, 200]
C = [45, 50, 65, 75]
D = [105, 195, 200, 205]

Andy reads "Mainstream Drivel" from 9:15 to 10:15
Andy reads "Daily Absurdities" from 10:20 to 10:50
Andy reads "Morning Brainwasher" from 10:50 to 10:52
Andy reads "Gutter News" from 11:15 to 11:20
Ben reads "Daily Absurdities" from 9:5 to 10:20
Ben reads "Morning Brainwasher" from 10:20 to 10:23
Ben reads "Mainstream Drivel" from 10:23 to 10:48
Ben reads "Gutter News" from 11:20 to 11:30
Carl reads "Morning Brainwasher" from 8:45 to 8:50
Carl reads "Daily Absurdities" from 8:50 to 9:5
Carl reads "Mainstream Drivel" from 9:5 to 9:15
Carl reads "Gutter News" from 9:15 to 9:25
Dusty reads "Gutter News" from 9:45 to 11:15
Dusty reads "Mainstream Drivel" from 11:15 to 11:20
```

```
Dusty reads "Daily Absurdities" from 11:20 to 11:25
Dusty reads "Morning Brainwasher" from 11:25 to 11:30
```

The reading order is this time slightly different from what we got before. Because of the non-uniqueness of the optimum solution, for the same minimum final reading time `11:30` , Andy and Ben swapped their readings of "Gutter News". As can be seen from the Gantt diagrams, this does not violate any constraint.

6.13 Reading newspapers 3

The way data was introduced in the previous two programs was unwieldy and and made their change difficult to handle.

Program `6_6_newspapers_3.ecl`[7] presents a more professional approach to data declaring; however, the price paid for this is poorer readability. The following important private predicates are used:

```
data(Data))
Data = [student(Name,Getting_ready_time,Papers)|Rest]
Papers = [Paper,Reading_time|Rest] =
               = [Paper_1, Reading_time1,
                       Paper_2, Reading_time2, etc.]
constrain(Data,Readings,Start_times,End)
Readings = [reading(Name,Paper,Start,Reading_time)|Rest]
constrain_single_paper(Paper,Readings)
constrain_with_accu(Data,Reading_accu,Readings,
              Start_times_accu,Start_times,End)
make_reading(Papers,Name,Getting_ready_time,Reading_accu,Readings,
              Start_times_accu,Start_times)
collect_papers(Readings,Paper,Start_times_accu,Start_times,
              Start_times_accu,Reading_times)
labeling(Start_times,End) - a private labeling predicate
```

The name `accu` denotes an `accumulator` for the relevant list.

The program `6_6_newspapers_3.ecl` reads as follows:

[7] This is an OST-type problem.

```
/*1*/ :- lib(ic).
/*2*/ :- lib(ic_edge_finder3).
/*3*/ :- lib(branch_and_bound).

/*4*/ top:-
/*5*/  data(Data),
/*6*/  constrain(Data,Readings,Start_times,End),
/*7*/  minimize(labeling(Start_times,End),End),
/*8*/  present_schedule(Readings).

 %  Getting_ready_time in "Data"
          %  is given by minutes after 08:00:
/*9*/ data([
  student("Andy",30,["MD",60,"DA",30,"MB",2,"GN",5]),
  student("Ben",45,["DA",75,"MB",3,"MD",25,"GN",10]),
  student("Carl",45,["MB",5,"DA",15,"MD",10,"GN",30]),
  student("Dusty",104,["GN",90,"MD",5,"DA",5,"MB",5])]).

/*10*/constrain(Data,Readings,Start_times,End):-
/*11*/ End :: 0..363,
/*12*/ constrain_with_accu(Data,[],Readings,[],Start_times,End),
/*13*/ constrain_single_paper("MD",Readings),
/*14*/ constrain_single_paper("DA",Readings),
/*15*/ constrain_single_paper("MB",Readings),
/*16*/ constrain_single_paper("GN",Readings).

 % Make a series of "readings" for all students:
/*17*/constrain_with_accu([],Readings,Readings,Start_times,
                Start_times,_).
/*18*/constrain_with_accu([student(Name,Getting_ready_time,
         [Paper,Reading_time|Rest])|Remaining],Reading_accu,
                Readings,Start_times_accu,Start_times,End):-

 % Make readings for "Name":
/*19*/ Start :: Getting_ready_time..363,
/*20*/ Next_time is Getting_ready_time + Reading_time,
/*21*/ make_reading(Rest,Name,Next_time,
         [reading(Name,Paper,Start,Reading_time)|Reading_accu],
               Read1,[Start|Start_times_accu],Start1),
/*22*/ Read1 = [reading(Name,_,S1,D1)|_],

 % Make "End" not less than the end_time for the
 % last reading of the student. Hence "End" will finally be equal
 % to the end of the last reading of the latest student:
/*23*/ End #>= S1+D1,
/*24*/ constrain_with_accu(Remaining,Read1,Readings,Start1,
           Start_times,End).

 % Make a "reading" for the  student "Name"
```

```
 % and constrain his sequence of readings:
/*25*/make_reading([],_,_,Readings,Readings,Start_times,
          Start_times).
/*26*/make_reading([Paper,Reading_time|Rest],Name,
          Next_time,Reading_accu,Read,Start_times_accu,Starts):-
/*27*/ Start :: Next_time..363,
/*28*/ Reading_accu = [reading(Name,_,S1,D1)|_],
/*29*/ Start #>= S1+D1,
/*30*/ Next_End is Next_time+Reading_time,
/*31*/ make_reading(Rest,Name,Next_End,
         [reading(Name,Paper,Start,Reading_time)|Reading_accu],
              Read,[Start|Start_times_accu],Starts).

 % Collect all "readings" for "Paper"
 % and enforce non-overlapping of readings
 % using the global constraint "cumulative/4":
/*32*/constrain_single_paper(Paper,Readings):-
/*33*/ collect_papers(Readings,Paper,[],Starts,[],
          Reading_times),
/*34*/ cumulative(Starts,Reading_times,[1,1,1,1],1).

 % Collect all start times and reading times for "Paper":
/*35*/collect_papers([],_,S,S,D,D).
/*36*/collect_papers([reading(_,Paper,S,D)|Rest],
          Paper,S0,S1,D0,D1):- !,
/*37*/ collect_papers(Rest,Paper,[S|S0],S1,[D|D0],D1).
/*38*/collect_papers([_|Rest],Paper,S0,S1,D0,D1):-
/*39*/ collect_papers(Rest,Paper,S0,S1,D0,D1).

 % Instantiate variables using the "firts fail" heuristic:
/*40*/labeling([],End):-
/*41*/ indomain(End,min),
/*42*/ convert_time(End,Hours,Minutes),
/*43*/ write("Readings are found for final time "),write(Hours),
          write(":"),write(Minutes),nl.

/*44*/labeling(Start_times,End):-
/*45*/ select(Variable,Start_times,Rest,0,first_fail),
/*46*/ indomain(Variable),
/*47*/ labeling(Rest,End).

 % The private predicate "select(Variable,List,Rest,Flag,Heuristic)"
 % uses the standard constraint "delete/5".
 % If "List" is not empty, backtrackings are possible:
/*48*/select(_, [], _, _, _):-
/*49*/ !,
/*50*/ fail.
/*51*/select(Variable, List, Rest, Flag, Heuristic):-
/*52*/ delete(Variable,List,Rest,Flag, Heuristic).
```

```
 % Present schedule:
/*53*/present_schedule([]).
/*54*/present_schedule([reading(Name,Paper,Start,
          Reading_time)|Rest]):-
/*55*/ convert_time(Start, FH, FM),
/*56*/ HH is Start+Reading_time,
/*57*/ convert_time(HH, TH, TM),
/*58*/ write(Name),write(" reads "),write(Paper),write(" from "),
          write(FH),write(":"),write(FM),
/*59*/ write(" to "),write(TH),write(":"),write(TM),nl,
/*60*/ present_schedule(Rest).

 % Convert time:
/*61*/convert_time(Time,Hours,Minutes) :-
/*62*/ div(Time, 60, G),
/*63*/ Hours is G + 8,
/*64*/ mod(Time,60,Minutes).
```

The following message is generated:

```
  Readings are found for final time 11:45
Found a solution with cost 225
Readings are found for final time 11:30
Found a solution with cost 210
Found no solution with cost 195.0 .. 209.0

  Dusty reads MB from 11:25 till 11:30
  Dusty reads DA from 11:20 till 11:25
  Dusty reads MD from 11:15 till 11:20
  Dusty reads GN from 9:45 till 11:15
  Carl reads GN from 9:15 till 9:45
  Carl reads MD from 9:5 till 9:15
  Carl reads DA from 8:50 till 9:5
  Carl reads MB from 8:45 till 8:50
  Ben reads GN from 11:15 till 11:25
  Ben reads MD from 10:23 till 10:48
  Ben reads MB from 10:20 till 10:23
  Ben reads DA from 9:5 till 10:20
  Andy reads GN from 11:25 till 11:30
  Andy reads MB from 10:50 till 10:52
  Andy reads DA from 10:20 till 10:50
```

```
Andy reads MD from 9:15 till 10:15
```

The schedule differs from what was obtained by `6_4_newspapers_1.ecl` and `6_5_newspapers_2.ecl`, the minimum reading time remains unchanged; this being a proof of multiple optimum solutions.

6.14 Assembling bicycles

This is yet another example of scheduling with cumulative and precedence constraints. It was inspired by the distinguished *discrete* mathematician Ronald Graham who wrote an enlightened essay on a fictitious bicycle assembly plant named *ACME*, see [Graham-78]. This essay will form the basis of an instructive scheduling program; 'instructive' means that it shows the unreliability and weakness of human intuition even if confronted with a simple scheduling problem. What follows is a large quote from Graham, slightly modified to make the problem more difficult and interesting:

"Things have not been going too well in the assembling section of the *ACME Bicycle Company*. For the past six month, the section had consistently failed to meet its quota and heads were beginning to roll. A newly appointed foreman of the assembling section has been brought in to remedy this sad state of affairs. He realizes that this is his big chance the catch the eye of upper management, so that the first day on the job he rolls up his sleeves and begins finding out everything about what goes on in the section.
The first thing he learns is that the overall job of assembling a bicycle is usually broken up into a number of specific smaller tasks:

```
A - Frame preparation which includes installation of the
    front fork and fenders.
B - Mounting and aligning front wheel.
C - Mounting and aligning back wheel.
D - Attaching the derailleur to the frame.
E - Installing the gear cluster.
F - Attaching the chain wheel to the crank.
G - Attaching the crank and chain wheel to the frame.
H - Mounting right pedal and toe clip.
I - Mounting left pedal and toe clip.
J - Final attachments which includes mounting and adjusting
```

handlebars, seat, brakes, etc.[8]

He also learns that his recently departed predecessor had collected reams of data on how long (in the mean, in minutes) each of these various tasks takes a trained assembler to perform, which he had conveniently summarized in the following table:

Tasks:	A	B	C	D	E	F	G	H	I	J
Time:	7	7	7	2	2	2	2	8	8	18

Because of space and equipment constraints in the shop, the 20 assemblers in the section are usually paired up into 10 teams of 2 assemblers each, with each team assembling one bicycle at a time. The foreman made a quick calculation: one bicycle requires altogether 63 minutes of total assembly time, so a team of two *should* manage this in 31.5 minutes. This means that in an eight-hour day, each team could assemble 15.23 bicycles and with all 10 teams doing this, the quota of 152 bicycles per day can be met. The new foreman can already taste his next year promotion.

His enthusiasm dwindles considerably, however, when he realizes that bicycles can't be put together in a random order. Certain tasks must be done before certain others. For example, it is extremely awkward to mount the front fork to the frame of a bicycle, if the handlebars have already been attached to the fork. Similarly, the crank must be mounted on the frame before the pedals can be attached. After lengthy discussion with several of the experienced assemblers, the new foreman prepares the following chart showing which tasks must be done before others during assembly:

A, B, C, D, E	must be done before	J
D, E, A	must be done before	C
D	must be done before	E, F
E, F, G	must be done before	H, I
F	must be done before	G
A	must be done before	B

In addition to this mechanical constraints on the work schedule, there are

[8] To paraphrase Benedykt Chmielowski (1700-1763), who in the first Polish encyclopedia "New Athens" for the entry "*Horse*" included only one short sentence: "*Everybody knows what a horse is like*", it can be said that "*Everybody knows what a bicycle is like*" and refrain from displaying the nice picture of the *ACME* bicycle, to be found in the original Graham publication.

also two rules (known locally as "busy" rules) that management requires to observe during working hours:

Rule 1: No assembler can be idle if there is some task he or she can be doing.

Rule 2: Once an assembler starts a task, he or she must continue working on the task until it is completed.

The customary order of assembling bicycles at *Acme Bicycles* has always been the following one:

```
Task          A  B  C  D  E  F  G  H  I  J
Start time    1, 8, 9, 1, 7, 3, 5, 15, 23, 16,
```

shown in the Gantt chart in Figure 6.10.

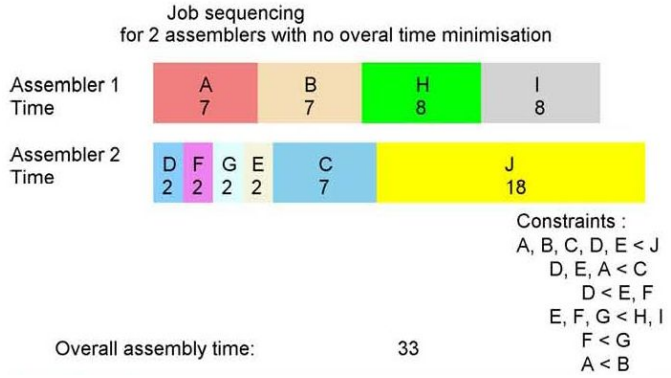


Figure 6.10: First (customary) schedule for bicycle assembling

The schedule shows the activity of each assembler of the team beginning at time `1` and progressing to the time of completed assembly, called the *overall assembling time*, some `33` minutes latter. Although this schedule obeys all the required order-of-assembly constraints given above, it allows each team to complete only `14.5` bicycles per day. Thus the total output of the section is `145` bicycles per day, well under the quota of `152`."[9].

After wasting numerous pieces of paper trying out various alternative schedules with no success, the foreman decided to ask a well-known CLP specialist for

[9]This ends for the time being the quotation from [Graham-78].

help. This specialist presented many solutions included in the `6_7_bicycles.clp`. The first solution generates a schedule that minimizes the *overall assembling time*. It is called by the query `top1`. This schedule (referred to as second schedule) may be described by the following lists, with consecutive positions corresponding to jobs `A`, `B`,...`J`:

```
Start times = [1, 8,  8,  1, 3, 5, 15, 17, 25, 15]
Durations =   [7, 7,  7,  2, 2, 2, 2,  8,  8,  18]
End times =   [8, 15, 15, 3, 5, 7, 17, 25, 33, 33]
End = 33
Assembling time = 32,
```

and visualized by the Gantt chart from Figure 6.11.

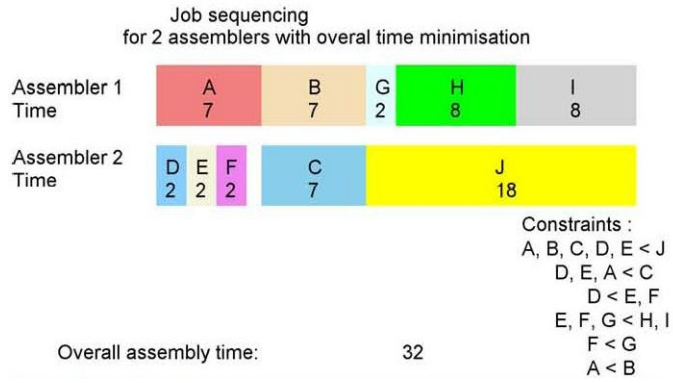


Figure 6.11: Second (optimum) schedule for bicycle assembling

Unfortunately, the overall assembling time is still over the expected `31.5` minutes. What's more - Rule 1 has been violated because between job `F` and job `C` an illegal 1-minute long inactivity is found. The foreman wants to eliminate it by changing the objective function: instead of minimizing the overall assembling time, the sum of end times for all jobs should be minimized. The CLP specialist wrote a program called by `top2` from `6_7_bicycles.clp`. The result is:

```
Start times = [1, 8,  9,  1, 3, 5, 7, 15, 16, 23]
Durations =   [7, 7,  7,  2, 2, 2, 2,  8,  8, 18]
```

```
End times = [8, 15, 16, 3, 5, 7, 9, 23, 24, 41]
End = 41
Assembling time = 40,
```

visualized by the Gantt chart from Figure 6.12.

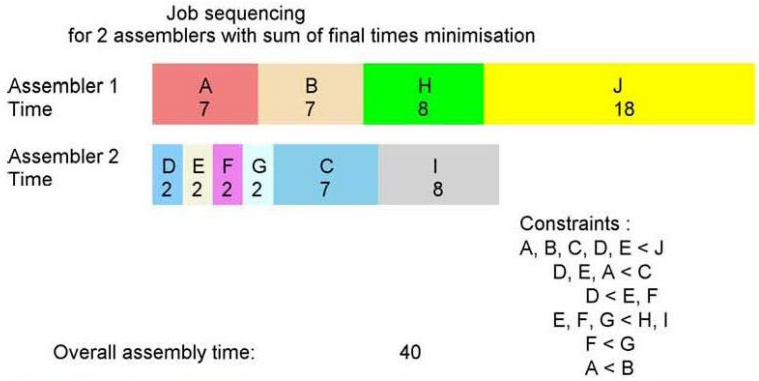


Figure 6.12: Third schedule for bicycle assembling

Obviously, this schedule is a calamity. Let us return to quoting from [Graham-78]: "The foreman decides, in haste, to furnish all the assemblers with rented electric powertools. This decreases the time of each of the jobs by exactly one minute, so the total time required for all jobs is only 53 minutes." Now the CLP specialist is checking what happens if - in order to eliminate idle times, the sum of end times for all jobs should be once more minimized. The CLP specialist wrote a program called by `top3` from `6_7_bicycles.clp`. The result is:

```
Start times = [1, 7,  12,  1, 2, 3, 4, 5, 13, 18]
Durations =   [6, 6,  6,  1, 1, 1, 1,  7,  7, 17]
End times =   [7, 13, 18, 2, 3, 4, 5, 12, 20, 35]
End = 35
Assembling time = 34,
```

visualized by the Gantt chart from Figure 6.13.

This is rather bad. The assembling time is 35 minutes, to say nothing about the 17 minutes long idle time at the end of job I.

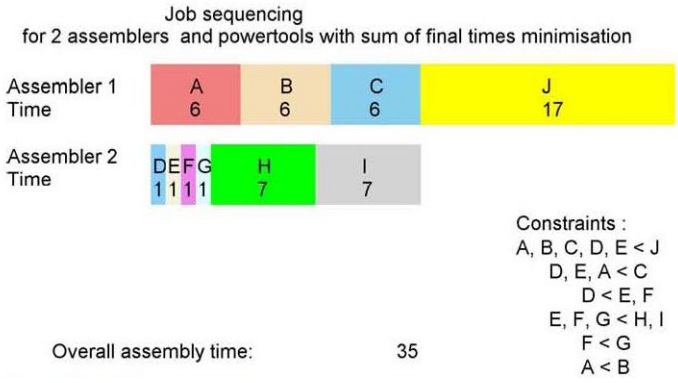


Figure 6.13: Fourth schedule for bicycle assembling

If instead the *overall assembling time* is minimized, the result is given by the `top4` part of the program, which generates the schedule:

```
Start times = [1, 7,  7,  1, 2, 3, 4, 13, 20, 13]
Durations =   [6, 6,  6,  1, 1, 1, 1,  7,  7, 17]
End times =   [7, 13, 13, 2, 3, 4, 5, 20, 27, 30]
End = 30
Assembling time = 29,
```

visualized by the Gantt chart from Figure 6.14.

Unfortunately, it contains a 2-minute long idle time between jobs `G` and `C`. The foreman resorts to a brute -force approach: he hires 10 extra assemblers and decree that from now on, each of the 10 teams will consists of three assemblers working together to put the miserable bicycle together. He realized that this increases the labor cost by `50%`, but he is determined to meet the quota. However, the CLP specialist warns him that additional `10` assemblers would not increase the production because of the order-of-assembly constraints. This he demonstrates by the `top4` part of this program, which generates a schedule:

```
Start times = [1, 8,  8,  1, 3, 3, 5, 7, 15, 15]
Durations =   [7, 7,  7,  2, 2, 2, 2,  8,  8, 18]
```

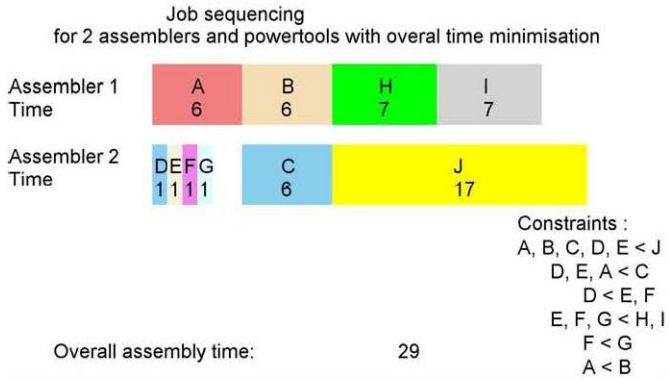


Figure 6.14: Fifth schedule for bicycle assembling

```
End times =   [8, 15, 15, 3, 5, 5, 7, 15, 23, 33]
End = 33
Assembling time = 32,
```

visualized by the Gantt chart from Figure 6.15.

It happens that the minimum assembling time for `3` assemblers is exactly the same as for two assemblers, see Figure 6.10. The foreman - desperate as he is - hires another 10 assemblers and decrees that from now on, each of the 10 teams will consists of four assemblers working together. The CLP specialist warns him again that this will be of no avail and demonstrates that by writing the `top5` part of his program, which generates the schedule:

```
Start times = [1, 8,  8,  1, 3, 3, 5, 7, 7, 15]
Durations =   [7, 7,  7,  2, 2, 2, 2,  8,  8, 18]
End times =   [8, 15, 15, 3, 5, 5, 7, 15, 15, 33]
End = 33
Assembling time = 32,
```

visualized by the Gantt chart from Figure 6.16.

The foreman, which brought *ACME* to the verge of bankruptcy, was fired on short notice: the termination notice arrived at the end of the week. I have been informed that after a number of sleepless nights he decided to offer his services

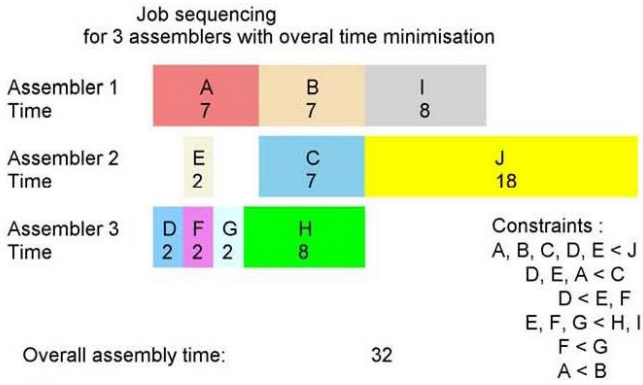


Figure 6.15: Sixth schedule for bicycle assembling

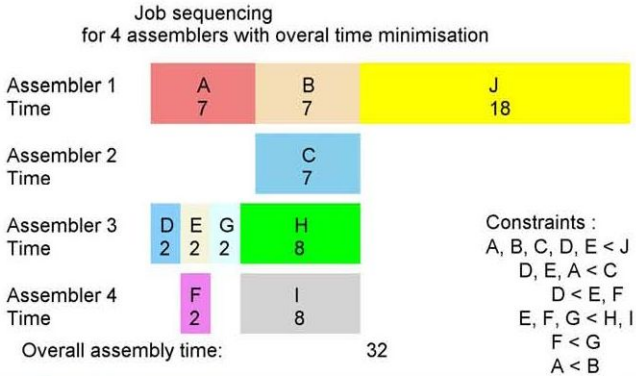


Figure 6.16: Seventh schedule for bicycle assembling

to the popular *All Things to All People* political party[10] which enthusiastically commissioned him - because of his industrial expertise - to create a lobbying service for providing manufacturing industries with financial bailouts by the

[10] A long time ago Alexis de Tocqueville (1805–1859) remarked in his famous "Democracy in America" book that "In America there are so many ways of making a living that a man doesn't usually enter politics until he has failed at everything else". Does it happen only in America? And only in such remote ages?

Absurdoland Government.

The conclusion is that getting more man-power to even such menial job as bicycle assembling is no guarantee of success. Let's quote [Graham-78] the last time: "One might well ask just where it was that our hypothetical foreman at *ACME Bicycle* did go wrong. It will turn out that he was a victim of Rules 1 and 2 (and a little bad luck). The short-sighted greediness resulted, as it often does, in an overall loss of performance of the system as a whole. In each case, assemblers were forced (by Rule 1) to start working on jobs that they couldn't interrupt (by Rule 2) when a more urgent job eventually cam up."

The program `6_7_bicycles.ecl`[11] is as follows:

```
/*1*/  :- lib(ic).
/*2*/  :- lib(ic_edge_finder3).
/*3*/  :- lib(branch_and_bound).

/*4*/  top:-
/*5*/    top1,
/*6*/    top2,
/*7*/    top3,
/*8*/    top4,
/*9*/    top5,
/*10*/   top6.

 % Minimizing assembly time for two assemblers:
/*11*/ top1:-
/*12*/   declare_domains(Start_Times,Resources),
/*13*/   Assembling_Times = [7, 7, 7, 2, 2, 2, 2, 8, 8,18],
/*14*/   End_Times =   [_, _, _, _, _, _, _, _, _, _],
/*15*/   End_Times :: 1..200,
/*16*/   Limit :: 2,

/*17*/   constraints(Start_Times),
/*18*/   cumulative(Start_Times,Assembling_Times,Resources,Limit),
/*19*/   end_times(Start_Times,Assembling_Times,End_Times,
          End),

/*20*/   bb_min(search(Start_Times,0,smallest,indomain_min,
         bbs(1),[]),End, bb_options{delta:1,timeout:60}),

/*21*/   Assembling_Time is End - 1,
/*22*/   write("Minimizing assembly time for two assemblers:"),
/*23*/   write_results(Start_Times,Assembling_Times,End_Times,
```

[11] This is an OST-type problem.

```
            End,Assembling_Time).

 % Minimizing sum of end times for two assemblers:
/*24*/ top2:-
/*25*/   declare_domains(Start_Times,Resources),
/*26*/   Assembling_Times = [7, 7, 7, 2, 2, 2, 2, 8, 8, 18],
/*27*/   End_Times =   [K1,K2,K3,K4,K5,K6,K7,K8,K9,K10],
/*28*/   End_Times :: 1..200,
/*29*/   Limit :: 2,

/*30*/   constraints(Start_Times),
/*31*/   cumulative(Start_Times,Assembling_Times,Resources,Limit),
/*32*/   end_times(Start_Times,Assembling_Times,End_Times,
         End),

/*33*/   Sum_of_End_Times #= K1+K2+K3+K4+K5+K6+K7+K8+K9+K10,

/*34*/   bb_min(search(Start_Times,0,smallest,indomain_min,
         bbs(1),[]),Sum_of_End_Times, bb_options{delta:1,timeout:60}),

/*35*/   Assembling_Time is End - 1,
/*36*/   write("Minimizing sum of end times for two assemblers:"),
/*37*/   write_results(Start_Times,Assembling_Times,End_Times,
            End,Assembling_Time).

 % Minimizing sum of end times for two assemblers and power tools:
/*38*/ top3:-
/*39*/   declare_domains(Start_Times,Resources),
/*40*/   Assembling_Times = [6, 6, 6, 1, 1, 1, 1, 7, 7, 17],
/*41*/   End_Times =   [K1,K2,K3,K4,K5,K6,K7,K8,K9,K10],
/*42*/   End_Times :: 1..200,
/*43*/   Limit :: 2,

/*44*/   constraints_for_power_tools(Start_Times),
/*45*/   cumulative(Start_Times,Assembling_Times,Resources,Limit),
/*46*/   end_times(Start_Times,Assembling_Times,End_Times,
         End),

/*47*/   Sum_of_End_Times #= K1+K2+K3+K4+K5+K6+K7+K8+K9+K10,

/*48*/   bb_min(search(Start_Times,0,smallest,indomain_min,
         bbs(1),[]),Sum_of_End_Times, bb_options{delta:1,timeout:60}),

/*49*/   Assembling_Time is End - 1,
/*50*/   write("Minimizing sum of end times for two assemblers
            and power tools:"),
/*51*/   write_results(Start_Times,Assembling_Times,End_Times,
            End,Assembling_Time).
```

```
 % Minimizing assembly time for two assemblers and power tools:
/*52*/ top4:-
/*53*/   declare_domains(Start_Times,Resources),
/*54*/   Assembling_Times = [6, 6, 6, 1, 1, 1, 1, 7, 7, 17],
/*55*/   End_Times =   [_, _, _, _, _, _, _, _, _, _],
/*56*/   End_Times :: 1..200,
/*57*/   Limit :: 2,

/*58*/   constraints_for_power_tools(Start_Times),
/*59*/   cumulative(Start_Times,Assembling_Times,Resources,Limit),
/*60*/   end_times(Start_Times,Assembling_Times,End_Times,
          End),

/*61*/   bb_min(search(Start_Times,0,first_fail,indomain,
         bbs(1),[]),End, bb_options{delta:1,timeout:60}),

/*62*/   Assembling_Time is End - 1,
/*63*/   write("Minimizing assembly time for two assemblers
              and power tools:"),
/*64*/   write_results(Start_Times,Assembling_Times,End_Times,
              End,Assembling_Time).

 % Minimizing assembly time for three assemblers:
/*65*/ top5:-
/*66*/   declare_domains(Start_Times,Resources),
/*67*/   Assembling_Times = [7, 7, 7, 2, 2, 2, 2, 8, 8,18],
/*68*/   End_Times =   [_, _, _, _, _, _, _, _, _, _],
/*69*/   End_Times :: 1..200,
/*70*/   Limit :: 3,

/*71*/   constraints(Start_Times),
/*72*/   cumulative(Start_Times,Assembling_Times,Resources,Limit),
/*73*/   end_times(Start_Times,Assembling_Times,End_Times,
          End),

/*74*/   bb_min(search(Start_Times,0,first_fail,indomain,
         bbs(1),[]),End, bb_options{delta:1,timeout:60}),

/*75*/   Assembling_Time is End - 1,
/*76*/   write("Minimizing assembly time for three assemblers:"),
/*77*/   write_results(Start_Times,Assembling_Times,End_Times,
              End,Assembling_Time).

 % Minimizing assembly time for four assemblers:
/*78*/ top6:-
/*79*/   declare_domains(Start_Times,Resources),
/*80*/   Assembling_Times = [7, 7, 7, 2, 2, 2, 2, 8, 8,18],
/*81*/   End_Times =   [_, _, _, _, _, _, _, _, _, _],
/*82*/   End_Times :: 1..200,
```

```
/*83*/   Limit :: 4,

/*84*/   constraints(Start_Times),
/*85*/   cumulative(Start_Times,Assembling_Times,Resources,Limit),
/*86*/   end_times(Start_Times,Assembling_Times,End_Times,
          End),

/*87*/   bb_min(search(Start_Times,0,first_fail,indomain,
         bbs(1),[]),End, bb_options{delta:1,timeout:60}),

/*88*/   Assembling_Time is End - 1,
/*89*/   write("Minimizing assembly time for four assemblers:"),
/*90*/   write_results(Start_Times,Assembling_Times,End_Times,
              End,Assembling_Time).

/*91*/ declare_domains(Start_Times,Resources):-
/*92*/   Start_Times =  [_, _, _, _, _, _, _, _, _, _],
/*93*/   Resources =       [_, _, _, _, _, _, _, _, _, _],
/*94*/   Start_Times :: 1..100,
/*95*/   Resources :: 1.

/*96*/ constraints([A,B,C,D,E,F,G,H,I,J]):-
/*97*/   A + 7 #=< J,
/*98*/   B + 7 #=< J,
/*99*/   C + 7 #=< J,
/*100*/   D + 2 #=< J,
/*101*/   E + 2 #=< J,
/*102*/   A + 7 #=< C,
/*103*/   D + 2 #=< C,
/*104*/   E + 2 #=< C,
/*105*/   D + 2 #=< E,
/*106*/   D + 2 #=< F,
/*107*/   E + 2 #=< H,
/*108*/   F + 2 #=< H,
/*109*/   G + 2 #=< H,
/*110*/   E + 2 #=< I,
/*111*/   F + 2 #=< I,
/*112*/   G + 2 #=< I,
/*113*/   F + 2 #=< G,
/*114*/   A + 7 #=< B.

/*115*/ constraints_for_power_tools([A,B,C,D,E,F,G,H,I,J]):-
/*116*/   A + 6 #=< J,
/*117*/   B + 6 #=< J,
/*118*/   C + 6 #=< J,
/*119*/   D + 1 #=< J,
/*120*/   E + 1 #=< J,
/*121*/   A + 6 #=< C,
/*122*/   D + 1 #=< C,
```

```
/*123*/   E + 1 #=< C,
/*124*/   D + 1 #=< E,
/*125*/   D + 1 #=< F,
/*126*/   E + 1 #=< H,
/*127*/   F + 1 #=< H,
/*128*/   G + 1 #=< H,
/*129*/   E + 1 #=< I,
/*130*/   F + 1 #=< I,
/*131*/   G + 1 #=< I,
/*132*/   F + 1 #=< G,
/*133*/   A + 6 #=< B.

/*134*/ end_times(Start_Times,Assembling_Times,End_Times,End):-
/*135*/   ( foreach(S,Start_Times),
/*136*/     foreach(D,Assembling_Times),
/*137*/     foreach(K,End_Times)
/*138*/   do
/*139*/   K #= S + D
/*140*/   ),
/*141*/   End #= max(End_Times).

/*142*/ write_results(Start_Times,Assembling_Times,End_Times,
              End,Assembling_Time):-
/*143*/   printf("%2n        Start times =
    [%d, %d,  %d,  %d, %d, %d, %d, %d, %d, %d].%n", Start_Times),
/*144*/   printf("        Assembling times =
    [%d, %d,  %d,  %d, %d, %d, %d,  %d,  %d, %d].%n", Assembling_Times),
/*145*/   printf("        End times =
    [%d, %d, %d, %d, %d, %d, %d, %d, %d, %d].%n", End_Times),
/*146*/   printf("        End = %d%2n", End),
/*147*/   printf("        Overall assembling time: =
    %d%2n", Assembling_Time).
```

The message is:

```
Found a solution with cost 41
Found a solution with cost 34
Found a solution with cost 33
Minimizing assembly time for two assemblers:
Start times = [1, 8,  8,  1, 3, 5, 15, 17, 25, 15].
Assembling times = [7, 7,  7,  2, 2, 2, 2,  8,  8, 18].
End times = [8, 15, 15, 3, 5, 7, 17, 25, 33, 33].
End = 33
Overall assembling time: = 32

Found a solution with cost 151
```

```
Found no solution with cost 121.0 .. 150.0
Minimizing sum of end times for two assemblers::
Start times = [1, 8,  9,  1, 3, 5, 7, 15, 16, 23].
Assembling times = [7, 7,  7,  2, 2, 2, 2,  8,  8, 18].
End times = [8, 15, 16, 3, 5, 7, 9, 23, 24, 41].
End = 41
Overall assembling time: = 40


Found a solution with cost 119
Found no solution with cost 97.0 .. 118.0
Minimizing sum of end times for two assemblers
and power_tools:
Start times = [1, 7,  12,  1, 2, 3, 4, 5, 13, 18].
Assembling times = [6, 6,  6,  1, 1, 1, 1,  7,  7, 17].
End times = [7, 13, 18, 2, 3, 4, 5, 12, 20, 35].
End = 35
Overall assembling time: = 34


Found a solution with cost 37
Found a solution with cost 30
Minimizing assembly time  for two assemblers and power tools:
Start times = [1, 7,  7,  1, 2, 3, 4, 13, 20, 13].
Assembling times = [6, 6,  6,  1, 1, 1, 1,  7,  7, 17].
End times = [7, 13, 13, 2, 3, 4, 5, 20, 27, 30].
End = 30
Overall assembling time: = 29


Found a solution with cost 33
Minimizing assembly time for three assemblers:
Start times = [1, 8,  8,  1, 3, 3, 5, 7, 15, 15].
Assembling times = [7, 7,  7,  2, 2, 2, 2,  8,  8, 18].
End times = [8, 15, 15, 3, 5, 5, 7, 15, 23, 33].
End = 33
Overall assembling time: = 32


Found a solution with cost 33
Minimizing assembly time for four assemblers:
Start times = [1, 8,  8,  1, 3, 3, 5, 7, 7, 15].
Assembling times = [7, 7,  7,  2, 2, 2, 2,  8,  8, 18].
End times = [8, 15, 15, 3, 5, 5, 7, 15, 15, 33].
End = 33
Overall assembling time: = 32
```

6.15 Ship unloading and loading

The built-in `cumulative` is also available with `5` arguments as `cumulative/5`:

```
cumulative(+StartTimes,+Durations,+Resources,+Areas,++Limit)
```

where `Areas` is a list of areas covered by tasks. The areas are given as products of duration and resource usage for all tasks. If:
`Durations = [D1,...,Dn]`, and
`Resources = [R1,...,Rn]`, then
`Areas = [A1,...,An]` with:
`Ai = Di*Ri` .
To program those products is up to the user. This global constraint will be useful to solve the following example first solved using *CHIP* by [Aggoun-93]:

The problem is to determine a schedule that minimizes the time to unload and load a ship. The work consists of 34 tasks, each one to be handled by a number of dockers during a given period of time. For each task the product of the number of dockers and time needed, expressed as *man-hours*[12], is given, see Table 6.3.

These man-hours correspond to the areas in `cumulative/5`. The constraint for man-hours is quite natural for this kind of tasks.

The job of unloading and loading should be done by a team of 12 dockers, each one to be employed no longer than 8 hours. The minimum-time schedule is determined by program `6_8_ship.ecl`[13]:

```
/*1*/  :- lib(ic).
/*2*/  :- lib(ic_edge_finder3).
/*3*/  :- lib(branch_and_bound).

/*4*/  top:-

  % List of task start times:
/*5*/    LS = [S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,S11,
              S12,S13,S14,S15,S16,S17,S18,S19,S20,S21,S22,
                  S23,S24,S25,S26,S27,S28,S29,S30,S31,S32,S33,S34],

  % List of task durations:
/*6*/    LD = [D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,
```

[12] A man-hour - the amount of work performed by an average docker in an hour.
[13] This is an OST-type problem.

Task	Man-hours	Next task
1	12	2, 4
2	16	3
3	12	5, 7
4	24	5
5	25	6
6	10	8
7	12	8
8	12	9
9	12	10, 14
10	16	11, 12
11	12	13
12	10	13
13	4	15,16
14	15	15
15	6	18
16	9	17
17	12	18
18	14	19, 20, 21
19	4	23
20	4	23
21	4	22
22	8	23
23	28	24
24	40	25
25	16	26,30,31,32
26	3	27
27	3	28
28	12	29
29	8	end
30	9	28
31	6	28
32	3	28
33	6	34
34	6	end

Table 6.3: Tasks for ship unloading and loading

```
              D12,D13,D14,D15,D16,D17,D18,D19,D20,D21,D22,
                  D23,D24,D25,D26,D27,D28,D29,D30,D31,D32,D33,D34],

  % List of task manpower requirements  - list of number of dockers
  % needed to accomplish the tasks:
/*7*/    LR = [R1,R2,R3,R4,R5,R6,R7,R8,R9,R10,R11,
              R12,R13,R14,R15,R16,R17,R18,R19,R20,R21,R22,
                  R23,R24,R25,R26,R27,R28,R29,R30,R31,R32,R33,R34],

  % List  of task surfaces - list of  man-hours needed to accomplish the tasks:
/*8*/    LF = [12,16,12,24,25,10,12,12,12,16,12,
              10,4,15,6,9,12,14,4,4,4,8,
                  28,40,16,3,3,12,8,9,6,3,6,6],

/*9*/    LS   :: 1..400,
/*10*/   LD :: 1..40,
/*11*/   LR :: 1..12,
/*12*/   End :: 1..400,
/*13*/   Limit :: 1..12,

/*14*/   cumulative(LS,LD,[R1,R2,R3,R4,R5,R6,R7,R8,R9,R10,R11,
              R12,R13,R14,R15,R16,R17,R18,R19,R20,R21,R22,
                  R23,R24,R25,R26,R27,R28,R29,R30,R31,R32,R33,R34],
                     LF,Limit),

/*15*/   S1 + D1 #=< S2,
/*16*/   S1 + D1 #=< S4,
/*17*/   S2 + D2 #=< S3,
/*18*/   S3 + D3 #=< S5,
/*19*/   S3 + D3 #=< S7,
/*20*/   S4 + D4 #=< S5,
/*21*/   S5 + D5 #=< S6,
/*22*/   S6 + D6 #=< S8,
/*23*/   S7 + D7 #=< S8,
/*24*/   S8 + D8 #=< S9,
/*25*/   S9 + D9 #=< S10,
/*26*/   S9 + D9 #=< S14,
/*27*/   S10 + D10 #=< S11,
/*28*/   S10 + D10 #=< S12,
/*29*/   S11 + D11 #=< S13,
/*30*/   S12 + D12 #=< S13,
/*31*/   S13 + D13 #=< S15,
/*32*/   S13 + D13 #=< S16,
/*33*/   S14 + D14 #=< S15,
/*34*/   S15 + D15 #=< S18,
/*35*/   S16 + D16 #=< S17,
/*36*/   S17 + D17 #=< S18,
/*37*/   S18 + D18 #=< S19,
/*38*/   S18 + D18 #=< S20,
```

```
/*39*/   S18 + D18 #=< S21,
/*40*/   S19 + D19 #=< S23,
/*41*/   S20 + D20 #=< S23,
/*42*/   S21 + D21 #=< S22,
/*43*/   S22 + D22 #=< S23,
/*44*/   S23 + D23 #=< S24,
/*45*/   S24 + D24 #=< S25,
/*46*/   S25 + D25 #=< S26,
/*47*/   S25 + D25 #=< S30,
/*48*/   S25 + D25 #=< S31,
/*49*/   S25 + D25 #=< S32,
/*50*/   S26 + D26 #=< S27,
/*51*/   S27 + D27 #=< S28,
/*52*/   S28 + D28 #=< S29,
/*53*/   S29 + D29 #=< 400,
/*54*/   S30 + D30 #=< S28,
/*55*/   S31 + D31 #=< S28,
/*56*/   S32 + D32 #=< S33,
/*57*/   S33 + D33 #=< S34,
/*58*/   S34 + D34 #=< 400,

  % Calculating list of task completion times:
/*59*/   (
/*60*/   foreach(I,LS),
/*61*/   foreach(J,LD),
/*62*/   foreach(K,LK)
/*63*/   do
/*64*/   K #= I + J
/*65*/   ),

  % Calculating list of task surfaces:
/*66*/   (
/*67*/   foreach(I,LD),
/*68*/   foreach(J,LR),
/*69*/   foreach(F,LF)
/*70*/   do
/*71*/   F #= I * J
/*72*/   ),

/*73*/   maxlist(LK,End),
/*74*/   minimize(labeling(LS,LD,LR),End),nl,

/*75*/   writeln("Number of dockers ":Limit),
/*76*/   writeln("End of unloading and loading ":End),nl,
/*77*/   writeln("Task   Start  Duration Dockers  End"),
/*78*/   present_results(LS,LD,LR,LK,01).

/*79*/ labeling([X1|X],[Y1|Y],[Z1|Z]):-
/*80*/   indomain(X1),
```

```
/*81*/   indomain(Y1),
/*82*/   indomain(Z1),
/*83*/   labeling(X,Y,Z).
/*84*/ labeling([],[],[]).

/*85*/ present_results([Sg|Sk],[Dg|Dk],[Rg|Rk],[Kg|Kk],N):-
/*86*/   printf("%d\t%d\t%d\t%d\t%d\t",[N,Sg,Dg,Rg,Kg]),nl,
/*87*/   N1 is N+1,
/*88*/   present_results(Sk,Dk,Rk,Kk,N1).
/*89*/ present_results([],[],[],[],_).
```

The message is:

```
  Found a solution with objective 54
  Found a solution with cost 53
  Found a solution with cost 44
  Found a solution with cost 43
  Found no solution with cost 37.0 .. 42.0
  Number of dockers  : 12
  End of unloading and loading  : 43

    Task  Start Duration  Dockers  End
     1      1        1      12      2
     2      2        2       8      4
     3      4        1      12      5
     4      5        2      12      7
     5      7        5       5     12
     6     12        1      10     13
     7      7        2       6      9
     8     13        1      12     14
     9     14        1      12     15
    10     15        2       8     17
    11     17        1      12     18
    12     18        1      10     19
    13     19        1       4     20
    14     19        3       5     22
    15     22        1       6     23
    16     20        3       3     23
    17     23        1      12     24
    18     24        2       7     26
    19     26        1       4     27
```

```
20      26          1          4      27
21      26          1          4      27
22      27          1          8      28
23      28          4          7      32
24      32          4         10      36
25      36          2          8      38
26      38          1          3      39
27      39          1          3      40
28      40          1         12      41
29      41          2          4      43
30      38          1          9      39
31      39          1          6      40
32      39          1          3      40
33      41          1          6      42
34      42          1          6      43,
```

where `Start` means the time to start the task, `Duration` is the time needed to accomplish the task, `Dockers` is the number of dockers employed for a task and `End` is the time the tasks is accomplished. This message is difficult to understand. Therefore its content is presented as Gantt chart in Figure 6.17.

To check for surface declarations and usage, see e.g. that for task `24` the number of man-hours is `40`; it amounts to `10` dockers working `4` hours.

6.16 What is a job-shop?

The newspaper reading problem from Sections 6.11, 6.12 and 6.13 belongs to a category of scheduling problems known as *job-shop* scheduling. It could be defined as follows: `n` *jobs* have to be done, each one consisting of `m` *tasks* performed in prescribed order by `m` *machines* on the *workshop floor*. It is assumed that:

- at any time, a machine can perform only one task;
- for all tasks on all machines the durations are known;
- for all jobs there is a prescribed order of tasks to be performed;
- the task performance cannot be interrupted;
- any machine is either available or unavailable;

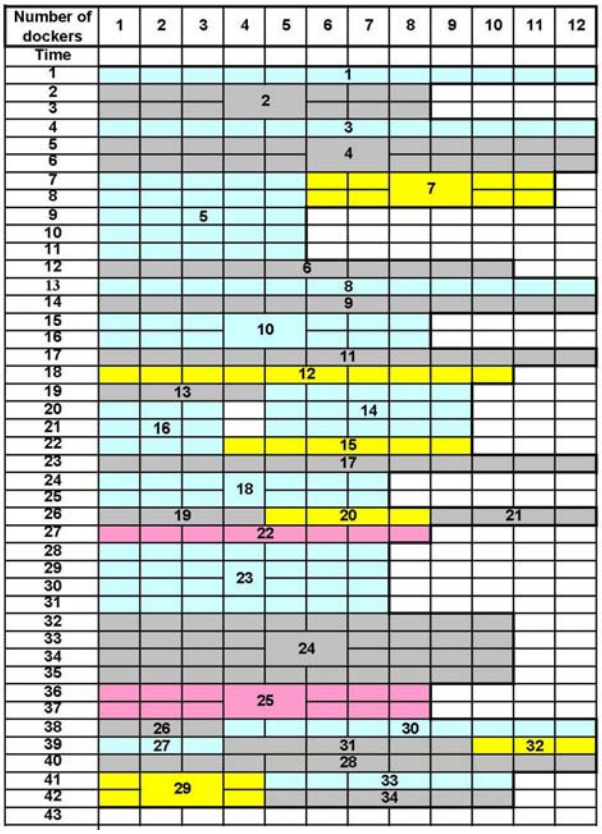


Figure 6.17: Gantt chart for optimum unloading and loading of a ship

- pauses between two consecutive tasks of a job are allowed;
- no machine can be swaped for any other;
- each machine functioning is independent from other machines functioning;
- each job is independent from other jobs.

The goal of *job-shop* problems is most often the determination of *starting times* for tasks of all jobs so that, under constraints of order and duration, the overall

time of performing all jobs (usually referred to as *makespan*) is minimized.

The terminology used above is due to early developments in the field of scheduling, which took place in manufacturing, done on some job floors in some job shops using some machines. Now, although scheduling is still a most important activity in managing manufacturing processes, especially in facilities that generate a variety of products in relatively low numbers and in batch lots, a great number of non-manufacturing (like business and military) scheduling applications have emerged, for which the old terminology (because of its relevance to any scheduling) is still used. So e.g. underwriters processing insurance policies could be considered as an insurance "shop" where underwriters ("machines") are processing policies ("jobs") by filling a number of documents ("tasks").
Let's attempt to define the *job-shop* problem in a more general way. Let

$$M = \{M_1, M_2, \ldots, M_i, \ldots, M_m\}$$

be the set of machines, and

$$J = \{J_1, J_2, \ldots, J_j, \ldots, J_n\}$$

be the set of jobs. Any job J_j consists of a sequence of m sequentially performed tasks:

$$T_j = \{T_{j,1}, T_{j,2}, \ldots, T_{j,i}, \ldots, T_{j,m}\},$$

each one needing a different machine:

$$T_{j,1} \rightarrow M_{j,1}$$

$$T_{j,2} \rightarrow M_{j,2}$$

.....

$$T_{j,i} \rightarrow M_{j,i}$$

.....

$$T_{j,m} \rightarrow M_{j,m}$$

where

$$M_{j,i} \in M,$$

and each one having a known duration:

$$\begin{gathered} T_{j,1} \to D_{j,1} \\ T_{j,2} \to D_{j,2} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ T_{j,i} \to D_{j,i} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ T_{j,m} \to D_{j,m}. \end{gathered}$$

Let

$$T = \{1, 2, \ldots, i, \ldots, m\}$$

be an ordered set of natural numbers corresponding to prescribed order of tasks. Associating with any element (j, i) of the Cartesian product $J \times T$ a pair $M_{j,i}, D_{j,i}$, leads to two functions defining a *job-shop* problem: a *machine function* and a *duration function.*

For an illustration of these concept the Reader is kindly asked to have another look at Table 6.2, where those two functions were defined for the newspaper reading problem: the rows correspond to student readings (i.e. to "jobs"), the columns correspond to reading order (i.e. to a prescribed order of "tasks"), and inside each cell of the table names of newspapers to be read (i.e. "machines" to be used) and durations of reading them (durations of "tasks" on those "machines") may be found.

Let us return to the general definition. Assume that all `n` jobs are performed using all `m` machines, and forget for a while the precedence constraints. Then the number of possible schedules is equal $(n!)^m$. It means that, while trying to solve the problem using exhaustive search, it is necessary to generate *all* $(n!)^m$ schedules, testing the precedence of tasks and when they are fulfilled - calculate the makespan[14].

Table 6.4 demonstrates how quickly the number of schedules increases.

[14]It is emphasized that all those schedules have to be generated and tested: one never knows whether the optimum makespan will occur for the last schedule generated.

m machines	n jobs	$(n!)^m$
1	5	120
3	5	1.7 million
5	5	25000 million

Table 6.4: Increase of job-shop schedule numbers

6.17 A job-shop scheduling problem - benchmark MT6

The benchmark MT6 is defined by the table from Figure 6.18. The problem has `6` *jobs* that have to be done, each one consisting of `6` *tasks*, performed in the order given by task numbers, by `6` *machines*. It is solved by program `6_13_MT6.ecl`:

```
/*1*/  :- lib(ic).
/*2*/  :- lib(ic_edge_finder3).
/*3*/  :- lib(branch_and_bound).
/*4*/  :- lib(lists).

/*5*/  top:-
      %     Sji - start time for task i of  job j:
/*5*/       S = [S00, S01, S02, S03, S04, S05,
                        S10, S11, S12, S13, S14, S15,
                        S20, S21, S22, S23, S24, S25,
                        S30, S31, S32, S33, S34, S35,
                        S40, S41, S42, S43, S44, S45,
                        S50, S51, S52, S53, S54, S55 ],

/*6*/       S :: 0..70,

     % End of  job times:
/*6*/       E = [E0, E1, E2, E3, E4, E5],
/*7*/       E :: 0..100,

     % Resources available for tasks:
/*8*/       R = [1, 1, 1, 1, 1, 1],
```

	Task 0		Task 1		Task 2		Task 3		Task 4		Task 5	
	M	D	M	D	M	D	M	D	M	D	M	D
Job 0	2	1	0	3	1	6	3	7	5	3	4	6
Job 1	1	8	2	5	4	10	5	10	0	10	3	4
Job 2	2	5	3	4	5	8	0	9	1	1	4	7
Job 3	1	5	0	5	2	5	3	3	4	8	5	9
Job 4	2	9	1	3	4	5	5	4	0	3	3	1
Job 5	1	3	3	3	5	9	0	10	4	4	2	1

Figure 6.18: Job-shop MT6 definition

```
    % Precedence constraints for tasks:
/*9*/      S01 #>= S00 + 1, S02 #>= S01 + 3, S03 #>= S02 + 6, S04 #>= S03 + 7,
/*10*/     S05 #>= S04 + 3, E0 #>= S05 + 6,
/*11*/     S11 #>= S10 + 8, S12 #>= S11 + 5, S13 #>= S12 + 10, S14 #>= S13 + 10,
/*12*/     S15 #>= S14 + 10, E1 #>= S15 + 4,
/*13*/     S21 #>= S20 + 5, S22 #>= S21 + 4, S23 #>= S22 + 8, S24 #>= S23 + 9,
/*14*/     S25 #>= S24 + 1, E2 #>= S25 + 7,
/*15*/     S31 #>= S30 + 5, S32 #>= S31 + 5, S33 #>= S32 + 5, S34 #>= S33 + 3,
/*16*/     S35 #>= S34 + 8, E3 #>= S35 + 9,
/*17*/     S41 #>= S40 + 9, S42 #>= S41 + 3, S43 #>= S42 + 5, S44 #>= S43 + 4,
/*18*/     S45 #>= S44 + 3, E4 #>= S45 + 1,
/*19*/     S51 #>= S50 + 3, S52 #>= S51 + 3, S53 #>= S52 + 9, S54 #>= S53 + 10,
/*20*/     S55 #>= S54 + 4, E5 #>= S55 + 1,

     %     Each machine is unique. Therefore it may
     %     perform at any time only a single task:

     % machine 0 may perform at any time only a single task:
/*21*/     cumulative([S01, S14, S23, S31, S44, S53], [3, 10, 9, 5, 3, 10], R, 1),

     % machine 1 may perform at any time only a single task:
/*22*/     cumulative([S02, S10, S24, S30, S41, S50],[6, 8, 1, 5, 3, 3], R, 1),

     % machine 2 may perform at any time only a single task:
/*23*/     cumulative([S00, S11, S20, S32, S40, S55],[1, 5, 5, 5, 9, 1], R, 1),

     % machine 3 may perform at any time only a single task:
/*24*/     cumulative([S03, S15, S21, S33, S45, S51],[7, 4, 4, 3, 1, 3], R, 1),

     % machine 4 may perform at any time only a single task:
```

```
/*25*/     cumulative([S05, S12, S25, S34, S42, S54],[6, 10, 7, 8, 5, 4], R, 1),

      % machine 5 may perform at any time only a single task:
/*26*/     cumulative([S04, S13, S22, S35, S43, S52],[3, 10, 8, 9, 4, 9], R, 1),

      %     Each job is a unique sequence of consecutive tasks.
      %     Therefore  at any time only one of its  tasks may ne performed:

      % job 0 is done by performing one of its task at any time:
/*27*/     cumulative([S00, S01, S02, S03, S04, S05],[1, 3, 6, 7, 3, 6], R, 1),

      % job 1 is done by performing one  of its task at any time:
/*28*/     cumulative([S10, S11, S12, S13, S14, S15],[8, 5, 10, 10, 10, 4], R, 1),

      % job 2 is done by performing one  of its task at any time:
/*29*/     cumulative([S20, S21, S22, S23, S24, S25],[5, 4, 8, 9, 1, 7], R, 1),

      % job 3 is done by performing one of its task at any time:
/*30*/     cumulative([S30, S31, S32, S33, S34, S35],[5, 5, 5, 3, 8, 9], R, 1),

      % job 4 is done by performing one of its task at any time:
/*31*/     cumulative([S40, S41, S42, S43, S44, S45],[9, 3, 5, 4, 3, 1], R, 1),

      % job 5 is done by performing one of its task at any time:
/*32*/     cumulative([S50, S51, S52, S53, S54, S55],[3, 3, 9, 10, 4, 1], R, 1),

/*33*/     append(S, E, SE),
/*34*/     maxlist(E, M),
/*35*/     bb_min(grounding(SE), M, bb_options with [strategy:continue,
                         from:0,to:100]),
/*36*/     write("End of job times = "),write(E),nl,
/*37*/     write("Minimal makespan = "),write(M),nl,nl,

/*38*/     write("  S00="),write(S00),
/*39*/     write("  S01="),write(S01),
/*40*/     write("  S02="),write(S02),
/*40*/     write("  S03="),write(S03),
/*41*/     write("  S04="),write(S04),
/*42*/     write("  S05="),write(S05),nl,

/*43*/     write("  S10="),write(S10),
/*44*/     write("  S11="),write(S11),
/*45*/     write("  S12="),write(S12),
/*46*/     write("  S13="),write(S13),
/*47*/     write("  S14="),write(S14),
/*48*/     write("  S15="),write(S15),nl,

/*49*/     write("  S20="),write(S20),
/*50*/     write("  S21="),write(S21),
```

```
/*51*/     write("  S22="),write(S22),
/*52*/     write("  S23="),write(S23),
/*53*/     write("  S24="),write(S24),
/*54*/     write("  S25="),write(S25),nl,

/*55*/     write("  S30="),write(S30),
/*56*/     write("  S31="),write(S31),
/*57*/     write("  S32="),write(S32),
/*58*/     write("  S33="),write(S33),
/*59*/     write("  S34="),write(S34),
/*60*/     write("  S35="),write(S35),nl,

/*61*/     write("  S40="),write(S40),
/*62*/     write("  S41="),write(S41),
/*63*/     write("  S42="),write(S42),
/*64*/     write("  S43="),write(S43),
/*65*/     write("  S44="),write(S44),
/*66*/     write("  S45="),write(S45),nl,

/*37*/    write("  S50="),write(S50),
/*68*/    write("  S51="),write(S51),
/*69*/    write("  S52="),write(S52),
/*70*/    write("  S53="),write(S53),
/*71*/    write("  S54="),write(S54),
/*72*/    write("  S55="),write(S55),nl.

/*73*/ grounding(All_Variables):-
/*74*/     middle_first(All_Variables, All_VariablesP),
/*75*/     (fromto(All_VariablesP, Variables, VariablesRem, []) do
/*76*/     delete(Variable, Variables, VariablesRem, 0, max_regret),
/*77*/     indomain(Variable, min)
/*78*/     ).

/*79*/ middle_first(List, Ord):-
/*80*/     halve(List, F, B),
/*81*/     reverse(F, RF),
/*82*/     splice(B, RF, Ord).
```

The message is:

```
Found a solution with cost 67
Found a solution with cost 64
Found a solution with cost 61
Found a solution with cost 60
Found a solution with cost 59
Found a solution with cost 58
Found a solution with cost 57
Found a solution with cost 56
Found a solution with cost 55
Found no solution with cost 47.0 .. 54.0
E = [55, 52, 45, 54, 53, 50]
Minimal makespan = 55

S00=5  S01=6  S02=16  S03=30  S04=42  S05=49
S10=0  S11=8  S12=13  S13=28  S14=38  S15=48
S20=0   S21=5  S22=9    S23=18  S24=27  S25=38
S30=8   S31=13  S32=22  S33=27  S34=30  S35=45
S40=13  S41=22  S42=25  S43=38  S44=48  S45=52
S50=13  S51=16  S52=19  S53=28  S54=45  S55=49
```

The solution is given by the Gantt charts from Figure 6.19.

6.18 A difficult job-shop scheduling problem - benchmark MT10

Obviously, solving job-shop problems must be a challenge to OR and CLP people. This is best shown by a range of job-shop benchmarks, more or less difficult, used over years to test various algorithms. One of the more celebrated and famous benchmark is the `10` jobs `10` machines job-shop benchmark known as *MT10*. It is defined by the table from Figure 6.20, where `M` denotes machines and `D` task durations. It was proposed in 1963 by J.F. Muth and G.L. Thompson in the book [Muth-63]. The problem has 100 integer variables - the start times for `10` tasks on `10` machines. Finding its solution was an open problem for more than 20 years. Until 1982 the best available upper bound for the makespan was equal `935` with no reasonable lower bound known. Using a highly specialized search algorithm the minimum makespan equal `930` was determined in 1987 by Carlier and Pinson, see [Carlier-89]. The problem is still considered as one of the most rewarding benchmarks for job-shop scheduling methods and constantly challenges designers of integer programming algorithms and CLP programs.

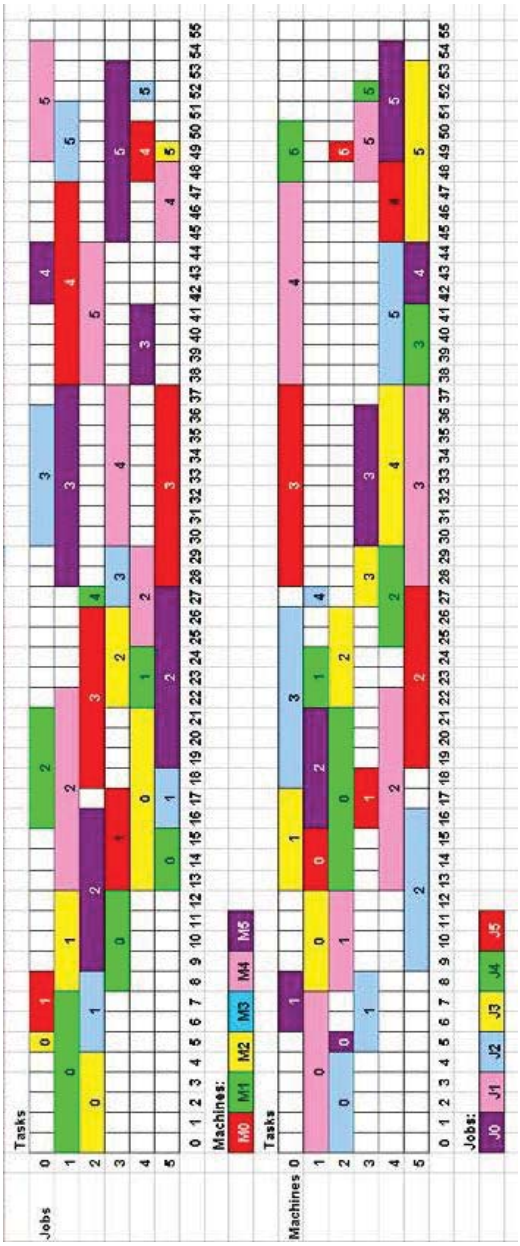


Figure 6.19: MT6 Gantt charts

The complexity of MT10 is the reason to first of all test the existence of a feasible solution. This may be done with the help of program `6_9_mt10_tes.ecl`[15]:

	Task 1		Task 2		Task 3		Task 4		Task 5		Task 6		Task 7		Task 8		Task 9		Task A	
	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D
Job 1	1	29	2	78	3	9	4	36	5	49	6	11	7	62	8	56	9	44	10	21
Job 2	1	43	3	90	5	75	10	11	4	69	2	28	7	46	6	46	8	72	9	30
Job 3	2	91	1	85	4	39	3	74	9	90	6	10	8	12	7	89	10	45	5	33
Job 4	2	81	3	95	1	71	5	99	7	9	9	52	8	85	4	98	10	22	6	43
Job 5	3	14	1	6	2	22	6	61	4	26	5	69	9	21	8	49	10	72	7	53
Job 6	3	84	2	2	6	52	4	95	9	48	10	72	1	47	7	65	5	6	8	25
Job 7	2	46	1	37	4	61	3	13	7	32	6	21	10	32	9	89	8	30	5	55
Job 8	3	31	1	86	2	46	66	74	5	32	7	88	9	19	10	48	8	36	4	79
Job 9	1	76	2	69	4	76	6	51	3	85	10	11	7	40	8	89	5	26	9	74
Job A	2	85	1	13	3	61	7	7	9	64	10	76	6	47	4	52	5	90	8	45

Figure 6.20: Job-shop MT10 definition

```
/*1*/  :- lib(ic).

/*2*/  top:-
/*3*/    S = [S11,S12,S13,S14,S15,S16,S17,S18,S19,S1A,
              S21,S22,S23,S24,S25,S26,S27,S28,S29,S2A,
              S31,S32,S33,S34,S35,S36,S37,S38,S39,S3A,
              S41,S42,S43,S44,S45,S46,S47,S48,S49,S4A,
              S51,S52,S53,S54,S55,S56,S57,S58,S59,S5A,
              S61,S62,S63,S64,S65,S66,S67,S68,S69,S6A,
              S71,S72,S73,S74,S75,S76,S77,S78,S79,S7A,
              S81,S82,S83,S84,S85,S86,S87,S88,S89,S8A,
              S91,S92,S93,S94,S95,S96,S97,S98,S99,S9A,
              SA1,SA2,SA3,SA4,SA5,SA6,SA7,SA8,SA9,SAA],

 %   Sji - start time for task i of job j

/*4*/    E = [E1,E2,E3,E4,E5,E6,E7,E8,E9,EA],
/*5*/    E :: 655..1000,
/*6*/    S :: 0..1000,
```

[15] This is an FS-type problem.

```
 %  Precedence constraints for operations: for all jobs,

/*7*/     S12 #>= S11+29,   /*8*/     S13 #>= S12+78,
/*9*/     S14 #>= S13+9,    /*10*/    S15 #>= S14+36,
/*11*/    S16 #>= S15+49,   /*12*/    S17 #>= S16+11,
/*13*/    S18 #>= S17+62,   /*14*/    S19 #>= S18+56,
/*15*/    S1A #>= S19+44,   /*16*/    E1  #>= S1A+21,

/*17*/    S22 #>= S21+43,   /*18*/    S23 #>= S22+90,
/*19*/    S24 #>= S23+75,   /*20*/    S25 #>= S24+11,
/*21*/    S26 #>= S25+69,   /*22*/    S27 #>= S26+28,
/*23*/    S28 #>= S27+46,   /*24*/    S29 #>= S28+46,
/*25*/    S2A #>= S29+72,   /*26*/    E2  #>= S2A+30,

/*27*/    S32 #>= S31+91,   /*28*/    S33 #>= S32+85,
/*29*/    S34 #>= S33+39,   /*30*/    S35 #>= S34+74,
/*31*/    S36 #>= S35+90,   /*32*/    S37 #>= S36+10,
/*33*/    S38 #>= S37+12,   /*34*/    S39 #>= S38+89,
/*35*/    S3A #>= S39+45,   /*36*/    E3  #>= S3A+33,

/*37*/    S42 #>= S41+81,   /*38*/    S43 #>= S42+95,
/*39*/    S44 #>= S43+71,   /*40*/    S45 #>= S44+99,
/*41*/    S46 #>= S45+9,    /*42*/    S47 #>= S46+52,
/*43*/    S48 #>= S47+85,   /*44*/    S49 #>= S48+98,
/*45*/    S4A #>= S49+22,   /*46*/    E4  #>= S4A+43,


/*47*/    S52 #>= S51+14,   /*48*/    S53 #>= S52+6,
/*49*/    S54 #>= S53+22,   /*50*/    S55 #>= S54+61,
/*51*/    S56 #>= S55+26,   /*52*/    S57 #>= S56+69,
/*53*/    S58 #>= S57+21,   /*54*/    S59 #>= S58+49,
/*55*/    S5A #>= S59+72,   /*56*/    E5  #>= S5A+53,

/*57*/    S62 #>= S61+84,   /*58*/    S63 #>= S62+2,
/*59*/    S64 #>= S63+52,   /*60*/    S65 #>= S64+95,
/*61*/    S66 #>= S65+48,   /*62*/    S67 #>= S66+72,
/*63*/    S68 #>= S67+47,   /*64*/    S69 #>= S68+65,
/*65*/    S6A #>= S69+6,    /*66*/    E6  #>= S6A+25,

/*67*/    S72 #>= S71+46,   /*68*/    S73 #>= S72+37,
/*69*/    S74 #>= S73+61,   /*70*/    S75 #>= S74+13,
/*71*/    S76 #>= S75+32,   /*72*/    S77 #>= S76+21,
/*73*/    S78 #>= S77+32,   /*74*/    S79 #>= S78+89,
/*75*/    S7A #>= S79+30,   /*76*/    E7  #>= S7A+55,

/*77*/    S82 #>= S81+31,   /*78*/    S83 #>= S82+86,
/*79*/    S84 #>= S83+46,   /*80*/    S85 #>= S84+74,
/*81*/    S86 #>= S85+32,   /*82*/    S87 #>= S86+88,
/*83*/    S88 #>= S87+19,   /*84*/    S89 #>= S88+48,
```

```
/*85*/    S8A #>= S89+36,   /*86*/    E8  #>= S8A+79,

/*87*/    S92 #>= S91+76,   /*88*/    S93 #>= S92+69,
/*89*/    S94 #>= S93+76,   /*90*/    S95 #>= S94+51,
/*91*/    S96 #>= S95+85,   /*92*/    S97 #>= S96+11,
/*93*/    S98 #>= S97+40,   /*94*/    S99 #>= S98+89,
/*95*/    S9A #>= S99+26,   /*96*/    E9  #>= S9A+74,

/*97*/    SA2 #>= SA1+85,   /*98*/    SA3 #>= SA2+13,
/*99*/    SA4 #>= SA3+61,   /*100*/   SA5 #>= SA4+7,
/*101*/   SA6 #>= SA5+64,   /*102*/   SA7 #>= SA6+76,
/*103*/   SA8 #>= SA7+47,   /*104*/   SA9 #>= SA8+52,
/*105*/   SAA #>= SA9+90,   /*106*/   EA  #>= SAA+45,

/*107*/   append(S,E,SE),
/*108*/   labeling(SE).
```

The program contains only precedence constraints and duration data. It generates the assuring message"

```
Yes (0.02s cpu, solution 1, maybe more),
```

indicating the existence of feasible solutions. So an optimum solution must exist as well.

An efficient, general and rather complex program for solving MT10 and other similar job-shop benchmarks using ECL^iPS^e, developed by J. Schimpf (see [Schimpf-10]) using algorithms presented in [Baptiste-95], is available on the website `http://www.eclipse-clp.org/eclipse/examples`, Section `Planning and Scheduling`, Subsection `Jobshop Scheduling`.

Considering the introductory nature of this book, the `6_10_mt10.ecl` program presented below is solely aimed at proofing the correctness of the minimal makespan being equal to `930`, while using elementary modeling and trying to get results quickly. This has been done using some *a priori* information about domains of start times. It has been obtained by calculating (outside of the program discussed) the earliest and latest start times of all tasks[16]. However, it was not sufficient to decrease the time needed to get the solution, so some *man-*

[16]The *earliest start time* for a task was calculated as the sum of durations of all tasks preceding it in the job. The *latest start time* was calculated as the difference between the upper bound of the End domain and the sum of durations of all following tasks in the job.

ual corrections were introduced for some domains to make them yet smaller. Obviously, a program tailored that way has no generality at all: any change of data will require fine tuning of domains. This makeshift brute-force approach conveys however an important and general principle: *the more we know about the variable domains, and the smaller they can be declared, the quicker solutions are obtained*. The program `6_10_mt10.ecl`[17] is as follows:

```
/*1*/  :- lib(ic).
/*2*/  :- lib(ic_edge_finder3).
/*3*/  :- lib(branch_and_bound).
/*4*/  :- lib(lists).

/*5*/  top:-
/*6*/    S = [S11,S12,S13,S14,S15,S16,S17,S18,S19,S1A,
              S21,S22,S23,S24,S25,S26,S27,S28,S29,S2A,
              S31,S32,S33,S34,S35,S36,S37,S38,S39,S3A,
              S41,S42,S43,S44,S45,S46,S47,S48,S49,S4A,
              S51,S52,S53,S54,S55,S56,S57,S58,S59,S5A,
              S61,S62,S63,S64,S65,S66,S67,S68,S69,S6A,
              S71,S72,S73,S74,S75,S76,S77,S78,S79,S7A,
              S81,S82,S83,S84,S85,S86,S87,S88,S89,S8A,
              S91,S92,S93,S94,S95,S96,S97,S98,S99,S9A,
              SA1,SA2,SA3,SA4,SA5,SA6,SA7,SA8,SA9,SAA],

    %     Sji - start time for task i of job j

/*7*/    E = [E1,E2,E3,E4,E5,E6,E7,E8,E9,EA],
/*8*/    E :: 655..1000,
/*9*/    R = [1,1,1,1,1,1,1,1,1,1],

/*10*/     S11 :: 0..605,     /*11*/     S21 :: 0..490,
/*12*/     S31 :: 0..432,     /*13*/     S41 :: 0..345,
/*14*/     S51 :: 0..607,     /*15*/     S61 :: 0..504,
/*16*/     S71 :: 0..584,     /*17*/     S81 :: 0..461,
/*18*/     S91 :: 0..403,     /*19*/     SA1 :: 0..460,

/*20*/     S12 :: 29..634,    /*21*/     S22 :: 43..533,
/*22*/     S32 :: 91..523,    /*23*/     S42 :: 81..426,
/*24*/     S52 :: 14..621,    /*25*/     S62 :: 84..588,
/*26*/     S72 :: 46..630,    /*27*/     S82 :: 31..492,
/*28*/     S92 :: 76..479,    /*29*/     SA2 :: 85..370,

/*30*/     S13 :: 107..712,   /*31*/     S23 :: 133..628,
/*32*/     S33 :: 176..608,   /*33*/     S43 :: 176..521,
/*34*/     S53 :: 20..627,    /*35*/     S63 :: 86..590,
```

[17] This is an OST-type problem.

```
/*36*/     S73 :: 83..667,    /*37*/     S83 :: 117..578,
/*38*/     S93 :: 145..548,   /*39*/     SA3 :: 98..548,

/*40*/     S14 :: 116..721,   /*41*/     S24 :: 208..698,
/*42*/     S34 :: 215..647,   /*43*/     S44 :: 247..592,
/*44*/     S54 :: 42..649,    /*45*/     S64 :: 138..642,
          % correction:                         % correction:
/*46*/     S74 :: 100..450,   /*47*/     S84 :: 100..450,
/*48*/     S94 :: 221..624,   /*49*/     SA4 :: 159..619,


/*50*/     S15 ::152..757,   /*51*/     S25 ::219..709,
/*52*/     S35 ::289..721,   /*53*/     S45 ::346..691,
/*54*/     S55 ::103..710,   /*55*/     S65 ::233..737,
/*56*/     S75 ::157..741,   /*57*/     S85 ::237..698,
/*58*/     S95 ::272..675,   /*59*/     SA5 ::166..626,

/*60*/     S16 ::201..806,   /*61*/     S26 ::288..778,
/*62*/     S36 ::300..700,   /*63*/     S46 ::355..700,
/*64*/     S56 ::129..736,   /*65*/     S66 ::281..736,
/*66*/     S76 ::189..736,   /*67*/     S86 ::269..730,
/*68*/     S96 ::250..600,   /*69*/     SA6 ::230..690,


/*70*/     S17 :: 212..817,   /*71*/     S27 :: 316..806,
/*72*/     S37 :: 389..821,   /*73*/     S47 :: 407..821,
/*74*/     S57 :: 198..805,   /*75*/     S67 :: 353..857,
/*76*/     S77 :: 210..793,   /*77*/     S87 :: 357..818,
/*78*/     S97 :: 368..771,   /*79*/     SA7 :: 306..766,

/*80*/     S18 :: 274..879,   /*81*/     S28 :: 362..852,
/*82*/     S38 :: 401..833,   /*83*/     S48 :: 492..837,
          % correction:
/*84*/     S58 :: 450..800,   /*85*/     S68 :: 400..904,
/*86*/     S78 :: 242..826,   /*87*/     S88 :: 376..837,
/*88*/     S98 :: 408..811,   /*89*/     SA8 :: 353..813,


/*90*/     S19 :: 330..935,   /*91*/     S29 :: 408..898,
/*92*/     S39 :: 490..922,   /*93*/     S49 :: 590..935,
/*94*/     S59 :: 268..875,   /*95*/     S69 :: 450..800,
/*96*/     S79 :: 450..800,   /*97*/     S89 :: 424..885,
/*98*/     S99 :: 497..900,   /*99*/     SA9 :: 405..865,

/*100*/    S1A :: 374..979,   /*101*/    S2A :: 480..970,
/*102*/    S3A :: 535..967,   /*103*/    S4A :: 612..957,
/*104*/    S5A :: 340..947,   /*105*/    S6A :: 471..975,
/*106*/    S7A :: 361..945,   /*107*/    S8A :: 460..921,
/*108*/    S9A :: 523..921,   /*109*/    SAA :: 495..955,
```

```
 %     Each machine is unique. Therefore it may
 %     perform at any time only a single task:

 %     Machine 1 may perform at any time only a single task:
/*110*/    cumulative([S11,S21,S32,S43,S52,S67,S72,S82,S91,SA2],
              [29,43,85,71,6,47,37,86,76,13],R,1),
 %     Machine 2 may perform at any time only a single task:
/*111*/    cumulative([S12,S26,S31,S41,S53,S62,S71,S83,S92,SA1],
              [78,28,91,81,22,2,46,46,69,85],R,1),
/*112*/    cumulative([S13,S22,S34,S42,S51,S61,S74,S81,S95,SA3],
              [9,90,74,95,14,84,13,31,85,61],R,1),
/*113*/    cumulative([S14,S25,S33,S48,S55,S64,S73,S8A,S93,SA8],
              [36,69,39,98,26,95,61,79,76,52],R,1),
/*114*/    cumulative([S15,S23,S3A,S44,S56,S69,S7A,S85,S99,SA9],
              [49,75,33,99,69,6,55,32,26,90],R,1),
/*115*/    cumulative([S16,S28,S36,S4A,S54,S63,S76,S84,S94,SA7],
              [11,46,10,43,61,52,21,74,51,47],R,1),
/*116*/    cumulative([S17,S27,S38,S45,S5A,S68,S75,S86,S97,SA4],
              [62,46,89,9,53,65,32,88,40,7],R,1),
/*117*/    cumulative([S18,S29,S37,S47,S58,S6A,S79,S89,S98,SAA],
              [56,72,12,85,49,25,30,36,89,45],R,1),
/*118*/    cumulative([S19,S2A,S35,S46,S57,S65,S78,S87,S9A,SA5],
              [44,30,90,52,21,48,89,19,74,64],R,1),
/*119*/    cumulative([S1A,S24,S39,S49,S59,S66,S77,S88,S96,SA6],
              [21,11,45,22,72,72,32,48,11,76],R,1),

 %     Precedence constraints for tasks: for all jobs,
 %     the next task may start no sooner than the previous
 %     task is completed:

/*120*/    S12 #>= S11+29,   /*121*/    S13 #>= S12+78,
/*122*/    S14 #>= S13+9,    /*123*/    S15 #>= S14+36,
/*124*/    S16 #>= S15+49,   /*125*/    S17 #>= S16+11,
/*126*/    S18 #>= S17+62,   /*127*/    S19 #>= S18+56,
/*128*/    S1A #>= S19+44,   /*129*/    E1  #>= S1A+21,

/*130*/    S22 #>= S21+43,   /*131*/    S23 #>= S22+90,
/*132*/    S24 #>= S23+75,   /*133*/    S25 #>= S24+11,
/*134*/    S26 #>= S25+69,   /*135*/    S27 #>= S26+28,
/*136*/    S28 #>= S27+46,   /*137*/    S29 #>= S28+46,
/*138*/    S2A #>= S29+72,   /*139*/    E2  #>= S2A+30,

/*140*/    S32 #>= S31+91,   /*141*/    S33 #>= S32+85,
/*142*/    S34 #>= S33+39,   /*143*/    S35 #>= S34+74,
/*144*/    S36 #>= S35+90,   /*145*/    S37 #>= S36+10,
/*146*/    S38 #>= S37+12,   /*147*/    S39 #>= S38+89,
/*148*/    S3A #>= S39+45,   /*149*/    E3  #>= S3A+33,
```

```
/*150*/    S42 #>= S41+81,   /*151*/    S43 #>= S42+95,
/*152*/    S44 #>= S43+71,   /*153*/    S45 #>= S44+99,
/*154*/    S46 #>= S45+9,    /*155*/    S47 #>= S46+52,
/*156*/    S48 #>= S47+85,   /*157*/    S49 #>= S48+98,
/*158*/    S4A #>= S49+22,   /*159*/    E4  #>= S4A+43,


/*160*/    S52 #>= S51+14,   /*161*/    S53 #>= S52+6,
/*162*/    S54 #>= S53+22,   /*163*/    S55 #>= S54+61,
/*164*/    S56 #>= S55+26,   /*165*/    S57 #>= S56+69,
/*166*/    S58 #>= S57+21,   /*167*/    S59 #>= S58+49,
/*168*/    S5A #>= S59+72,   /*169*/    E5  #>= S5A+53,

/*170*/    S62 #>= S61+84,   /*171*/    S63 #>= S62+2,
/*172*/    S64 #>= S63+52,   /*173*/    S65 #>= S64+95,
/*174*/    S66 #>= S65+48,   /*175*/    S67 #>= S66+72,
/*176*/    S68 #>= S67+47,   /*177*/    S69 #>= S68+65,
/*178*/    S6A #>= S69+6,    /*179*/    E6  #>= S6A+25,

/*180*/    S72 #>= S71+46,   /*181*/    S73 #>= S72+37,
/*182*/    S74 #>= S73+61,   /*183*/    S75 #>= S74+13,
/*184*/    S76 #>= S75+32,   /*185*/    S77 #>= S76+21,
/*186*/    S78 #>= S77+32,   /*187*/    S79 #>= S78+89,
/*188*/    S7A #>= S79+30,   /*189*/    E7  #>= S7A+55,

/*190*/    S82 #>= S81+31,   /*191*/    S83 #>= S82+86,
/*192*/    S84 #>= S83+46,   /*193*/    S85 #>= S84+74,
/*194*/    S86 #>= S85+32,   /*195*/    S87 #>= S86+88,
/*196*/    S88 #>= S87+19,   /*197*/    S89 #>= S88+48,
/*198*/    S8A #>= S89+36,   /*199*/    E8  #>= S8A+79,


/*200*/    S92 #>= S91+76,   /*201*/    S93 #>= S92+69,
/*202*/    S94 #>= S93+76,   /*203*/    S95 #>= S94+51,
/*204*/    S96 #>= S95+85,   /*205*/    S97 #>= S96+11,
/*206*/    S98 #>= S97+40,   /*207*/    S99 #>= S98+89,
/*208*/    S9A #>= S99+26,   /*209*/    E9  #>= S9A+74,

/*210*/    SA2 #>= SA1+85,   /*211*/    SA3 #>= SA2+13,
/*212*/    SA4 #>= SA3+61,   /*213*/    SA5 #>= SA4+7,
/*214*/    SA6 #>= SA5+64,   /*215*/    SA7 #>= SA6+76,
/*216*/    SA8 #>= SA7+47,   /*217*/    SA9 #>= SA8+52,
/*218*/    SAA #>= SA9+90,   /*219*/    EA  #>= SAA+45,


 %     Each job is unique. Therefore at any time
 %     only one of its task may be performed:
```

```
/*219*/    cumulative([S11,S12,S13,S14,S15,S16,S17,S18,S19,S1A],
               [29,78,9,36,49,11,62,56,44,21],R,1),
/*220*/    cumulative([S21,S22,S23,S24,S25,S26,S27,S28,S29,S2A],
               [43,90,75,11,69,28,46,46,72,30],R,1),
/*221*/    cumulative([S31,S32,S33,S34,S35,S36,S37,S38,S39,S3A],
               [91,85,39,74,90,10,12,89,45,33],R,1),
/*222*/    cumulative([S41,S42,S43,S44,S45,S46,S47,S48,S49,S4A],
               [81,95,71,99,9,52,85,98,22,43],R,1),
/*223*/    cumulative([S51,S52,S53,S54,S55,S56,S57,S58,S59,S5A],
               [14,6,22,61,26,69,21,49,72,53],R,1),
/*224*/    cumulative([S61,S62,S63,S64,S65,S66,S67,S68,S69,S6A],
               [84,2,52,95,48,72,47,65,6,25],R,1),
/*225*/    cumulative([S71,S72,S73,S74,S75,S76,S77,S78,S79,S7A],
               [46,37,61,13,32,21,32,89,30,55],R,1),
/*226*/    cumulative([S81,S82,S83,S84,S85,S86,S87,S88,S89,S8A],
               [31,86,46,74,32,88,19,48,36,79],R,1),
/*227*/    cumulative([S91,S92,S93,S94,S95,S96,S97,S98,S99,S9A],
               [76,69,76,51,85,11,40,89,26,74],R,1),
/*228*/    cumulative([SA1,SA2,SA3,SA4,SA5,SA6,SA7,SA8,SA9,SAA],
               [85,13,61,7,64,76,47,52,90,45],R,1),

/*229*/    append(S,E,SE),
/*230*/    maxlist(E,M),

/*231*/    bb_min(my_labeling(SE), M, bb_options with
               [strategy:continue,from:900,to:930]),

/*232*/    write("E  = "),write(E),nl,
/*233*/    write("Minimum makespan = "),write(M),nl,nl,

/*234*/    write("S11="),write(S11),write("  S12="),write(S12),
           write("  S13="),write(S13),write("  S14="),write(S14),
           write("  S15="),write(S15),nl,write("S16="),write(S16),
           write("  S17="),write(S17),write("  S18="),write(S18),
           write("  S19="),write(S19),write("  S1A="),write(S1A),nl,nl,

/*235*/    write("S21="),write(S21),write("  S22="),write(S22),
           write("  S23="),write(S23),write("  S24="),write(S24),
           write("  S25="),write(S25),nl,write("S26="),write(S26),
           write("  S27="),write(S27),write("  S28="),write(S28),
           write("  S29="),write(S29),write("  S2A="),write(S2A),nl,nl,

/*236*/    write("S31="),write(S31),write("  S32="),write(S32),
           write("  S33="),write(S33),write("  S34="),write(S34),
           write("  S35="),write(S35),nl,write("S36="),write(S36),
           write("  S37="),write(S37),write("  S38="),write(S38),
           write("  S39="),write(S39),write("  S3A="),write(S3A),nl,nl,
```

```
/*237*/    write("S41="),write(S41),write("  S42="),write(S42),
           write("  S43="),write(S43),write("  S44="),write(S44),
           write("  S45="),write(S45),nl,write("S46="),write(S46),
           write("  S47="),write(S47),write("  S48="),write(S48),
           write("  S49="),write(S49),write("  S4A="),write(S4A),nl,nl,

/*238*/    write("S51="),write(S51),write("  S52="),write(S52),
           write("  S53="),write(S53),write("  S54="),write(S54),
           write("  S55="),write(S55),nl,write("S56="),write(S56),
           write("  S57="),write(S57),write("  S58="),write(S58),
           write("  S59="),write(S59),write("  S5A="),write(S5A),nl,nl,

/*239*/    write("S61="),write(S61),write("  S62="),write(S62),
           write("  S63="),write(S63),write("  S64="),write(S64),
           write("  S65="),write(S65),nl,write("S66="),write(S66),
           write("  S67="),write(S67),write("  S68="),write(S68),
           write("  S69="),write(S69),write("  S6A="),write(S6A),nl,nl,

/*240*/    write("S71="),write(S71),write("  S72="),write(S72),
           write("  S73="),write(S73),write("  S74="),write(S74),
           write("  S75="),write(S75),nl,write("S76="),write(S76),
           write("  S77="),write(S77),write("  S78="),write(S78),
           write("  S79="),write(S79),write("  S7A="),write(S7A),nl,nl,

/*241*/    write("S81="),write(S81),write("  S82="),write(S82),
           write("  S83="),write(S83),write("  S84="),write(S84),
           write("  S85="),write(S85),nl,write("S86="),write(S86),
           write("  S87="),write(S87),write("  S88="),write(S88),
           write("  S89="),write(S89),write("  S8A="),write(S8A),nl,nl,

/*242*/    write("S91="),write(S92),write("  S92="),write(S92),
           write("  S93="),write(S93),write("  S94="),write(S94),
           write("  S95="),write(S95),nl,write("S96="),write(S96),
           write("  S97="),write(S97),write("  S98="),write(S98),
           write("  S99="),write(S99),write("  S9A="),write(S9A),nl,nl,

/*243*/    write("SA1="),write(SA1),write("  SA2="),write(SA2),
           write("  SA3="),write(SA3),write("  SA4="),write(SA4),
           write("  SA5="),write(SA5),nl,write("SA6="),write(SA6),
           write("  SA7="),write(SA7),write("  SA8="),write(SA8),
           write("  SA9="),write(SA9),write("  SAA="),write(SAA),nl.

/*244*/ my_labeling(All_Variables):-
/*245*/    middle_first(All_Variables,All_VariablesP),
/*246*/    ( fromto(All_VariablesP, Variables, VariablesRem, []) do
/*247*/    delete(Variable, Variables, VariablesRem, 0, max_regret),
/*248*/    indomain(Variable,min)
/*249*/    ).
```

```
/*250*/ middle_first(List,Ord):-
/*251*/    halve(List,F,B),
/*252*/    reverse(F,RF),
/*253*/    splice(B,RF,Ord).
```

The message is:

```
Found a solution with cost 930
Found no solution with cost 900.0 .. 929.0
E  = [908, 915, 920, 842, 895, 655, 753, 892, 792, 930]
Minimum makespan = 930

S11=119  S12=445  S13=523  S14=532  S15=568
S16=617  S17=645  S18=721  S19=792  S1A=887

S21=76  S22=224  S23=355  S24=430  S25=568
S26=637  S27=707  S28=753  S29=813  S2A=885

S31=308  S32=408  S33=493  S34=532  S35=609
S36=699  S37=709  S38=753  S39=842  S3A=887

S41=0  S42=84  S43=185  S44=256  S45=359
S46=368  S47=420  S48=637  S49=766  S4A=799

S51=179  S52=256  S53=286  S54=308  S55=370
S56=430  S57=499  S58=530  S59=593  S5A=842

S61=0  S62=84  S63=86  S64=138  S65=233
S66=281  S67=361  S68=408  S69=499  S6A=505

S71=86  S72=148  S73=233  S74=314  S75=327
S76=421  S77=442  S78=520  S79=668  S7A=698

S81=193  S82=275  S83=399  S84=445  S85=519
S86=557  S87=699  S88=718  S89=777  S8A=813

S91=217  S92=217  S93=294  S94=370  S95=421
S96=506  S97=517  S98=579  S99=668  S9A=718

SA1=132  SA2=262  SA3=327  SA4=388  SA5=420
SA6=517  SA7=628  SA8=735  SA9=787  SAA=885
```

This message is highly uninformative. It has to be converted to Gantt charts, as was previously done in Section 6.11.

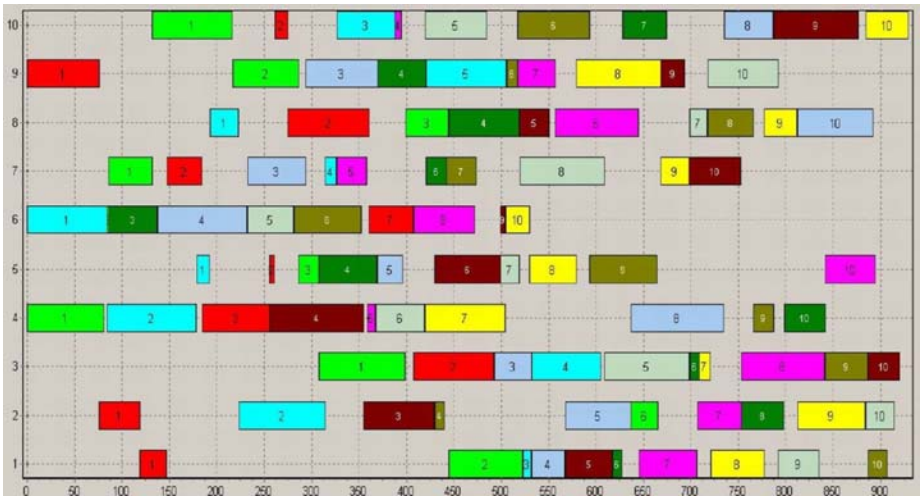


Figure 6.21: Gantt charts for MT10 jobs

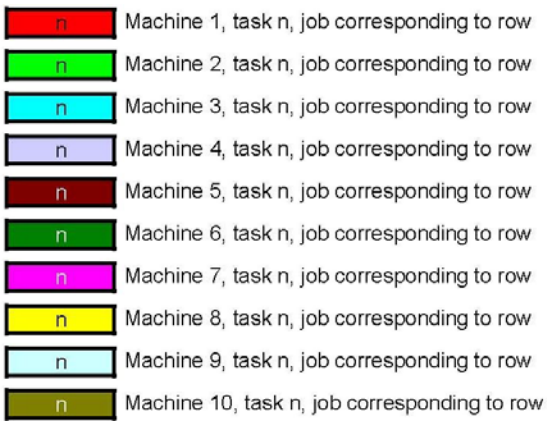


Figure 6.22: Machine coloring codes for the jobs Gantt chart

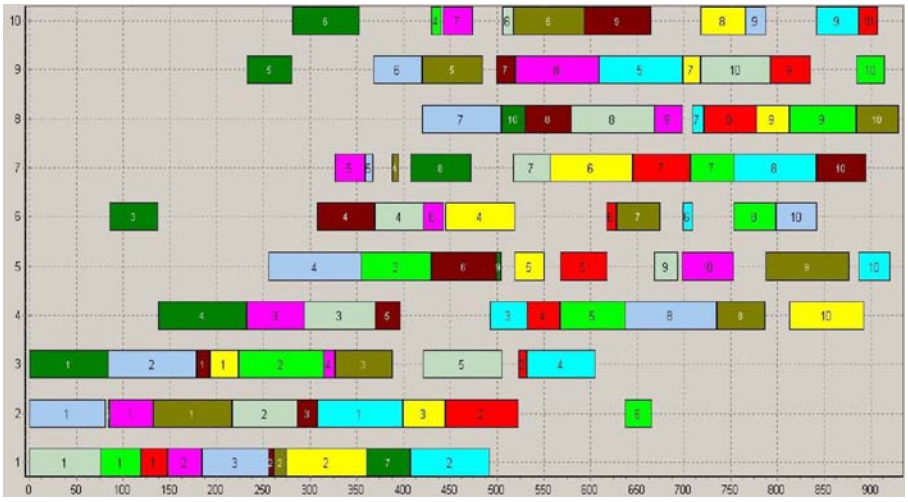


Figure 6.23: Gantt charts for MT10 machines

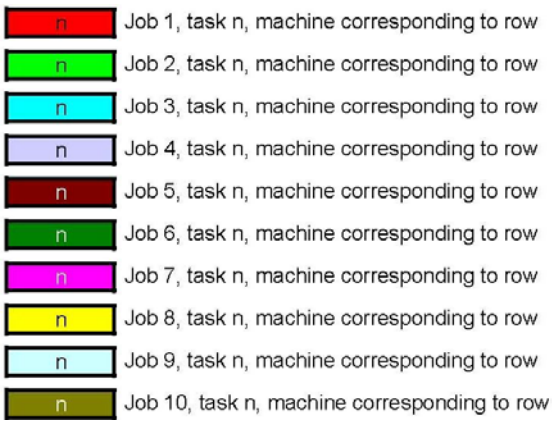


Figure 6.24: Job coloring codes for the machines Gantt charts

Figure 6.21 is the Gantt chart for jobs, with Figure 6.22 explaining the colour coding for machines.

On Figure 6.21 it may be difficult to spot task `2` for job `6`, because - as follows from line `/*170*/` of the program, he duration of this task is equal `2`, and this cannot be properly shown for the scale used. The meaning of Figure 6.21 is obvious. E.g. for job `4` the consecutive tasks are the tasks performed by the: light-green machine (machine `2`), light-blue machine (machine `3`), red machine (machine 1) etc. etc.

In order to make the Gantt chart for machines communicative, the color coding used for the job Gantt chart cannot be used any longer: otherwise all *boxes* in a row will be of the same color. Figure 6.23 is the Gantt chart for machines and Figure 6.24 shows the color coding used for jobs[18].

The meaning of Figure 6.23 is as follows, e.g. the red *box* for machine `1` corresponds to task `1` for job `1` in Figure 6.21 (also a red *box*), the red *box* formachine `2` corresponds to task 2 from job 1 in Figure 6.21 (light-green *box*), the red *box* (quite narrow) for machine 3 corresponds to task 3 of job 1 in Figure 6.21, depicted by an also rather narrow light-blue *box*, etc. etc.

Scheduling problems are rightly considered to belong to the most difficult combinatorial decision problems. They are ubiquitous. There is hardly any human activity where they may not be found. They are important as being one of the tools to control cost and time. Sometimes they may be of exorbitant size: the Viking NASA mission to Mars is believed to be based on scheduling activities of over 20.000 people. The techniques to solve them evolved considerably over time, starting with classical, often heuristic approaches (see [Muth-63]), but still used (see e.g. [Baker-09]), to constraint programming techniques, see [Baptiste-95] and [Baptiste-01].

6.19 Traveling Salesman Problems

The *Traveling Salesman Problem* (TSP) can be stated as follows: a salesman based at some city (say, city `1`) must travel to cities `2,3,...,n` visiting each city only once and then return to city `1`. The person wishes to do it in the most efficient way, i.e. covering the minimum total distance. No general method (i.e. for any `n`) of solving this problem is known, and the problem exhibits a

[18]The charts from Figures 6.21 and 6.23 have been generated using a program developed in the M.Sc. thesis by my student Bartosz Wójcik, presented in [Wójcik-05].

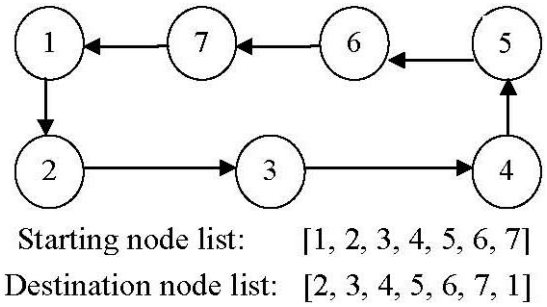


Figure 6.25: A graph that is a Hamiltonian circuit for nodes 1,2,3,4,5,6,7.

strong combinatorial explosion, or - as theoreticians prefer to call it - is *NP-hard*: the second city may be chosen in $n-1$ ways, the third city in $n-2$ ways, so for the second and third cities we have $(n-1) \times (n-2)$ choices, added the fourth city we arrive at $(n-1) \times (n-2) \times (n-3)$ choices etc. So any attempt to solve the problem by exhaustive search requires generally (i.e. while between cities `i` and `j` is a different distance than between cities `j` and `i`) $(n-1)!$ distance evaluations, which seems practical for no more than `10` cities. However, a large number of heuristics and exact methods are known at present (most of them utilizing parallel computations), which solve TSP instances with tens of thousands of cities.

6.19.1 Hamiltonian circuits

A basic concept in TSP is the concept of *Hamiltonian circuit*, defined as such circuit (i.e., closed loop) through a set of nodes that visits each node exactly once. This is illustrated by Figure 6.25.

To discuss Hamiltonian circuits in the CLP perspective, it pays to consider two lists:

1. *Starting node list* that is a list of numbers for all relevant nodes, each node represented only once. For the sake of convenience it may be an ordered list.

2. *Destination node list* that is a list of numbers of those nodes that are visited from nodes occupying the same position in the starting node list. So, in Figure 6.25, the node `3` is the destination node for starting node `2`.

To sharpen the concept, Figure 6.26 presents a graph that is not a Hamiltonian circuit: the destination node list is not a permutation of the starting node list.

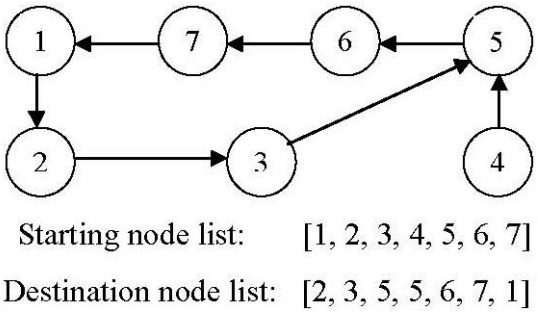


Figure 6.26: A graph that is not a Hamiltonian circuit for nodes 1,2,3,4,5,6,7.

The program `6_11_hamilton.ecl`[19] may be used to verify the nature of both graphs using the built-in `circuit/1`:

```
/*1*/    :-lib(ic).

/*2*/  top:-
/*3*/        circuit([2, 3, 4, 5, 6, 7, 1]),
/*4*/        ~(circuit([2, 3, 5, 5, 6, 7, 1])).
 % ~Goal is the sound negation operator, which delays if +Goal is not grounded.+

/*5*/  circuit(DestinationNodeList):-
/*6*/        length(DestinationNodeList,NodeCount),
/*7*/        dim(DestinationNodeArray,[NodeCount]),
/*8*/        DestinationNodeArray=..[[]|DestinationNodeList],
/*9*/        (
/*10*/       count(StartingNodeNr,1,NodeCount),
/*11*/       param(DestinationNodeArray,NodeCount)
/*12*/       do
/*13*/       arg(StartingNodeNr,DestinationNodeArray,DestinationNode),
/*14*/       CycleLength is NodeCount -2 ,
/*15*/          (
/*16*/          count(_,1,CycleLength),
/*17*/          fromto(DestinationNode,DestinationNodeIn,DestinationNodeOut,_),
```

[19]This program and the program defining the built-in `circuit/1` that forces a Hamiltonian cycle in a directed graph, has been proposed by Łukasz Domagała.

```
/*18*/          param(StartingNodeNr,DestinationNodeArray)
/*19*/          do
/*20*/          arr_element(DestinationNodeIn, DestinationNodeArray,
                          DestinationNodeOut),
/*21*/          DestinationNodeOut #\= StartingNodeNr
/*22*/          )
/*23*/       ).

/*24*/  arr_element(Index,Array,Value):-
/*25*/       (
/*26*/       ground(Index)->
/*27*/       arg(Index,Array,Value)
/*28*/       ;
/*29*/       suspend(
/*30*/       arg(Index,Array,Value),
/*31*/       0,
/*32*/       [Index->inst],
/*33*/       _ThisSusp
/*34*/           )
/*35*/       ).
```

The program describes an FS-type problem. It generates a `Yes` message.

6.19.2 Scheduling a process line

The TSP has several applications that seem far removed from the original salesman problem. They are to be found in planning and scheduling various production installations, in the manufacture of microchips and even in DNA sequencing. In these applications, the concept *city* (or more generally - node) represents, for example, installation set-ups, soldering points, or DNA fragments, and the concept *distance* represents set-up times or set-up costs, or a similarity measure between DNA fragments.

To start with a small-size problem, an installation set-up will be considered first. A process line may manufacture any of 7 types of gasoline, provided it is properly set-up. The set-up time depends upon the sequence in which these fuels are produced. In a full production cycle, during which one batch is devoted to each product, the amount of non-productive time (the set-up time)is given by Table 6.5[20].

[20]The example was inspired by a simpler one presented by [Baker-09]. There it was solved

Gasoline	1	2	3	4	5	6	7
Diesel 1	0	30	67	50	60	70	90
Regular 2	20	0	88	43	39	11	74
Premium 3	47	88	0	42	32	20	47
Ethanol_5% 4	38	43	62	0	41	59	57
Racing 5	46	39	32	41	0	52	29
Unleaded 6	40	11	20	59	52	0	69
Aviation 7	30	45	37	40	19	55	0

Table 6.5: Set-up times for gasoline production changes

The table means that e.g. to switch from producing *Premium* to producing *Aviation*, the installation has to be properly set-up which takes 47 time units. Similar problems may be found in different industries, e.g. in car body paint-shops at car assembling lines.

Consider a simple program (`6_12_TSP_small.ecl`[21]) that does the job of sequencing the gasoline manufacturing processing using the *circuit()* predicate embedded in the module `circuit.ecl`:

```
/*1*/ :-use_module(circuit).
/*2*/ :-lib(ic).
/*3*/ :-lib(branch_and_bound).

/*4*/    top:-
/*5*/        DestinationNodeList = [X1, X2, X3, X4, X5, X6, X7],
          %  Xi - number of instalation setup following installation setup i
/*6*/        DestinationNodeList :: 1..7,

/*7*/        element(X1, [ 0, 30, 67, 50, 60, 70, 90], C1),
/*8*/        element(X2, [20,  0, 88, 43, 39, 11, 74], C2),
/*9*/        element(X3, [47, 88,  0, 42, 32, 20, 47], C3),
/*10*/       element(X4, [38, 43, 62,  0, 41, 59, 57], C4),
/*11*/       element(X5, [46, 39, 32, 41,  0, 52, 29], C5),
/*12*/       element(X6, [40, 11, 20, 59, 52, 0,  69], C6),
/*13*/       element(X7, [30, 45, 37, 40, 19, 55,  0], C7),

/*14*/       circuit(DestinationNodeList),
 /*15*/       Sum_of_setup_times #= C1+C2+C3+C4+C5+C6+C7,
/*16*/       SearchGoal=search(DestinationNodeList, 0, most_constrained,
```

using classical OR techniques.

[21] This is an OST-type problem.

```
                    indomain_split, complete, []),
/*17*/      BBOptions=bb_options{strategy:dichotomic, timeout:_},
/*18*/      bb_min(SearchGoal, Sum_of_setup_times, BBOptions),
/*19*/      writeln(" Starting installation setup list   ":
                            [ 1,  2,  3,  4,  5,  6,  7]),
/*20*/      writeln(" Destination installation setup list":
                            [X1, X2, X3, X4, X5, X6, X7]),
/*21*/      writeln(" Minimum overall setup time ": Sum_of_setup_times).
```

The program generates the following message:

```
Found a solution with cost 352
Found no solution with cost 0.0 .. 176.0
Found a solution with cost 263
Found a solution with cost 203
Found no solution with cost 176.0 .. 189.5
Found no solution with cost 189.5 .. 196.25
Found no solution with cost 196.25 .. 199.625
Found a solution with cost 200
Starting installation setup list    : [1, 2, 3, 4, 5, 6, 7]
Destination installation setup list : [2, 6, 5, 1, 7, 3, 4]
Minimum overall setup time  : 200
```

The solution corresponds to the Hamiltonian circuit from Figure 6.27.

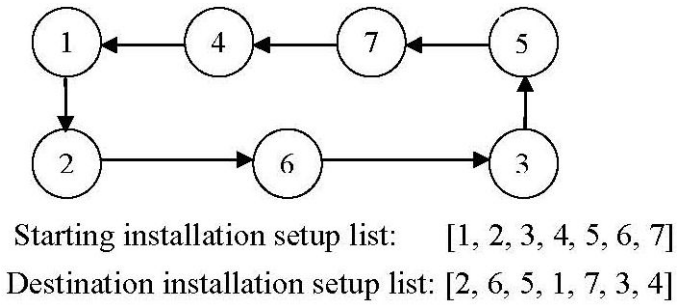


Figure 6.27: Hamiltonian circuit for optimum sequencing of set-ups.

6.19.3 Scheduling a salesman

Consider the problem of optimal scheduling a salesman visiting all 16 Absurdoland's district capitals. For this problem the approach used in the already presented example `6_12_TSP_small.ecl` turns out to be hopelessly inefficient. This is mostly due to the bad propagation properties of the `element/3` predicate. A more efficient solution is given by program `6_13_TSP_large.ecl`[22], where the module `circuit.ecl` has been evoked once more, but where no `element/3` built-ins where used to define the geometry of places to be visited. The program `6_13_TSP_large.ecl`is as follows :

```
/*1*/ :-use_module(circuit).
/*2*/ :-lib(ic).
/*3*/ :-lib(branch_and_bound).
/*4*/ :-lib(ic_global).

  % Absurdoland's district capitals are named by numbers 1,2,...16.
  % The 'Distance_matrix' below has rows assigned to starting district capitals,
  % and columns assigned to destination district capitals. It is symmetric,
  % but this is  just a happy coincidence. The program works equally well for
  % non-symmetric distance matrices.

/*5*/    distance_matrix(Distance_matrix):-
/*6*/       Distance_matrix=[](
```

[22]This is an OST-type problem.

```
                                %Destination district capitals:
             %   1   2   3   4   5   6   7   8   9   10  11  11  13  14  15  16
% Starting
% district
% capitals:
/*7,  1*/     [](  0,384,484,214,234,267,524,656,446,371,459,561,585,683,634,751),
/*8,  2*/     [](384,  0,156,411,296,167,339,379,340,432,485,545,483,500,565,642),
/*9,  3*/     [](484,156,  0,453,323,217,213,223,281,442,452,479,394,370,500,516),
/*10, 4*/     [](214,411,453,  0,130,259,413,601,303,157,245,356,422,542,427,585),
/*11, 5*/     [](234,296,323,130,  0,129,310,491,212,178,261,335,354,465,403,517),
/*12, 6*/     [](267,167,217,259,129,  0,255,389,205,265,318,391,348,421,430,516),
/*13, 7*/     [](524,339,213,413,310,255,  0,188,134,344,319,297,181,161,295,303),
/*14, 8*/     [](656,379,223,601,491,389,188,  0,322,532,507,485,363,260,477,430),
/*15, 9*/     [](446,340,281,303,212,205,134,322,  0,204,181,196,143,242,220,306),
/*16, 10*/    [](371,432,442,157,178,265,344,532,204,  0, 86,199,300,428,268,433),
/*17, 11*/    [](459,485,452,245,261,318,319,507,181, 86,  0,113,220,382,182,347),
/*18, 12*/    [](561,545,479,356,335,391,297,485,196,199,113,  0,156,323, 75,244),
/*19, 13*/    [](585,483,394,422,354,348,181,363,143,300,220,156,  0,167,114,163),
/*20, 14*/    [](683,500,370,542,465,421,161,260,242,428,382,323,167,  0,269,170),
/*21, 15*/    [](634,565,500,427,403,430,295,477,220,268,182, 75,114,269,  0,165),
/*22, 16*/    [](751,642,516,585,517,516,303,430,306,433,347,244,163,170,165,  0)
/*23*/      ).

/*24*/   top:-
/*25*/      distance_matrix(Distance_matrix),
/*26*/      dim(Distance_matrix,[CityCount,CityCount]),

     %  A variable corresponds to each destination city,
     %  its domain is given by the numbers of all cities:
/*27*/      length(DestinationCityList,CityCount),
/*28*/      DestinationCityList#::1..CityCount,

/*29*/      (foreach(DestinationCity,DestinationCityList),
     %  construct a distance array and distance list for pairs
     %  StartingCity - DestinationCity:
/*30*/      count(StartingCity,1,CityCount),
     %  construct a list of all  distances:
/*31*/      foreach(Distance,DistanceList),
     %  construct a list of all starting city numbers:
/*32*/      foreach(StartingCity,StartingCityList),
/*33*/      param(Distance_matrix) do
     %  Destination city must be different from starting city:
/*34*/      DestinationCity#\=StartingCity,
     %  The distance between starting city and destination city:
/*35*/      arg(StartingCity,Distance_matrix,DistanceArray),
/*36*/      DistanceArray=..[[]|DistanceList],
/*37*/      element(DestinationCity, DistanceList, Distance)
/*38*/      ),
```

```
    %  Each destination city must be  visited only once:
/*39*/     ic_global:    alldifferent(DestinationCityList),
    %  This is an implementation with the same semantics as the standard
    %  alldifferent/1 constraint, but with stronger propagation properties.

    %  A destination city must exist for each starting city:
/*40*/     sorted(DestinationCityList, StartingCityList),

    %  sum of distances between corresponding starting and destination cities:
/*41*/     sumlist(DistanceList,SumOfDistances),

/*42*/     circuit(DestinationCityList),

/*43*/     SearchGoal=search(DestinationCityList, 0, most_constrained,
                      indomain_split, complete, []),
/*44*/     BBOptions=bb_options{strategy:dichotomic, timeout:_},
/*45*/     bb_min(SearchGoal, SumOfDistances, BBOptions),

/*46*/     write("Overall distance = "),writeln(SumOfDistances),
/*47*/     write("Starting capitals =    "), writeln([1,2,3,4,5,6,7,8,9,10,
                      11,12,13,14,15,16]),
/*48*/     write("Destination capitals = "),writeln(DestinationCityList).
```

The program solves the problem in `1.75` seconds and generates the message:

```
Found a solution with cost 3928
Found a solution with cost 3021
Found a solution with cost 2565
Found no solution with cost 2130.0 .. 2347.5
Found no solution with cost 2347.5 .. 2456.25
Found no solution with cost 2456.25 .. 2510.625
Found a solution with cost 2521
Found no solution with cost 2510.625 .. 2515.8125
Found no solution with cost 2515.8125 .. 2518.40625
Found no solution with cost 2518.40625 .. 2519.703125
Found no solution with cost 2519.703125 .. 2520.3515625
Overall distance = 2521

Starting capitals:   [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]
Destination capitals:[4, 6, 2,10, 1, 5, 8, 3, 7, 11, 12, 15,  9, 13, 16, 14]
Search time = 1.75
```

The optimum Hamiltonian circuit is presented in Figure 6.28.

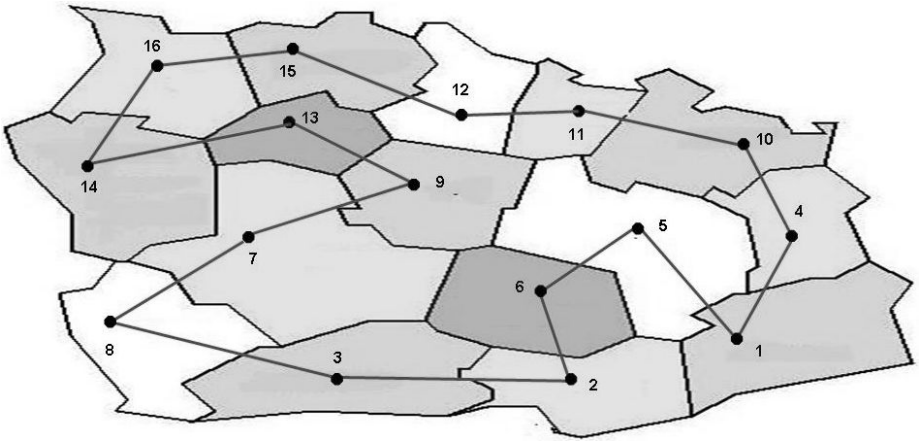


Figure 6.28: Hamiltonian circuit for the TSP solution for Absurdoland's district capitals.

We may improve the propagation properties for this problem by using a global `cycle/3` predicate[23]:

```
cycle(+DestinationCityList,++Distance_matrix,-SumOfDistances)
```

It forces a Hamiltonian cycle in a directed graph, but does it more efficiently than `circuit/1`. This is shown by example `6_14_TSP_with_cycle.ecl`[24], where the distance matrix has been put into the module `distance_matrix`:

```
/*1*/ :-use_module(distance_matrix).
/*2*/ :-lib(ic).
/*3*/ :-lib(branch_and_bound).
/*4*/ :-lib(cycle).

/*5*/ top:-
/*6*/      distance_matrix(Distance_matrix),
/*7*/      dim(Distance_matrix,[CityCount,CityCount]),
/*8*/      length(DestinationCityList,CityCount),
/*9*/      DestinationCityList#::1..CityCount,
```

[23]This global predicate has been designed by Łukasz Domagała.
[24]This is an OST-type problem.

```
/*10*/     cycle(DestinationCityList,Distance_matrix,SumOfDistances),

/*11*/     cputime(StartTime),
/*11*/     SearchGoal=search(DestinationCityList, 0, most_constrained,
                 indomain_max, complete, []),
/*12*/     bb_min(SearchGoal, SumOfDistances, bb_options{strategy:dichotomic}),
/*13*/     cputime(EndTime),
/*14*/     SearchTime is EndTime - StartTime,

/*15*/     write("Overall distance = "),writeln(SumOfDistances),
/*16*/     write("Starting capitals =    "), writeln([1,2,3,4,5,6,7,8,9,10,
                    11,12,13,14,15,16]),
/*17*/     write("Destination capitals = "),writeln(DestinationCityList),
/*18*/     write("Search time = "),writeln(SearchTime).
```

This time the program solves the problem in in shorter time (0.906) seconds and generates the message:

```
Found a solution with cost 4914
Found a solution with cost 3701
Found a solution with cost 3072
Found a solution with cost 2781
Found a solution with cost 2644
Found a solution with cost 2521
Overall distance = 2521
Starting capitals =    [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]
Destination capitals = [4, 6, 2,10, 1, 5, 8, 3, 7, 11, 12, 15,  9, 13, 16, 14]
Search time = 0.906
```

6.20 Appendices

The definition of modules `circuit.ecl` and `distance matrix.ecl` needed in programs `6_12_TSP_small.ecl` and `6_14_TSP_with_cycle.ecl` respectively are given below.

6.20.1 The "circuit.ecl" module

```
/*1*/  :- module(circuit).
/*2*/  :- export(circuit/1).
/*3*/  :- lib(ic).

/*4*/  circuit(DestinationNodeList):-
/*5*/     length(DestinationNodeList,NodeCount),
/*6*/     dim(DestinationNodeArray,[NodeCount]),
/*7*/     DestinationNodeArray=..[[]|DestinationNodeList],
/*8*/     (
/*9*/     count(StartingNodeNr,1,NodeCount),
/*10*/   param(DestinationNodeArray,NodeCount)
/*11*/   do
/*12*/   arg(StartingNodeNr,DestinationNodeArray,DestinationNode),
/*13*/   CycleLength is NodeCount -2 ,
/*14*/        (
/*15*/        count(_,1,CycleLength),
/*16*/        fromto(DestinationNode,DestinationNodeIn,
                        DestinationNodeOut,_),
/*17*/        param(StartingNodeNr,DestinationNodeArray)
/*18*/        do
/*19*/        arr_element(DestinationNodeIn, DestinationNodeArray,
                        DestinationNodeOut),
/*20*/        DestinationNodeOut #\= StartingNodeNr
/*21*/         )
/*22*/    ).

/*23*/   arr_element(Index,Array,Value):-
/*24*/        (
/*25*/        ground(Index)->
/*26*/        arg(Index,Array,Value)
/*27*/        ;
/*28*/        suspend(
/*29*/            arg(Index,Array,Value),
/*30*/            0,
/*31*/            [Index->inst],
/*32*/            _ThisSusp
```

```
/*33*/           )
/*34*/       ).
```

6.20.2 The "distance matrix.ecl" module

```
/*1*/    :- module(distance_matrix).
/*2*/ :- export(distance_matrix/1).
/*3*/ :- lib(ic).

/*4*/  distance_matrix(Distance_matrix):-
/*5*/     Distance_matrix=[](
      [](  0,384,484,214,234,267,524,656,446,371,459,561,585,683,634,751),
      [](384,  0,156,411,296,167,339,379,340,432,485,545,483,500,565,642),
      [](484,156,  0,453,323,217,213,223,281,442,452,479,394,370,500,516),
      [](214,411,453,  0,130,259,413,601,303,157,245,356,422,542,427,585),
      [](234,296,323,130,  0,129,310,491,212,178,261,335,354,465,403,517),
      [](267,167,217,259,129,  0,255,389,205,265,318,391,348,421,430,516),
      [](524,339,213,413,310,255,  0,188,134,344,319,297,181,161,295,303),
      [](656,379,223,601,491,389,188,  0,322,532,507,485,363,260,477,430),
      [](446,340,281,303,212,205,134,322,  0,204,181,196,143,242,220,306),
      [](371,432,442,157,178,265,344,532,204,  0, 86,199,300,428,268,433),
      [](459,485,452,245,261,318,319,507,181, 86,  0,113,220,382,182,347),
      [](561,545,479,356,335,391,297,485,196,199,113,  0,156,323, 75,244),
      [](585,483,394,422,354,348,181,363,143,300,220,156,  0,167,114,163),
      [](683,500,370,542,465,421,161,260,242,428,382,323,167,  0,269,170),
      [](634,565,500,427,403,430,295,477,220,268,182, 75,114,269,  0,165),
       [](751,642,516,585,517,516,303,430,306,433,347,244,163,170,165,  0)
  ).
```

6.21 Exercises

Simple scheduling

There are 4 identical machines, on which seven tasks should be performed with durations given in Table 6.6[25]. Write a program for a minimum makespan schedule provided there are no precedence constraints among tasks.

[25] This exercise is from [Baker-09].

Task	1	2	3	4	5	6	7
Duration	3	3	3	1	1	1	4

Table 6.6: Task durations

More complicated scheduling

Three machines, one of type M1 and two of type M2, have to process four jobs Ja, Jb, Jc and Jd. Each job is different and is broken up into one or more tasks that must be performed on various machines, in the order determined by the task number, as shown in Table 7.7.

Job	Task	Machine	Duration
Ja	Ta1	M1	2
Ja	Ta2	M2	6
Jb	Tb1	M2	5
Jb	Tb2	M1	3
Jb	Tb3	M2	3
Jc	Tc	M2	4
Jd	Td1	M1	5
Jd	Td2	M2	2

Table 6.7: Three machines - three jobs data

E.g. task Tb3 may begin only when task Tb2 is completed. Write a program to determine a minimum makespan schedule.

Constructing a pizzeria once more

Consider once more table 5.19 for pizzeria constructing activities. The data presented there tacitly assumed that there are unlimited resources available for the construction. Now it is assumed that there is only a 6-man strong workforce available for all activities of the job. Write a program to determine a schedule that minimizes the time to construct the pizzeria.

Five tasks

Consider a five tasks problem, in which each task is characterized by release time, duration and delivery time, as shown in Table 6.8[26].

[26]This exercise is from [Baker-09].

Task	1	2	3	4	5
Release time	0	2	3	0	6
Duration	2	1	2	3	2
Delivery time	5	2	6	3	1

Table 6.8: Five tasks data

Write a program for a minimum makespan schedule provided there are no precedence constraints among tasks.

Project

Consider the project described in Table 6.9[27].

Task	Duration	Predecessors	Resource requirement
A	6	-	2
B	8	-	3
C	4	-	3
D	4	A	4
E	4	A	2
F	12	B,E	3
G	14	B,E	1
H	6	B,C,E	4
I	8	D,F	2
J	16	D,F,G	1
K	2	D,F,G	1
L	12	H,K	3

Table 6.9: Project data

For each task its duration is known as well as its predecessors and shared resource requirement. The total number of resource units available is `5`. Write a program to minimize the project makespan.

Drilling holes

A manufacturer of printed circuit boards uses programmable drill machines to drill six holes in each board. The `x` and `y` coordinates of each hole are given in Table 6.10[28].

[27] This exercise is from [Baker-09].

[28] This exercise is from [Winston-94].

x	y	Hole
1	2	1
3	1	2
5	3	3
7	2	4
8	3	5

Table 6.10: Hole coordinates

The time (in seconds) it takes the drill machine to move from one hole to the next is equal to the distance between the points. Write a program to determine the drilling order that minimizes the total time the drilling machine spends moving between holes.

Four jobs

Four jobs must be processed on a single machine. The time required to process each job and the date the job is due are shown in Table 7.10[29].

Job number	Job duration (in days)	Due date
1	6	End of day 8
2	4	End of day 4
3	5	End of day 12
4	8	End of day 16

Table 6.11: Job durations and due dates

The delay of a job is the number of days after the due date that a job is completed. If a job is completed on time or early, the jobs delay is zero. Write a program that determines the order the jobs be processed to minimize the total delay of the four jobs.

Due date jobs [30]

JobCo uses a single machine to process three jobs. The job durations, due date and late penalties are given in Table 6.12.

Determine a schedule that minimizes the overall late penalty.

[29] This exercise is from [Winston-94].
[30] This exercise is from [Taha-08].

Job number	Job duration (in days)	Due date (in days)	Late penalty (in MU/day)
1	5	End of day 25	19
2	20	End of day 20	12
3	15	End of day 35	34

Table 6.12: Job durations, due dates and late penalties

ABZ5 benchmark

Check whether for the ABZ5 job-shop benchmark defined in Figure 6.29 there is a feasible solution. If the check is positive, write a program to solve the job-shop problem using an approach similar to that used in Section 6.18.

	Task 0		Task 1		Task 2		Task 3		Task 4		Task 5		Task 6		Task 7		Task 8		Task 9	
	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D	M	D
Job 0	4	88	8	68	6	94	5	99	1	67	2	89	9	77	7	99	0	86	3	92
Job 1	5	72	3	50	6	69	4	75	2	94	8	66	0	92	1	82	7	94	9	63
Job 2	9	83	8	61	0	83	1	65	6	64	5	85	7	78	4	85	2	55	3	77
Job 3	7	94	2	68	1	61	4	99	3	54	6	75	5	66	0	76	9	63	8	67
Job 4	3	69	4	88	9	82	8	95	0	99	2	67	6	95	5	68	7	67	1	86
Job 5	1	99	4	81	5	64	6	66	8	80	2	80	7	69	9	62	3	79	0	88
Job 6	7	50	1	86	4	97	3	96	0	95	8	97	2	66	5	99	6	52	9	71
Job 7	4	98	6	73	3	82	2	51	1	71	5	94	7	85	0	62	8	95	9	79
Job 8	0	94	6	71	3	81	7	85	1	66	2	90	4	76	5	58	8	93	9	97
Job 9	3	50	0	59	1	82	8	67	7	56	9	96	6	58	4	81	5	59	2	96

Figure 6.29: Job-shop ABZ5 definition

Chapter 7

CLP for continuous variables

7.1 CCSP and CCOP

All examples discussed so far were for discrete variables. The search trees were of finite depth and the state spaces had a finite number of points, which could be explored state after state, to search for feasible states.

ECL^iPS^e provides also tools for solving constraint satisfaction problems and constraint optimization problems for *continuous variables*, i.e. variables having continuous domains, like e.g. $0 \leq X \leq 150$;

1. A *continuous constraint satisfaction problem* (CCSP) is defined by:

   - a finite set S of continuous decision variables $X_1, ..., X_n$, with values from continuous *domains* $D_1, ..., D_n$, where $D_i = Max_i \diamond X_i \diamond Min_i$, $\diamond \in \{<, \leq, =\}$;
   - a set of constraints between variables. The constraint $C_i(X_{i_1}, ..., X_{i_k})$ between k variables from S is given by a *relation* defined as subset of the Cartesian product $D_{i_1} \times, ..., \times D_{i_k}$ that determines variable values *corresponding* to each other in a sense defined by the problem. Constraints for continuous variables are most often stated by means of equations and inequalities.

Continuous domains would make the search tree infinitely deep if the approach used for discrete domains as we know it from Chapters 2,..., 6 would be applied. To avoid this, the depth of search trees is limited by using constraint propagation methods that successfully narrow the variable domains, e.g. for some initial domain $10 \leq X \leq 90$ the next domain may be $5 \leq X \leq 65$, etc. Such narrowing never results in a single value, but in a comparatively narrow domain. Therefore the results obtained have the form:

```
X = Lower_bound__Upper_bound,
```

e.g.:

```
X = 36345.099404108__36345.099448266.
```

So bounded real results are written as two floating point bounds separated by a double underscore`__`. They may also be written as:

```
X{Lower_bound .. Upper_bound}
```

e.g.:

```
X{36345.099404108 .. 36345.099448266},
```

with two floating point bounds separated by a double full stop `...` The apparatus needed to accomplish this is known as *interval arithmetic*[1]. So a CCSP *solution* is given by any assignment of domain subintervals to variables so that all constraints are satisfied. It may be *non-unique* or *unique*. As for discrete CSP, CCSP's may be divided into feasible state determination problems and feasible trajectory determination problems. For CCSP problems there is no need to evoke the *eplex* library, the library *ic* being just right. However, symbols of arithmetic operations and relations for continuous variables have to be prefixed by `$`.

[1]Interval arithmetic - as contrasted with "normal" arithmetic - deals with arithmetic operations on intervals. The result of arithmetic interval operations is not given by some set of state variable values, but by some set of state variable intervals.

2. A *continuous constraint optimization problem* (CCOP) is defined by:

   - a finite set S of continuous and discrete decision variables $X_1, ..., X_n$, with values often from a standard domain declared as `0.0..1.0Inf`;
   - a set of constraints between variables, often stated by means of equations and inequalities;
   - an *objective function*, expressed as linear function of decision variables with not necessarily integer coefficients, to be minimized or maximized by choosing proper values for the decision variables from their domains;
   - a set of declaration for parameterizing the solver and the [print-out.

   For solving CCOP's, the ECL^iPS^e platform is integrated with incremental interval solvers of linear equations, of linear programming, integer programming and mixed programming problems. They may be used by evoking the ECL^iPS^e library named *eplex*. This library enables the use of ECL^iPS^e for the design of interfaces to commercial solvers like *XPRESS-MP* by Dash Optimization or *CPLEX* by ILOG. Although both companies provide students with free academic versions, the following examples will makes use of only ECL^iPS^e-provided *eplex* solver.

   Similar to discrete COP, CCOP may be divided into optimum state determination problems and optimum trajectory determination problems.

   As for discrete domains, it is worthwhile to start the discussion of ECL^iPS^e applications to continuous domains with feasible state determination problems.

7.2 The blessing and curse of compound interest

7.2.1 Basic

Compound interest arises when interest is added to the principal of a deposit or loan, so that, from that moment on, the interest that has been added also earns interest. This addition of interest to the principal is called *compounding*.

Assume that some initial capital C_o is deposited on a bank account with interest rate $0 < i < 1$ compounded yearly. After the first year the value of the deposit equals $C_1 = C + i * C_o = C_o(1 + i)$. After the second year the value is $C_2 = C_1 + i * C_1 = C_1 * (1 + i) = C_o * (1 + i)^2$. So after n ears the

deposit is worth $C_n = C_o * (1+1)^n$. Of course, if a loan of C_o is made at a bank under the same conditions, after n years the debt increases to $C_n = C_o * (1+i)^n$.

What does it mean? Suppose that in the year 1 A.D. our forefather borrowed 1 MU with 1% interest rate compounded yearly, and he as well as all his descendants forgot about it, but the bank survived all historical calamities keeping account of this loan, we would - in year 2010 - inherit a debt equal 485245261.49 MU[2].

7.2.2 Calculating compound interest in CLP

Mariott [Marriott-98] presented a useful recursive predicate defining compound interest. It is used in the `7_1_compound_interest.clp` program:

```
/*1*/ :- lib(ic).
/*2*/ top :-
/*3*/                 List = [PV,T,I],
/*4*/                 List :: 0.0..1000000.0,
/*5*/                 T $= 24,    % Number of periods
/*6*/                 I $= 8/100, % Interest rate per period
/*7*/                 PV $= 1000,  % Initial prinicple
/*8*/                 compound_interest(PV,T,I).

/*9*/ compound_interest(PV,T,I):-
/*10*/               T $>= 1,   % There are still some periods of payments,
/*11*/               NT $= T-1, % but each period it is one period less.
/*12*/               FV $= PV + (PV * I), % Updated principle
/*13*/               compound_interest(FV,NT,I).

/*14*/ compound_interest(FV,T,_):- %
/*15*/               T $= 0,
/*16*/               write("Future Value = "),write(FV), nl.
```

It essence is to define recursively `compound_interest/3` by itself with updated number of periods and updated principle, and continuing this process until we run out of time periods. The result is given i interval arithmetic:

[2]May be this explosion is responsible for the known fact of politicians not minding much about paying national debts, but simply waiting until it reaches exorbitant "unpayable" proportions. May be this is the reason banks like nurturing creditors debts to wait for the moment the debt is sufficiently high but still payable by the debtor.

```
Principle = 6341.18073133441__6341.18073724012
```

So the domain of the variable `Principle` has been reduced to a suitably small interval.

7.2.3 To retire as millionaire - 1

Consider the following example: assume that while being 20 years old we decided to retire at 65 being a millionaire. How large should the initial (and only) deposit be at our personal account with a yearly compounded interest of 6% to achieve this goal?

The solution is given by program `7_2_millionaire_1.ecl`[3] :

```
/*1*/  :- lib(ic).

/*2*/  top :-
/*3*/           LD = [K,T,I],
/*4*/           LD :: 0.0..100000,
/*5*/           T $= 45.0,   % Number of saving years
/*6*/           I $= 6/100,  % Interest rate per year
/*7*/            pension(K,T,I),
/*8*/           write("First and only deposit (present value) = "), write(K), nl.

/*9*/  pension(K,T,I) :-
/*10*/          T $>= 1.0,
/*11*/           NK $= K + (K * I), % New state of pensioners account
/*12*/          NT $= T-1,  % Yearly decrease of saving years
/*13*/          pension(NK, NT, I).

/*14*/ pension(K,T,_):-
/*15*/          T $= 0.0,
/*16*/          K $= 1000000.  % State of pensioners account after 45 years.
```

The message is:

```
First and only deposit (present value) = 72650.0743490985__72650.0743562801
```

So a one-time deposit of 72650 MU at the age of 20 years will give - at 6%

[3]This is an FS-type problem.

compound yearly interest - a pension equal 1 million MU at the age of 65[4].

The most important part of the program is the elegant recursive definition in lines `/*9*/...\verb`/*16*/+: with each recursion the number of years decreases by 1 i and correspondingly our account increases until the number of saving years equal 0 (line `/*15*/`), when the account reaches the value of 1000000, see line `/*16*/`.

7.2.4 To retire as millionaire - 2

Assume now that we can't afford to make a deposit of `72650` MU at the age of `20`, but in order to retire as millionaire at `65` we will deposit each year for `45` years a fixed amount on our personal account with a yearly compounded interest of `6%`. How large must that yearly deposit be?
This is settled by program `7_3_millionaire_2.ecl`[5]:

```
/*1*/  :- lib(ic).

/*2*/  top:-
/*3*/     LD = [K,T,I,R],
/*4*/     LD :: 0.0..1000000,
/*5*/     K $= 0.0,   % Initial state of pensioners account
/*6*/     T $= 45.0,  % Number of saving years
/*7*/     I $= 6/100, % Interest rate per year
/*8*/     R =< 4701, % Yearly payment
/*9*/     pension(K,T,I,R),
/*10*/    write("Yearly payment = "), write(R), nl.

/*11*/ pension(K,T,I,R):-
/*12*/    T  $>= 1.0,
/*13*/    NK $= K + (K * I) + R, % New state of pensioners account
/*14*/    NT $= T-1, % Yearly decrease of saving years
/*15*/    pension(NK, NT,I,R).

/*16*/ pension(K,T,_,_):-
/*17*/    T $= 0.0,
/*18*/    K $= 1000000.0.
```

The private predicate `pension(K,T,I,R)` has - compared with the version from Section 7.2.3 - one more argument. This is the yearly payment `R` that according

[4]Of course providing we enjoy political and economic stability.
[5]This is an FS-type problem.

to line `/*13*/` augments yearly our account. Because the problem is nonlinear, ECL^iPS^e has to be helped by a *trial and error* determination of the bound in line `/*8*/`.

The message generated is:

```
Yearly payment = _11419{4696.74977810691 .. 4701.0}
```

The variable `_11419` is an internal variable used by ECL^iPS^e to store the final result. Rounding up a little bit, the yearly deposit is between `4696.75` and `4701.0`. So depositing yearly `4701` MU, which corresponds roughly to 392 MU per month, we could retire after 45 years of toil having at our pensioners account one million of MU[6].

Notice that your overall payments would be this time $45 \times 4701 = 211545$, which is roughly three times as much as for the one-time deposit from Section 7.2.3.

7.2.5 Those cursed mortgages!

We got a mortgage to be payed for the next 24 years, the yearly payment being 12000 MU, the mortgage being at yearly interest `8%`[7]. How large was the mortgage we got? What is the price of this mortgage? This will be clarified by program `7_4_mortgage.ecl`[8]:

```
/*1*/  :- lib(ic).

/*2*/  top:-
/*3*/         List = [K,T,I,R],
/*4*/         List :: 0.0..1000000.0,
/*5*/         T $= 24.0,          % Number of paying years
/*6*/         I $= 8/100,         % Interest rate per year
/*7*/         R $= 12000.0,       % Yearly mortgage payments
/*8*/         mortgage(K,T,I,R),
/*9*/         Cost $= T * R,
/*10*/        write("Principle = "),write(K),nl,
/*11*/        write("Cost of mortgage = "),write(Cost),nl.
```

[6] It would be really nice if governments stopped looking after our well-fare and stopped being good to us.

[7] Being young, healthy and having a secure academic employment, we surly can afford a mortgage like this!

[8] This is an FS-type problem.

```
/*12*/ mortgage(K,T,I,R):-
/*13*/       T $>= 1.0,  % There are still some years of payments,
/*14*/       NT $= T-1,  % but each year it is one year less.
/*15*/       NK $= K + (K * I) - R, % Updated principal or amount of loan
/*16*/       mortgage(NK,NT,I,R).

/*17*/ mortgage(K,T,_,_):-  % uf! - finally the end of the ordeal!
/*18*/       T $= 0,
/*19*/       K $= 0.
```

The message is:

```
Principle = 126345.099404108__126345.099448266
Cost of mortgage = 288000.0__288000.0
```

So for a mortgage of roughly 126346 MU we have to pay over 24 years 288000 MU. Nothing better illustrates the saying "Time is Money".

The lines `/*12*/,...,/*19*/` correspond this time to the recursive decrease of our principal as the result of yearly payments, up to the final principal equal 0. According to line `/*15*/` the current principal to be repaid increases yearly by the interest rate term, and decreases yearly by the payment term.

7.2.6 Net Present Value or how much we make (or loose) really?

While making business it is sometimes worthwhile to remember lost opportunities. Lost, because we could not engage in them just because of this business. But in order to make a true balance of what has been gained (or lost), we better take those lost opportunities into account. This is done by a concept known as *Net Present Value* (NPV), which is estimating our future gains (or loses) in terms of its present values, while considering the most likely (and most certain) lost opportunity. The basic fact underlying NPV is that the value of money changes in time because it may be invested and bring profit. So some m MU gained (or lost) a year from now have a different present value, equal to:

$$\frac{m}{(1+r)},$$

referred to as *Net Present Value* of those m MU gained (or lost) a year from now. This means that $\frac{m}{(1+r)}$ MU invested now will yield (with certainty) m MU a year

later, r being the *annual rate of return* (or *discount rate*) . The words "with certainty" mean that there is a market commodity (e.g. government bonds) guaranteeing an annual rate of return equal to r. A simple extension fo this reasoning shows that $\frac{m}{(1+r)^k}$ MU invested now will yield (with certainty) m MU k years later. The concept is readily generalized to any future cash flows, and therefore is used for comparing the desirability of different investments. Let's consider the following example of two investments:

1. Investment A requires a cash outlay of 8 million MU at time 0, will yield a return of 26 million MU a year from now and requires yet another cash outlay of 18 million MU two years from now in order to clear some environmental damage due to our business activities. The net cash flow for this investment is:

   $$-8 + 26 - 18 = 0,$$

   and that looks rather discouraging. Suppose there are government bonds guaranteeing an annual rate of return equal to 0.25. This makes the NPV of our investment equal to:

   $$-8 + \frac{26}{(1+0.25)} - \frac{18}{(1+0.25)^2} = -8 + 20.8 - 11.52 = 1.2,$$

   and that is not so bad, because it means investment A will increase the company's value expressed in time 0 by 1.2 million MU.

2. Investment B requires a cash outlay of 6 million MU at time 0, will yield a return of 8 million MU a year from now and requires yet another cash outlay of 18 million MU two years from now in order to clear some environmental damage due to our business activities. The net cash flow for this investment is:

   $$-6 + 8 - 1 = 1,$$

   however its NPV is

   $$-6 + \frac{8}{(1+0.25)} - \frac{1}{(1+0.25)^2} = -6 + 6.4 - 0.65 = -0.25,$$

   So investment B, in spite of the positive cash flow, will decrease the company's value expressed in time by 0 by 0.25 million MU.

Consider the following example highlighting the concept[9]:

Star Oil Company is considering five different investment opportunities.The cash outflows and NPV-s (in milliones of MU) are given by Table 7.1:

Financial parameters	Inv. 1	Inv. 2	Inv. 3	Inv. 4	Inv. 5
Time 0 cash outflow	11	53	5	5	29
Time 1 cash outflow	3	6	5	1	14
NPV	13	10	16	14	39

Table 7.1: Financial parameters for investment options

Star Oil has 40 million MU available for investment at the present time (time `0`), it estimates that one year from now (time `1`) 20 million MU will be available for investment. Star Oil may purchase any fraction of each investment. In this case, cash outflows and NPV are adjusted accordingly. For example, if Star Oil purchases one fifth of investment `3`, then a cash outflow of $\frac{1}{5}5 = 1 million MU$ would be required at time `0`, and a cash outflow of $\frac{1}{5}5 = 1 million MU$ would be required at time `1`. The one-fifth of investment `3` would yield an NPV of $\frac{1}{5}16 = 3.2 million MU$. Star Oil wants to maximize the NPV that can be obtained by investing in investment `1-5`, while assuming that any funds left over at time 0 cannot be used at time 1. This is done by program `7_5_NPV.ecl`:

```
/*1*/ :- lib(eplex).
/*2*/ top :-
     %  Xn - fraction of investment n purchased
/*3*/      Variables = [X1,X2,X3,X4,X5],
/*4*/      Variables $:: 0.0..1.0Inf,
/*5*       X1 $=< 1,
/*6*/      X2 $=< 1,
/*7*/      X3 $=< 1,
/*8*/      X4 $=< 1,
/*9*/      X5 $=< 1,
/*10*/     11*X1 + 53*X2  + 5*X3 + 5*X4 + 29*X5 $=< 40,      % time 0 constraint
/*11*/     3*X1 + 6*X2  + 5*X3 + X4 + 34*X5 $=< 20,          % time 1 constraint

/*12*/     eplex_solver_setup(max(13*X1 + 16*X2 + 16*X3 +14*X4 + 39*X5)),
/*13*/     eplex_solve(Profit),

/*14*/     (foreach(Name,["X1","X2","X3","X4","X5"]),
/*15*/     foreach(V, Variables)  do
```

[9]This example is from [Winston-94].

```
/*16*/     eplex_var_get(V, typed_solution, V),
/*17*/     write(Name),write(" = "),write(V),nl),
/*18*/     write("Profit = "),write(Profit).
```

The solution is:

```
X1 = 1.0                  X2 = 0.200859950859951
X3 = 1.0                  X4 = 1.0
X5 = 0.288083538083538    Profit = 57.4490171990172
```

7.3 Warehouses - suppliers

Linear programming problems are problems of minimizing an objective function being a linear form of decision variables under constraints being linear forms of decision variables. They are ubiquitous in OR applications. Their solution using ECL^iPS^e may be illustrated by the transportation problem for `3` warehouses and `4` suppliers:

Some volumes of *Important Raw Material* (IRM) have been contracted from four suppliers S1, S2, S3 and S4. The material should be delivered to three warehouses W1, W2 and W3 of limited capacity and different delivery costs due to different delivery distances, see Figure 7.1.

The following data is known:
Contracts signed with supplier j:
`Contract_j, j=1,2,3,4`
Delivery cost for a unit of IRM to warehouse i from supplier j:
`Cost_i_j, i=1,2,3, j=1,2,3,4`
Space available in warehouse i:
`Capacity_i, i=1,2,3`
The delivery sizes:
`Delivery_i_j, i=1,2,3, j=1,2,3,4`
to each warehouse i from each supplier j should be determined in a way minimizing the entire delivery cost while fulfilling the contracted quotas and respecting available warehouse capacities.
The solution is given by program `7_4_warehouses_clients_1.ecl`[10] :

[10] This is an OS-type problem.

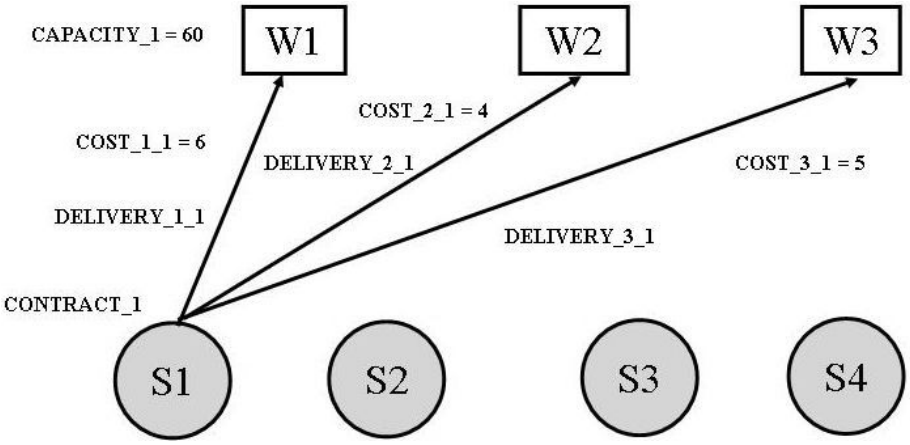


Figure 7.1: Warehouses - suppliers data

```
/*1*/  :- lib(eplex).
/*2*/  top :-
/*3*/      solve(_,_).
 /*4*/  solve(Cost,Variables):-
% Declaring variables and their domains:
/*5*/      Variables = [Delivery_1_1,Delivery_2_1,Delivery_3_1,
                       Delivery_1_2,Delivery_2_2,Delivery_3_2,
                       Delivery_1_3,Delivery_2_3,Delivery_3_3,
                       Delivery_1_4,Delivery_2_4,Delivery_3_4],
/*6*/       Variables $:: 0.0..1.0Inf,
/*7*/     %  (integers(Variables)),
/*8*/       Cost_1_1 is 6.5,
/*9*/       Cost_1_2 is 2,
/*10*/      Cost_1_3 is 6.3,
/*11*/      Cost_1_4 is 7.3,
/*12*/      Cost_2_1 is 4,
/*13*/      Cost_2_2 is 9.7,
/*14*/      Cost_2_3 is 5.2,
/*15*/      Cost_2_4 is 3,
/*16*/      Cost_3_1 is 5.8,
/*17*/      Cost_3_2 is 2.4,
/*18*/      Cost_3_3 is 1.7,
/*19*/      Cost_3_4 is 9,
/*20*/      Capacity_1 is 60,
```

```
/*21*/     Capacity_2 is 55,
/*22*/     Capacity_3 is 51,
/*23*/     Contract_1 is 35.5,
/*24*/     Contract_2 is 37,
/*25*/     Contract_3 is 22.7,
/*26*/     Contract_4 is 32,
% Contract constraints for clients:
/*27*/     Delivery_1_1 + Delivery_2_1 + Delivery_3_1 $=
                    Contract_1,
/*28*/     Delivery_1_2 + Delivery_2_2 + Delivery_3_2 $=
                    Contract_2,
/*29*/     Delivery_1_3 + Delivery_2_3 + Delivery_3_3 $=
                    Contract_3,
/*30*/     Delivery_1_4 + Delivery_2_4 + Delivery_3_4 $=
                    Contract_4,
% Space constraints for warehouses:
/*31*/     Delivery_1_1 + Delivery_1_2 + Delivery_1_3 +
                    Delivery_1_4 $=< Capacity_1,
/*32*/     Delivery_2_1 + Delivery_2_2 + Delivery_2_3 +
                    Delivery_2_4 $=< Capacity_2,
/*33*/     Delivery_3_1 + Delivery_3_2 + Delivery_3_3 +
                    Delivery_3_4 $=< Capacity_3,

  % Configuring  eplex solver for
  % minimizing the performance index:
/*34*/     eplex_solver_setup(min(
            Cost_1_1 * Delivery_1_1 + Cost_2_1 * Delivery_2_1 +
            Cost_3_1 * Delivery_3_1 + Cost_1_2 * Delivery_1_2 +
            Cost_2_2 * Delivery_2_2 + Cost_3_2 * Delivery_3_2 +
            Cost_1_3 * Delivery_1_3 + Cost_2_3 * Delivery_2_3 +
            Cost_3_3 * Delivery_3_3 + Cost_1_4 * Delivery_1_4 +
            Cost_2_4 * Delivery_2_4 + Cost_3_4 * Delivery_3_4
             )),

% % Solving the problem:
/*35*/     eplex_solveCost),

% Displaying results:
/*36*/    (foreach(Name,[
              "Delivery_1_1","Delivery_2_1","Delivery_3_1",
              "Delivery_1_2","Delivery_2_2","Delivery_3_2",
              "Delivery_1_3","Delivery_2_3","Delivery_3_3",
              "Delivery_1_4","Delivery_2_4","Delivery_3_4"]),
/*37*/    foreach(V, Variables)
/*38*/    do
/*39*/     eplex_var_get(V, typed_solution, V),
```

```
/*40*/    write(Name),write(" = "),write(V),nl
/*41*/    ),
/*42*/    write("Cost"),write(" = "),writeCost),nl.
```

The message is:

```
Delivery_1_1 = 0.0
Delivery_2_1 = 23.0
Delivery_3_1 = 12.5
Delivery_1_2 = 37.0
Delivery_2_2 = 0.0
Delivery_3_2 = 0.0
Delivery_1_3 = 0.0
Delivery_2_3 = 0.0
Delivery_3_3 = 22.7
Delivery_1_4 = 0.0
Delivery_2_4 = 32.0
Delivery_3_4 = 0.0
Cost = 373.09
```

Decommenting the code in line `/*7*/` makes the solver an integer programming solver. If additionally in lines `/*27*/`, `/*28*/`, `/*29*/` and `/*30*/` the relations "`$=` are swapped for "`$>=` (i.e. some over-realization of contracts will be acceptable), the programm changes into `7_5_warehouses_clients _2.ecl` giving the solution:

```
Delivery_1_1 = 0
Delivery_2_1 = 23
Delivery_3_1 = 13
Delivery_1_2 = 37
Delivery_2_2 = 0
Delivery_3_2 = 0
Delivery_1_3 = 0
Delivery_2_3 = 0
Delivery_3_3 = 23
Delivery_1_4 = 0
Delivery_2_4 = 32
Delivery_3_4 = 0
Cost = 376.5
```

This solution is intuitively obvious. Because in order to get an integer solution the constraints in lines `/*27*/`, `/*28*/`, `/*29*/` and `/*30*/` have been relaxed (contracts could be over-realized), the minimum cost has to increase.

7.4 Refining and blending oils

Consider another classical OR problem traditionally solved using linear programming:

To manufacture some food, refining and blending of five oils: two common vegetable oils (C1 and C2) and three tropical oils (T1, T2 and T3), must be performed. The blend must maximize profit under constraint of hardiness.

To refine common vegetable oils, a different production line is needed than for tropical oil refining.

The monthly refinery lines production cannot exceed 200 ton of common plant oil and 250 ton tropical oils. The production losses are negligible.

The purchase costs, refinery costs and hardiness for 1 ton of oils may be found in Table 7.2.

	Oils				
Parameters	C1	C2	T1	T2	T3
Cost	110	120	130	110	115
Hardness	8.8	6.1	2.0	4.2	5.0

Table 7.2: Oil data

The hardiness of the blend is a linear function of the component hardiness and should be in the range [3,...,6]. A ton of the food may be sold for 150 MU. The amount of monthly purchased oils and the monthly food production should be determined so as to maximize the objective function given by the monthly profit.

Following variables are defined:
`XC1, XC2, XT1, XT2, XT3` - amount (in tons) of oils purchased monthly,
`Y` – amount (in tons) of food produced monthly.

The solution is given by program `7_6_mixing_oils_1.ecl`[11] :

```
/*1*/  :- lib(eplex).
/*2*/  top :-
/*3*/     oils_1(_,_).

/*4*/  oils_1(Profit, Variables) :-
% Declaring variables and their domains:
/*5*/     Variables = [XC1,XC2,XT1,XT2,XT3,Y],
/*6*/    % integers(Variables),
/*7*/     Variables $:: 0.0..1.0Inf,

% Declaring constraints for the eplex instance:
/*8*/     XC1 + XC2 $=< 200,
/*9*/     XT1 + XT2 + XT3 $=< 250,
/*10*/    8.8*XC1 + 6.1*XC2 + 2*XT1 + 4.2*XT2 +5*XT3 $=< 6*Y,
/*11*/    8.8*XC1 + 6.1*XC2 + 2*XT1 + 4.2*XT2 +5*XT3 $>= 3*Y,
/*12*/    XC1 + XC2 + XT1 + XT2 + XT3 $= Y,

% Configuring  eplex solver for
% maximizing the performance index:
+
/*13*/     eplex_solver_setup(max(
              150*Y - 110*XC1 - 120*XC2 -130*XT1 - 110*XT2 - 115*XT3),

% Solving the problem:
/*14*/     eplex_solve(Profit),

% Displaying results:
/*15*/    (foreach(Name,["XC1","XC2","XT1","XT2","XT3","Y"]),
/*16*/    foreach(V, Variables) do
/*17*/    prob: eplex_var_get(V, typed_solution, V),
/*18*/    write(Name),write(" = "),write(V),nl
/*19*/    ),
/*20*/    write("Profit = "),write(Profit).
```

The message is:

```
XC1 = 159.259259259259      XC2 = 40.7407407407409
XT1 = 0.0                   XT2 = 250.0
XT3 = 0.0                   Y = 450.0
Profit = 17592.5925925926
```

[11] This is an OS-type problem.

If the code in line `/*6*/` is decommented, the resulting program `7_7_mixing_oils_2.ecl` gives an integer solution:

```
XC1 = 159       XC2 = 41
XT1 = 0         XT2 = 250
XT3 = 0         Y = 450
Profit = 17590.0
```

7.5 How to make easy money?

The previous examples show that ECL^iPS^e is tolerating LP models that do not exactly conform to the classical LP canonical form. This tolerance is really far reaching, as demonstrated by the next example:

A tireless public servant and distinguished member of the Absurdoland's Upper House of Parliament, the *Celebrated Senator*, firmly convinced that ethanol in automotive fuels would save the Earth, for a number of years did all he could to satisfy the legislating wishes and suggestions of the well-known ethanol producer *Corny Fuels*. Appreciating his relentless efforts, the friendly CEO of *Corny Fuels* ordered its *Banking Outlet* to provide to the company of the *Celebrated Senators Wife* a 100 million MU loan for some shady investment, for 4 years at very decent interests of `2%` per year[12].

The *Friendly Manager* of the *Banking Outlet* suggested the *Celebrated Senators Wife* should herself determine the yearly payments provided they are not lower than 10 million MU. The *Celebrated Senators Wife* very wisely did not pursue the shady investment, but deposited the entire loan in a *Less Friendly Bank*, where yearly deposits could be kept at a yearly compound interest, always `2%` higher than the forecasted inflation. The yearly inflation was forecasted for the first year to be `5%`, for the second year - `4%`, for the third year `3%`, and for the fourth year - `2%`.

However the *Celebrated Senators Wife* had quite a headache with managing the loan. Her *Financial Advisor* suggested two different investment strategies:
1. To maximize the profit from the deposit while sticking to the *Friendly Managers* suggestions.
2. To minimize the forecasted real costs of the loan while sticking to the *Friendly*

[12]The CEO was firmly convinced that it is always cheaper to buy legislatures than to buy the majority of voters.

Managers suggestions.

The *Financial Advisor* could not satisfactory explain the difference in outcome (if any) of both strategies. In appreciation of services rendered to the Society by the *Celebrated Senator*, a befriended CLP programmer wrote a program (`7_8_getting_rich.ecl`[13]) designed to dispel any doubts about the outcomes of both strategies:

```
/*1*/  :- lib(eplex).

/*2*/  top:-
/*4*/     write("Choose the version (1 or 2):"),nl,
/*4*/     read_token(Number, integer),
/*5*/     solve(Number).
 /*6*/  solve(1):-
/*7*/     A $>= 10,    % payment after first year
/*8*/     B $>= 10,    % payment after second year
/*9*/     C $>= 10,    % payment after third year
/*10*/    D $>= 10,    % payment after fourth year

/*11*/    payments([A,B,C,D], 100),
/*12*/    Profit_A $= 100*(1.07)-A,
/*13*/    Profit_B $= Profit_A*(1.06)-B,
/*14*/    Profit_C $= Profit_B*(1.05)-C,
/*15*/    Profit_D $= Profit_C*(1.04)-D,
/*16*/    Profit_D $:: 0..250,

/*17*/    eplex_solver_setup(max(Profit_D)),
/*18*/    eplex_solve(Profit_D),
/*19*/    eplex_get(vars, Vars),
/*20*/    eplex_get(typed_solution, Vals),
/*21*/    Vars = Vals,

/*22*/    Cost_of_credit is
              A*0.95+B*0.95*0.96+C*0.95*0.96*0.97+
                        D*0.95*0.96*0.97*0.98,

/*23*/    write("Payment after first year = "),write(A),write(" MM."),nl,
/*24*/    write("Payment after second year = "),write(B),write(" MM."),nl,
/*25*/    write("Payment after third year = "),write(C),write(" MM."),nl,
/*26*/    write("Payment after fourth year = "),write(D),write(" MM."),nl,
/*27*/    write("Projected real cost of credit = "),  write(Cost_of_credit),
                         write(" MM. "),nl,
/*28*/    write("Maximum profit after 4 years = "), write(Profit_D),
                         write(" MM. "),nl,nl.
```

[13]This is an OST-type problem.

```
/*29*/  solve(2):-
/*39*/     A $>= 10,    % payment after firts year
/*31*/     B $>= 10,    % payment after second year
/*32*/     C $>= 10,    % payment after third year
/*33*/     D $>= 10,    % payment after fourth year
/*34*/     payments([A,B,C,D], 100),
/*35*/     Cost_of_credit $=
              A*0.95+B*0.95*0.96+C*0.95*0.96*0.97+
                        D*0.95*0.96*0.97*0.98,

/*36*/     eplex_solver_setup(min(Cost_of_credit)),
/*37*/     eplex_solve(Cost_of_credit),
/*38*/     eplex_get(vars, Vars),
/*39*/     eplex_get(typed_solution, Vals),
/*40*/     Vars = Vals,
/*41*/     Profit_A is 100*(1.07)-A,
/*42*/     Profit_B is Profit_A*(1.06)-B,
/*43*/     Profit_C is Profit_B*1.05-C,
/*44*/     Profit_D is Profit_C*1.04-D,

/*45*/     write("Payment after first year = "),write(A),write(" MM."),nl,
/*46*/     write("Payment after second year = "),write(B),write(" MM."),nl,
/*47*/     write("Payment after third year = "),write(C),write(" MM."),nl,
/*48*/     write("Payment after fourth year = "),write(D),write(" MM."),nl,
/*49*/     write("Minimum projected cost of credit  = "),  write(Cost_of_credit),
                       write(" MM."),nl,
/*50*/     write("Profit after 4 years ="),write(Profit_D),write(" MM."),nl.

/*51*/  payments([],Loan) :-
/*52*/     Loan $=0.

/*53*/  payments([Payment|List_of_payments],Loan) :-
/*54*/     Updated_principal $=(1+2/100)*Loan-Payment,
/*55*/     payments(List_of_payments,Updated_principal).
```

The message is:

```
Choose the version (1 or 2): choice 1
 Payment after first year = 10.0 MM.
 Payment after second year = 10.0 MM.
 Payment after third year = 10.0 MM.
 Payment after fourth year = 77.027136 MM.
 Projected real cost of credit =
          94.2448598792192 MM.
 Maximum profit after 4 years: 13.932304 MM.

Choose the version (1 or 2): choice 2
```

```
Payment after first year = 10.0 MM.
Payment after second year = 10.0 MM.
Payment after third year = 10.0 MM.
Payment after fourth year = 77.027136 MM.
Minimum projected cost of credit  =
         94.2448598792192 MM.
Profit after 4 years: 13.932304 MM.
```

It follows that no matter what strategy the *Celebrated Senators Wife* will opt for, the profit (a nice `13.932304` MM MU) and the payment strategy will always be the same.

7.6 Making shrewd investments

ECL^iPS^e formulations of LP problems may be far, far removed from the conventional LP canonical form. This is illustrated by the following example:

The chief accountant of some small company has forecast the cash requirements for the next five years. It turned out that the company would have some free cash in the future. He considered the following investments options:

1. Short term (one-year bonds) with interest rates (return after a year) 20%.
2. Intermediate term (two-year bonds) with interest rates (return after two years) 47%.
3. Long term (three-year bonds) with interest rates (return after three years) 78%.

He wishes to plan the investments over five years taking into account the cash requirements and one of the following three investment options, given the initial cash of 100000 MU:

Option 1: satisfy yearly demands and maximize the amount of cash at the end of the five years period.

Option 2: satisfy yearly demands and minimize initial cash.

Option 3: satisfy yearly demands and determine the minimum initial cash needed to have at the end of the five years period the same amount of cash.

Cash requirements at the beginnings of each year are given by Table 7.3:

Year	*Amount*	*Variable*
1	10000	Crx1
2	10000	Crx2
3	20000	Crx3
4	20000	Crx4
5	20000	Crx5

Table 7.3: Cash requirements for consecutive years

To model the problem, following variables are defined :

Sinx : Short term investment at the beginning of year x
Iinx : Intermediate term investment at the beginning of year x
Linx : Long term investment at the beginning of year x
Tinx : Total investment at the beginning of year x

Srex : Short term revenue at the beginning of year x
Irex : Intermediate term revenue at the beginning of year x
Lrex : Long term revenue at the beginning of year x
Trex : Total revenue at the beginning of year x

Crx : cash requirement at the beginning of year x
Ebcx: ending cash balance at the beginning of year x

What decisions with respect to forms of investment have to be made each year? The answer is given by `7_9_inwestments.ecl`[14] :

```
/*1*/  :- lib(eplex).
 /*2*/  top:-
/*3*/     writeln("Option 1:"),
/*4*/     not( not(top(100000.0,_))),
/*5*/     writeln("Option 2:"),
/*6*/     not( not(top(_,150000.0))),
/*7*/     writeln("Option 3:"),
/*8*/     not( not(top(X,X))).
```

[14]This is an OST-type problem.

```
/*9*/  top(Initial_cash,Final_cash):-
      % we start with a dummy performance index:
/*10*/    eplex_solver_setup(max(0)),
/*11*/    investments(Initial_cash, Variables, Names, Final_cash),
      % find minimum Initial_cash and fix it:
/*12*/    eplex_probe(min(Initial_cash), Initial_cash),
      % find maximum Final_cash and fix it:
/*13*/    eplex_probe(max(Final_cash), Final_cash),
/*14*/    eplex_get(typed_solution, Vs), eplex_get(vars, Vs),

/*15*/    writelist(Names, Variables),
/*16*/    write("Initial cash:  "),writeln(Initial_cash),
/*17*/    write("Final cash:     "),writeln(Final_cash),nl.

/*18*/    investments(Initial_cash, Variables, Names, Final_cash) :-
/*19*/    Variables =   [
                Sin1,Sin2,Sin3,Sin4,Sin5,
                Iin1,Iin2,Iin3,Iin4,Iin5,
                Lin1,Lin2,Lin3,Lin4,Lin5,
                Tin1,Tin2,Tin3,Tin4,Tin5,
                Sre2,Sre3,Sre4,Sre5,Sre6,
                Ire3,Ire4,Ire5,Ire6,
                Lre4,Lre5,LreC,
                Tre2,Tre3,Tre4,Tre5,Tre6,
                Ebc1,Ebc2,Ebc3,Ebc4,Ebc5
                ],

/*20*/    Names =   [
                "Sin1","Sin2","Sin3","Sin4","Sin5",
                "Iin1","Iin2","Iin3","Iin4","Iin5",
                "Lin1","Lin2","Lin3","Lin4","Lin5",
                "Tin1","Tin2","Tin3","Tin4","Tin5",
                "Sre2","Sre3","Sre4","Sre5","Sre6",
                "Ire3","Ire4","Ire5","Ire6",
                "Lre4","Lre5","LreC",
                "Tre2","Tre3","Tre4","Tre5","Tre6",
                "Ebc1","Ebc2","Ebc3","Ebc4","Ebc5"
                 ],

/*21*/    Variables $:: 0..inf,

/*22*/    Tin1 $= Iin1 + Sin1 + Lin1,
/*23*/    Tin2 $= Iin2 + Sin2 + Lin2,
/*24*/    Tin3 $= Iin3 + Sin3 + Lin3,
/*25*/    Tin4 $= Iin4 + Sin4 + Lin4,
/*26*/    Tin5 $= Iin5 + Sin5 + Lin5,

/*27*/    Ire3 $= 1.47 * Iin1,
/*28*/    Ire4 $= 1.47 * Iin2,
```

```
/*29*/    Ire5 $= 1.47 * Iin3,
/*30*/    Ire6 $= 1.47 * Iin4,

/*31*/    Lre4 $= 1.78 * Lin1,
/*32*/    Lre5 $= 1.78 * Lin2,
/*33*/    Lre6 $= 1.78 * Lin3,

/*34*/    Sre2 $= 1.2 * Sin1,
/*35*/    Sre3 $= 1.2 * Sin2,
/*36*/    Sre4 $= 1.2 * Sin3,
/*37*/    Sre5 $= 1.2 * Sin4,
/*38*/    Sre6 $= 1.2 * Sin5,

/*39*/    Tre2 $= Sre2,
/*40*/    Tre3 $= Sre3 + Ire3,
/*41*/    Tre4 $= Sre4 + Ire4 + Lre4,
/*42*/    Tre5 $= Sre5 + Ire5 + Lre5,
/*43*/    Tre6 $= Sre6 + Ire6 + Lre6,

/*44*/    Cr1 $>= 10000,
/*45*/    Cr2 $>= 10000,
/*46*/    Cr3 $>= 20000,
/*47*/    Cr4 $>= 20000,
/*48*/    Cr5 $>= 20000,

/*49*/    Ebc1 $= Initial_cash - Tin1 - Cr1,
/*50*/    Ebc2 $= Ebc1 + Tre2 - Tin2 - Cr2,
/*51*/    Ebc3 $= Ebc2 + Tre3 - Tin3 - Cr3,
/*52*/    Ebc4 $= Ebc3 + Tre4 - Tin4 - Cr4,
/*53*/    Ebc5 $= Ebc4 + Tre5 - Tin5 - Cr5,
/*54*/    Final_cash $= Ebc5 + Tre6.

/*55*/    writelist([], []).
/*56*/    writelist([FN|RN], [FV|RV]) :-
/*57*/    write(FN), write(" = "), writeln(FV),
/*58*/    writelist(RN, RV).
```

To enhance the expressive power of the results they are presented in table 7.4, which makes it easy to compare various investment options. The "double negations" in lines `/*4*/`, `/*6*/` and `/*8*/` may be astonishing at first sight.

Variable	*Option 1*	*Option 2*	*Option 3*
Sin1	8333.33333333333	8333.33333333333	8333.33333333333
Sin2	0.0	0.0	0.0
Sin3	0.0	0.0	0.0
Sin4	0.0	0.0	0.0
Sin5	0.0	0.0	0.0
Iin1	22860.8450182794	80187.1463253183	55293.192790018
Iin2	0.0	0.0	0.0
Iin3	13605.4421768707	13605.4421768696	13605.4421768707
Iin4	84674.3625341293	0.0	0.0
Iin5	0.0	0.0	0.0
Lin1	58805.8216483872	11235.9550561798	11235.9550561798
Lin2	0.0	0.0	0.0
Lin3	0.0	84269.6629213483	47675.5512244557
Lin4	0.0	0.0	0.0
Lin5	0.0	0.0	0.0
Tin1	90000.0	99756.4347148322	74862.4811795311
Tin2	0.0	0.0	0.0
Tin3	13605.4421768707	97875.105098218	61280.9934013264
Tin4	84674.3625341293	0.0	0.0
Tin5	0.0	0.0	0.0
Sre2	10000.0	10000.0	10000.0
Sre3	0.0	0.0	0.0
Sre4	0.0	0.0	0.0
Sre5	0.0	0.0	0.0
Sre6	0.0	0.0	0.0
Ire3	33605.4421768707	117875.105098218	81280.9934013264
Ire4	0.0	0.0	0.0
Ire5	20000.0	19999.9999999984	20000.0
Ire6	124471.31292517	0.0	0.0
Lre4	104674.362534129	20000.0	20000.0
Lre5	0.0	0.0	0.0
Lre6	0.0	150000.0	84862.4811795311

Table 7.4: Results for investment options

Variable	*Option 1*	*Option 2*	*Option 3*
Tre2	10000.0	10000.0	10000.0
Tre3	33605.4421768707	117875.105098218	81280.9934013264
Tre4	104674.362534129	20000.0	20000.0
Tre5	20000.0	19999.9999999984	20000.0
Tre6	124471.31292517	150000.0	84862.4811795311
Gnad1	0.0	0.0	0.0
Gnad2	0.0	0.0	0.0
Gnad3	0.0	0.0	0.0
Gnad4	0.0	0.0	0.0
Gnad5	0.0	0.0	0.0
Initial cash	100000.0	109756.434714832	84862.4811795311
Final cash	124471.31292517	150000.0	84862.4811795311

Table 7.5: Results for investment options - continuation

However, it is an old Prolog trick[15] that serves to call the same predicate with changing data: the satisfaction of the first `top(_,_)` predicate makes the internal negation fail. That removes all results got so far, which in turn makes the external negation true, and enables the second `top(_,_)` predicate to be activated, and so on.

The basic model is given by cash balance equations and investment updates equations from lines `/*22*/` - `/*54*/`. They correspond directly to the investment options as stated in the problem and are - by all standards - far removed from the conventional LP canonical form.

7.7 Yet another financial *Perpetuum Mobile*!

The mathematical model used for LP are mainly various balances. The nature of those balances may sometimes be strange indeed. This is illustrated by the following example. inspired by one presented by [Taha-08]:

Clever Young, a computer prodigy as well as a mathematical and business whiz-kid, has been thinking a long time about how to make money on-line. He finally decided to make some currency speculations using general available

[15]Thanks are due to Joachim Schimpf from ECLiPSe for drawing the Authors attention to this trick.

real-time currency sites information and on-line facilities of foreign exchange spot trading. He started with simulating speculations on five currencies: the USD, EUR, GBP, JPY and PLN (that's Polish Zloty). The exchange rates he assembled for this purpose are mid-market rates derived from the mid-point between the "buy" and "sell" rates from global currency markets for some day and hour, and are given by Table 7.6.

	USD	EUR	GBP	JPY	PLN
USD	1	0.8353	0.692	91.89	3.4872
EUR	1.1972	1	0.8283	110.03	4.181
GBP	1.445	1.2073	1	132	5.0414
JPY	0.01088	0.009088	0.007575	1	3.7950
PLN	0.2867	0.2392	0.1983	0.2635	1

Table 7.6: Currency exchange rates for March 10, 2010

The meaning of this table is obvious: e.g. 1 EUR could be sold (bought) for 1.1972 USD. Clever Young thinks it is possible to increase the USD holdings (above some initial `A` MM USD) by circulating currencies throughout the currency market. The problem is what and how much to buy, and what and how much to sell in order to maximize the profit from the initial investment of `A` MM USD?

The speculation is constrained by the regulation that sets the following limits on the amount of any single transaction: USD ¡= 5, EUR ¡= 3, GBP ¡= 3.5, JPY ¡= 100, PLN ¡= 40 (in millions)

Transaction are denoted by by variables of type:

`CURRENCYitoCURRENCYj`

denoting the amount in `CURRENCYi` converted to `CURRENCYj`. Exchange rates from Table 7.6 are presented using variables:

`Ex_rate_CURRENCYi_to_CURRENCYj`.

Variable `A` denotes the initial USD amount (in MM), variable `Z` - the final USD holdings (in MM).

The program to test the speculation effectiveness is based on currency balances for all currencies. They have the form:

```
Currency accumulation + Currency out = Currency in
```

The program `7_10_currency_speculations.ecl`[16] is as follows:

```
/*1*/  :- lib(eplex).
/*2*/  top:-
/*3*/      % A = 5.0,
/*3a*/      A = 0.0,

   % what currencies are USD converted to:
/*4*/      USD = [USDtoEUR,USDtoGBP,USDtoJPY,USDtoPLN],
/*5*/      USD :: 0.0..5.0,
/*6*/      Z :: 0.0..6.0,

   %  what currencies are EUR converted to:
/*7*/      EUR = [EURtoUSD,EURtoGBP,EURtoJPY,EURtoPLN],
/*8*/      EUR :: 0.0..3.0,

   %  what currencies are GBP converted to:
/*9*/      GBP = [GBPtoUSD,GBPtoEUR,GBPtoJPY,GBPtoPLN],
/*10*/     GBP :: 0.0..3.5,

   %  what currencies are JPY converted to:
/*11*/     JPY = [JPYtoUSD,JPYtoEUR,JPYtoGBP,JPYtoPLN],
/*12*/     JPY :: 0.0..100.0,

   %  what currencies are PLN converted to:
/*13*/     PLN = [PLNtoUSD,PLNtoEUR,PLNtoGBP,PLNtoJPY],
/*14*/     PLN :: 0.0..40.0,

/*15*/     Ex_rate_USD_to_EUR = 0.8353,
/*16*/     Ex_rate_USD_to_GBP = 0.692,
/*17*/     Ex_rate_USD_to_JPY = 91.89,
/*18*/     Ex_rate_USD_to_PLN = 3.4872,
/*19*/     Ex_rate_EUR_to_GBP = 0.8283 ,
/*20*/     Ex_rate_EUR_to_JPY = 110.03,
/*21*/     Ex_rate_EUR_to_PLN = 4.181,
/*22*/     Ex_rate_GBP_to_JPY = 132,
/*23*/     Ex_rate_GBP_to_PLN = 5.0414,
/*24*/     Ex_rate_JPY_to_PLN = 3.7950,

/*25*/     Ex_rate_EUR_to_USD is 1/(Ex_rate_USD_to_EUR),
/*26*/     Ex_rate_GBP_to_USD is 1/(Ex_rate_USD_to_GBP),
/*27*/     Ex_rate_JPY_to_USD is 1/(Ex_rate_USD_to_JPY),
/*28*/     Ex_rate_PLN_to_USD is 1/(Ex_rate_USD_to_PLN),
/*29*/     Ex_rate_GBP_to_EUR is 1/(Ex_rate_EUR_to_GBP),
/*30*/     Ex_rate_JPY_to_EUR is 1/(Ex_rate_EUR_to_JPY),
/*31*/     Ex_rate_PLN_to_EUR is 1/(Ex_rate_EUR_to_PLN),
```

[16] This is an OS-type problem.

```
/*33*/    Ex_rate_JPY_to_GBP is 1/(Ex_rate_GBP_to_JPY),
/*33*/    Ex_rate_PLN_to_GBP is 1/(Ex_rate_GBP_to_PLN),
/*34*/    Ex_rate_PLN_to_JPY is 1/(Ex_rate_JPY_to_PLN),

     % USD balance:
/*35*/    Z + USDtoEUR + USDtoGBP + USDtoJPY + USDtoPLN  $=
     A + (Ex_rate_EUR_to_USD)*EURtoUSD + (Ex_rate_GBP_to_USD)*GBPtoUSD +
                 (Ex_rate_JPY_to_USD)*JPYtoUSD + (Ex_rate_PLN_to_USD)*PLNtoUSD,

     % EUR balance:
/*36*/    EURtoUSD + EURtoGBP + EURtoJPY + EURtoPLN $=
     (Ex_rate_USD_to_EUR)*USDtoEUR + (Ex_rate_GBP_to_EUR)*GBPtoEUR +
                  (Ex_rate_JPY_to_EUR)*JPYtoEUR + (Ex_rate_PLN_to_EUR)*PLNtoEUR,

     % GBP balance:
/*37*/    GBPtoUSD + GBPtoEUR + GBPtoJPY + GBPtoPLN $=
     (Ex_rate_USD_to_GBP)*USDtoGBP + (Ex_rate_EUR_to_GBP)*EURtoGBP +
                  (Ex_rate_JPY_to_GBP)*JPYtoGBP + (Ex_rate_PLN_to_GBP)*PLNtoGBP,

     % JPY balance:
/*38*/    JPYtoUSD + JPYtoEUR + JPYtoGBP + JPYtoPLN $=
     (Ex_rate_USD_to_JPY)*USDtoJPY + (Ex_rate_EUR_to_JPY)*EURtoJPY +
                  (Ex_rate_GBP_to_JPY)*GBPtoJPY + (Ex_rate_PLN_to_JPY)*PLNtoJPY,

     % PLN balance:
/*39*/    PLNtoUSD + PLNtoEUR + PLNtoGBP + PLNtoJPY $=
     (Ex_rate_USD_to_PLN)*USDtoPLN + (Ex_rate_EUR_to_PLN)*EURtoPLN +
                  (Ex_rate_GBP_to_PLN)*GBPtoPLN + (Ex_rate_JPY_to_PLN)*JPYtoPLN,

/*40*/    eplex_solver_setup(max(Z)),
/*41*/    eplex_solve(Z),
/*42*/    eplex_get(vars, Vars),
/*43*/    eplex_get(typed_solution, Vals),
/*44*/    Vars = Vals,

/*45*/    writeln("A ": A),
/*46*/    writeln("Final USD holdings (in MM)) ": Z),
/*47*/    write_positive("USDtoEUR", USDtoGBP),
/*48*/    write_positive("USDtoGBP", USDtoGBP),
/*49*/    write_positive("USDtoJPY", USDtoJPY),
/*50*/    write_positive("USDtoPLN", USDtoPLN),
/*51*/    write_positive("EURtoUSD", EURtoUSD),
/*52*/    write_positive("EURtoGBP", EURtoGBP),
/*53*/    write_positive("EURtoJPY", EURtoJPY),
/*54*/    write_positive("EURtoPLN", EURtoPLN),
/*55*/    write_positive("GBPtoUSD", GBPtoUSD),
/*56*/    write_positive("GBPtoEUR", GBPtoEUR),
/*57*/    write_positive("GBPtoJPY", GBPtoJPY),
/*58*/    write_positive("GBPtoPLN", GBPtoPLN),
```

```
/*59*/    write_positive("JPYtoUSD", JPYtoUSD),
/*60*/    write_positive("JPYtoEUR", JPYtoEUR),
/*61*/    write_positive("JPYtoGBP", JPYtoGBP),
/*62*/    write_positive("JPYtoPLN", JPYtoPLN),
/*63*/    write_positive("PLNtoUSD", PLNtoUSD),
/*64*/    write_positive("PLNtoEUR", PLNtoEUR),
/*65*/    write_positive("PLNtoGBP", PLNtoGBP),
/*66*/    write_positive("PLNtoJPY", PLNtoJPY),

/*67*/    writeln("Ex_rate_USD_to_EUR":Ex_rate_USD_to_EUR),
/*68*/    writeln("Ex_rate_USD_to_GBP":Ex_rate_USD_to_GBP),
/*69*/    writeln("Ex_rate_USD_to_JPY":Ex_rate_USD_to_JPY),
/*70*/    writeln("Ex_rate_USD_to_PLN":Ex_rate_USD_to_PLN),
/*71*/    writeln("Ex_rate_EUR_to_USD":Ex_rate_EUR_to_USD),
/*72*/    writeln("Ex_rate_EUR_to_GBP":Ex_rate_EUR_to_GBP),
/*73*/    writeln("Ex_rate_EUR_to_JPY":Ex_rate_EUR_to_JPY),
/*74*/    writeln("Ex_rate_EUR_to_PLN":Ex_rate_EUR_to_PLN),
/*75*/    writeln("Ex_rate_GBP_to_USD":Ex_rate_GBP_to_USD),
/*76*/    writeln("Ex_rate_GBP_to_EUR":Ex_rate_GBP_to_EUR),
/*77*/    writeln("Ex_rate_GBP_to_JPY":Ex_rate_GBP_to_JPY),
/*78*/    writeln("Ex_rate_GBP_to_PLN":Ex_rate_GBP_to_PLN),
/*79*/    writeln("Ex_rate_JPY_to_USD":Ex_rate_JPY_to_USD),
/*80*/    writeln("Ex_rate_JPY_to_EUR":Ex_rate_JPY_to_EUR),
/*81*/    writeln("Ex_rate_JPY_to_GBP":Ex_rate_JPY_to_GBP),
/*82*/    writeln("Ex_rate_JPY_to_PLN":Ex_rate_JPY_to_PLN),
/*83*/    writeln("Ex_rate_PLN_to_USD":Ex_rate_PLN_to_USD),
/*84*/    writeln("Ex_rate_PLN_to_EUR":Ex_rate_PLN_to_EUR),
/*85*/    writeln("Ex_rate_PLN_to_GBP":Ex_rate_PLN_to_GBP),
/*86*/    writeln("Ex_rate_PLN_to_JPY":Ex_rate_PLN_to_JPY).

/*87*/    write_positive(A, B):-
/*88*/    (B > 0 -> writeln(A:B); true).
```

The message is:

```
A  : 5.0
Final USD holdings (in MM))  : 6.0
EURtoJPY : 0.00843576360267486
JPYtoPLN : 0.928187069202315
PLNtoUSD : 3.4872
PLNtoEUR : 0.0352699276227836

Ex_rate_USD_to_EUR : 0.8353
Ex_rate_USD_to_GBP : 0.692
Ex_rate_USD_to_JPY : 91.89
Ex_rate_USD_to_PLN : 3.4872
Ex_rate_EUR_to_USD : 1.19717466778403
Ex_rate_EUR_to_GBP : 0.8283
```

```
Ex_rate_EUR_to_JPY : 110.03
Ex_rate_EUR_to_PLN : 4.181
Ex_rate_GBP_to_USD : 1.44508670520231
Ex_rate_GBP_to_EUR : 1.20729204394543
Ex_rate_GBP_to_JPY : 132
Ex_rate_GBP_to_PLN : 5.0414
Ex_rate_JPY_to_USD : 0.0108825769942322
Ex_rate_JPY_to_EUR : 0.00908843042806507
Ex_rate_JPY_to_GBP : 0.00757575757575758
Ex_rate_JPY_to_PLN : 3.795
Ex_rate_PLN_to_USD : 0.286763019041064
Ex_rate_PLN_to_EUR : 0.239177230327673
Ex_rate_PLN_to_GBP : 0.198357599079621
Ex_rate_PLN_to_JPY : 0.263504611330698
```

The result was so astonishing that Clever Young decided to check all balances:

The USD check:

```
  6+0+0+0+0 =  5+0+0+0+0.286763019041064*3.4872
```

gives:

```
 6 = 6
```

The EUR check:

```
  0+0+0.00843576360267486+0 = 0+0+0+
          0.239177230327673*0.0352699276227836
```

gives:

```
  0.008435763602674869 = 0.008435763602674869
```

The GBP check is trivial:

```
  0+0+0+0 = 0+0+0+0
```

The JPY check:

```
  0+0+0+0.928187069202315 = 0+110.03*0.00843576360267486+0+0
```

gives:

```
  0.928187069202315 = 0.9281870692023148458
```

The PLN check:

```
  3.4872+0.0352699276227836+0+0 = 0+0+0+3.795*0.928187069202315
```

gives:

```
3.5224699 = 3.5224699
```

So everything is O.K.! If the obtained solution is submitted to the currency dealer as *one order*, there is no need for waiting until some other currencies are accumulated to make a buy. However the problem remains: where to get the `5` MM of USD from in order to convert them to `6` MM USD. Clever Young, having an enterprising nature, made one more try, this time with `A=0`, i.e. with no initial USD at all. The result was as follows, the exchange rates messages being omitted:

```
A  : 0.0
Final USD holdings (in MM))  : 6.0
EURtoJPY : 0.0506145816160492
JPYtoPLN : 5.56912241521389
PLNtoUSD : 20.9232
PLNtoEUR : 0.211619565736702
```

with GBP not participating in the deal.

Clever Young just couldn't believe his eyes. To get 6 million USD just out of thin air, starting with nothing at all! He made a check of balances:

The USD check:

```
6+0+0+0+0 =  0+0+0+0+20.9232*3.4872
```

gives:

```
6 = 6
```

The EUR check:

```
0+0+0.0506145816160492+0 = 0+0+0+
        0.239177230327673*0.211619565736702
```

gives:

```
0.0506145816160492 = 0.0506145816160492
```

The GBP check is trivial:

```
0+0+0+0 = 0+0+0+0
```

The JPY check:

```
0+0+0+5.56912241521389 = 0+110.03*0.0506145816160492+0+0
```

gives:

```
5.56912241521389 = 5.56912241521389
```

The PLN check:

```
20.9232+0.211619565736702+0+0 = 0+0+0+3.795*5.56912241521389
```

gives:

```
21.13481956573 = 21.1348195657
```

So everything is O.K. one more time. However, Clever Young still wondered about getting the same 6 MM USD as before. Because those 6 MM USD were equal to the (somewhat arbitrarily) upper level in line `/*6*/` (domain definition for `Z`), he decided to make yet one more simulation, but this time with the upper level equal to 120 MM USD. he result was as follows wit the exchange rates messages being omitted:

```
A  : 0.0
Final USD holdings (in MM))  : 21.208105932626
EURtoUSD : 3.0
EURtoJPY : 1.08782427718034
GBPtoUSD : 3.5
JPYtoUSD : 100.0
JPYtoPLN : 19.6933052181531
PLNtoUSD : 40.0
PLNtoEUR : 17.091193302891
PLNtoGBP : 17.6449
```

with all currencies participating in the deal.
This time Clever Young got more than 21 million USD. Another check proves everything is O.K.: a new financial *Perpetuum Mobile*[17] has been invented! The only problem Clever Young is facing now is to find a currency dealer willing to accept such order from someone having no money at all[18].

[17] A hypothetical machine that produces more energy than it consumes, no matter how long it operates. Scientists agree that a Perpetuum Mobile is unfeasible: its existence would violate fundamental laws of physics.

[18] Well, the mathematics used by Clever Young was O.K., but his data was faulty. He inputed wrong numbers for the JPYtoPLN and PLNtoJPY exchange rates into Table 7.6.

7.8 Exercises

Building a factory

An enterprizing businessman decided to build a factory producing XYZ Gizmos, which - he believed - will be much in demand the moment they appear on the market. The first thing to do was to get financing. To secure a loan of 6 MM MU costs him 0.1 MM MU and took 4 months to arrange. The loan was at a bargain-basement price of 7% per year, to be repaid in 3 years. If defaulted, the balance after three years had to be repaid at 12% per year. Next, he bought a piece of property for 1 MM MU. To arrange the purchase took an unbelievable short time of 4 months. He began to pay property taxes on it immediately, which goes to pay for fire, police and roads, etc., to the amount of 0.01 MM MU per month. Next he had to get an environmental impact study done, which normally may take as long as 1.5 years. Unfortunately, he was challenged by an Environmental Group claiming that his property is the habitat of some very rare micro-rodents, now on the verge of extinction. The businessman had to defend himself in a court of law for 2 years, and finally settled with the Environmental Group to drop their charges by paying them 0.2 MM MU for resettling the entire micro-rodent population from his property. He also needed to place microphones and cameras in his grass to monitor if some remnants of the micro-rodent population did not remain there and are not disturbed by business activities. This contributed a yearly cost of 0.05 MM MU to the budget. Only then could he provide electric, water and sewage hookups for his property. It took 3 month at the cost of 0.2 MM MU. At the same time he started to design and pay for sidewalks, roads, drainage swales, green belts; because his property was near an established road, he had to pay to have it widened. This took 5 months and costed 0.2 MM MU. Next he hired an architect to do the drawings. They were submitted for approval, and rejected because of protests from the religious community of Boo-Woo Worshipers demanding a Room for Prayer at the factory for their Brothers/Sisters in Faith (in case they are employed at the factory), and because of protests from the Nursing Mother Association demanding a Nursing Mother Rest-and-Care Room at the factory for nursing mothers to be surely employed at the factory. Ultimately, after 3 months and multiple checks and revisions, at the overall cost of 0.5 MM MU, the drawings were approved by proper Authorities. Only then (i.e. 3 years after getting the loan, for which no nickel has been repaid yet) the businessman hired a general contractor who agreed

to build the factory in 7 months (at the overall cost of 3.2 MM MU), and at the same time the businessman started to buy several permits: building permit, electrical permit, plumbing permit, and had multiple inspections all along the way, each inspection costing him a fee; all the fees for permits and inspections amount to 0.3 MM MU; unfortunately, they have to be renewed at 3-months intervals. Once the factory was built, the businessman began outfitting it with necessary tools, machines and office equipment, which took 3 month and costed 1.2 MM MU). At the same time he started to staff his factory with employees; because of the large percentage of unemployable unemployed in the working age population, and because of the local LGBT Community accusing him of discriminatory hiring practices, it was quite a job and took 6 month at the cost of 0.1 MM MUs. Now, everything was ready to start producing those XYZ Gizmos. However, by chance entirely, the businessman visited a local World Market Mall outlet, and found that the Famous Eastern Global Company has already flooded the market with a large spectrum of various XYZ Gizmos at ridiculously low prices. In seeing that, the businessman suffered a fatal heart attack. Write a program to determine how long did it take and how much did it cost to arrive at this situation, assuming no payment of the purchase loan (neither the principal nor the interest rate) has ever been made and any money needed by the businessman above the initial loan of 6 MM has been granted by the loan provider but at `12%` per year. In order to make the time-structure of events evident, it has been shown in Figure 7.2.

Private investments

A private investor wishes to invest 15000 MU over the next year in two types of investment: investment `I1` yields `5%` and investment `I2` yields `9%`. The broker advises to allocate at least `30%` in `I1` and at most `55%` in `I2`. Besides, the investment in `I1` should be at least half the investment in `I2`. How to invest to maximize the yearly return?

PR campaign

The well-known party *All Things to All People* is misleading the electorate by a well-organized PR campaign on radio and television. Its PR budget is limited to 15000 MU per month. Each minute of radio hype costs 15 MU, and each minute of TV hype costs 300 MU. The party likes radio hype at least twice as much as it likes TV hype. Research indicates that it is not practical to broadcast party hype on radio more than 400 minutes

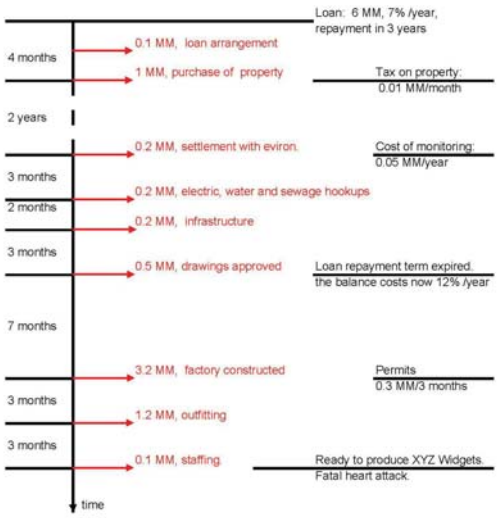


Figure 7.2: Time structure of business events

per month. The same research show that TV hype is 25 times more effective than radio hype. How to allocate the PR budget to maximize effectiveness of the PR campaign?

Assembling phones

An assembling line, consisting of four consecutive workstations, is used to assemble 2 phones, `Handy_1` and `Handy_2`. The assembly data is given by Table 7.7. The shift-wise maintenance of workstations consumes a given percentage of the overall 480 minutes work-time on a shift. Write a program to determine the optimum numbers of `Handy_1` and `Handy_2` phones produced at a shift that will minimize the overall idle time for a shift.

Station number	Assembly time in minutes per unit for `Handy_1`	Assembly time in minutes per unit for `Handy_2`	Daily maintenance in % of 480 minutes
l	6	4	10
2	4	6	12
3	5	5	14
4	7	8	16

Table 7.7: Assembly line data

Homes and apartments

The Lotus Point Condo Project will contain both homes and apartments. The site can accommodate up to 10.000 dwelling units. The project must contain a recreation project: either a swimming-tennis complex or a sailboat marina, but not both. If a marina is built, the number of homes in the project must be at least triple the number of apartments in the project. A marina will cost 1.2 MM MU, and a swimming-tennis complex will cost 2.8 MM MU. The developers believe that each apartment will yield revenues with an net present value (NPV) of 48000 MU, and each home will yield revenues with an NPV 46000 MU. Each home (or apartment) costs 40000 MU to build. Write a program to maximize profits.

Personal computers

Orange Co owns four production plants at which personal computers are produced. The Company can sell up to 20.000 computers per year at a price of 3500 MU per computer. For each plant the production capacity, the production cost per computer, and the fixed cost of operating a plant for a year are given in Table.

Write a program to determine how *Orange Co* can maximize its yearly profit from computer production.

Constructing a bridge

For the construction of a new bridge over the Large River a financing plan has to be established. Table 7.9 gives the estimated cost over the 6 years of construction. The City of Riverside plans to raise the funds needed to pay these costs by issuing bonds. Such a bond is valid up to 6 years. It can be taken out every 1st of January and is due on the 31st December of the year that it is due—the validity period is fixed beforehand. Of

Plant number	Production capacity	Plant fixed cost (MM MU)	Cost per computer (MU)
1	10000	9	1000
2	8000	5	1700
3	9000	3	2300
4	6000	1	2900

Table 7.8: Computer production data

course, interest has to be paid on bonds when they are due, depending on how long they are valid, see Table 7.9[19]. Money that is not used for construction can be invested at the National Bank at an interest rate of `6.8%` annually. Write a program to find out how many bonds to which terms should be issued each year to keep the outstanding debts at the end as low as possible.

Year	Cost MM MU	Length of validity, years	Overall interest rate, %
1	20	1	7
2	17	2	15
3	23	3	23
4	24	4	32
5	25	5	41
6	21	6	50

Table 7.9: Construction costs each year and interest rates for bonds

Buses

Two Bus Depots (D1 and D2) are dispatching buses to four Bus Stations (S1, S2, S3 and S4). Table 7.10 shows the distances between the depots and stations, the number of buses available at the depots and the demands of the bus stations. Write a program allocating buses from depots to stations so as to minimize the overall distance traveled between depots and stations.

[19] This exercise is from ftp.math.tu-berlin.de/pub/Lehre/LinOpt/WS09/linoptWS09-08.pdf

Depot	Station				Buses available
	S1	S2	S3	S4	
D1	15	12	10	17	100
D2	5	18	24	7	150
Demand	40	65	45	60	

Table 7.10: Bus allocation data

Farmland management

A farmer can choose to grow wheat or corn in his fields, each crop produces a different yield per hectare but also requires a different amount of time for its care[20]. There is a limit to the maximum number of working days the farmer has available for these crops. Write a program to determine the maximum yield achievable from his 100 hectares and 40 working days. The yield of wheat pro acre is 2.5, while that of corn is 3.5. The time necessary for cultivating wheat compared with that for corn is 1:2.

Paying bills on time

E.J.Korvair Department Store has 10000 MU in available cash.[21] At the beginning of each of the next six month, E.J. will receive revenues and pay bills as shown in Table 7.11:

Month	Revenues (in MU)	Bills (in MU)
July	10000	50000
August	20000	50000
September	20000	60000
October	40000	20000
November	70000	20000
December	90000	10000

Table 7.11: Revenues and bills for for six months

It is clear that E.J. will have a short-term cash flow problem until the store receives revenues from the Christmas shopping season. To solve this problem, E.J. must borrow money.

[20]This exercise is from http://www.ifcomputer.com/IFProlog/

[21]This exercise is from [Winston-94].

At the beginning of July, E.J. may take out a six-month loan. Any money borrowed for a six-month period must be paid back at the end of December along with `9%` interest (early payback does not reduce the the interest cost of the loan). E.J. may also meet cash needs through month-to-month borrowing. Any money borrowed for a one-month period incurs an interest cost of `4%` per month. Write a program to determine how E.J. can minimize the cost of paying its bills on time.

Loan policy

The *Famous Bank* is in the process of designing a loan policy for maximum 12 million MU. Table 7.12[22] provides pertinent data about types of loans available at the bank.

Type of loan	Interest rate	Bad-debt ratio
Personal	0.140	0.1
Car	0.130	0.07
Home	0.120	0.03
Farm	0.125	0.05
Commercial	0.100	0.02

Table 7.12: Loan types data

It is assumed that bad debts are unrecoverable and produce no interest revenue. Competition with other financial institutions requires that the bank allocate at least `40%` of the funds to farm and commercial loans. To assist the housing industry in the region, home loans must equal at least `50%` of the personal, car and home loans. The bank also has a stated policy of not allowing the overall ratio of bad debts on all loans to exceed `4%`. Write a program to maximize the net return of the *Famous Bank*, i.e. the difference between interest revenue and lost bad debts.

Healing the *No Symptoms Disorder*

Researchers at the famous *BigPharma* company, working around the clock to make a pill for every ill, have eventually designed a breakthrough habit-forming drug (*BHFD*), which does not generate any pleasant sensations and has no proven healing effects, but - after a few usages - creates a formidable craving for more and more, which - if not satisfied - is the source of acute discomfort bordering on suffering, but if satisfied leads

[22] This exercise is from [Taha-08].

to a number of degenerative illnesses. The *BigPharma* Board of Directors had analyzed in depth a number of possible strategies to market this stuff and had finally decided to sell it for supposedly healing the invented (by their marketing people) disease named *No Symptoms Disorder* (*NSD*). The marketing people have described *NSD* as a fatal-outcome disease to be endemic in *Normal People*, i.e. people who seem to be quite healthy, enjoy their life, family and work, lead a healthy life style with no cigarettes, recreational drugs or alcohol, devote some time to risk-free sporting activities, steer clear of high-carb low-fat nutrition and prefer natural saturated fat food over industry-manufactured concoctions, avoid ridiculous expenses and stressful occupations, are strong proponents of various natural healing methods (including legally banned hydrotherapy cures designed by Sebastian Kneipp). Now the job was to convince *Normal People* they need pharmaceuticals to treat their disorder, the best being obviously *BHFD*. Therefore *Main-Stream Media*, generously financed by *BigPharma*, started a hysterical campaign highlighting all *No Symptoms Disorder* fatalities, usually forgetting to mention the rather advanced age of those who died, but praising the healing-power of *BHFD*. This has been supported financially by the *Department of Longevity* of the *World Institute for Wellness*, worried about the constantly rising number of ageing *Normal People*, being a drain on all pension schemes and not contributing enough to any taxing schemes. The cynical quotation from one of its high-level functionaries: *"How can you control a population if you don't keep them medicated?"* was quickly and thoroughly swept under the carpet.

To use its monopoly on *BHFD*, *BigPharma* started to work on *BHFD* technologies, aiming at producing *BHFD* pills, suppositories, syrups, vaccines and patches, so as to satisfy the preferences of a wide range of customers. However, the production of all those *BHFD* articles was hampered by some constraints:

- The *Basic Raw Material* (*BRM*) for *BHFD* turned out to be an extract from some tropical plant found only in the *Famous Tropical (*Forest), which - for the time being - could be harvested at no more than 1000 kg monthly.
- The production of pills and suppositories run - for the time being - on the same production line, which constraints the overall monthly output of pills and suppositories to 1000 standard packages;
- The production of patches needed a special textile fabric which was

available up to 10 m^2 monthly;

- The production of vaccines and syrups depended upon the same solvent available up to 100 liters monthly.

Write a program determining the monthly production volume of all *BHFD* articles in order to maximize *BigPharma* profit provided that:

- for producing a single pill package 0.01 kg *BRM* was needed, for producing a single suppository package 0.015 kg *BRM* was needed, for producing a single syrup bottle 0.01 kg *BRM* was needed, for producing a single vaccine package 0.02 kg *BRM* was needed, and for producing a single package of patches - 0.012 kg *BRM* was needed,;
- for producing a single package of patches 10 cm^2 special textile fabric was needed;
- for producing a single syrup container 0.001 liters, and for producing a single package of vaccines 0.01 liters of solvent was needed.

and that:

- selling a single pill package gives profit equal to 1.5 MU;
- selling a single suppository package gives profit equal to 1.8 MU;
- selling a single sirup bottle gives profit equal to 2.0 MU;
- selling a single vaccine package gives profit equal to 3.5 MU;
- selling a single package of patches gives profit equal to 2.5 MU.

Afterword

"Well, in our country," said Alice, still panting a little, "you'd generally get to somewhere else – if you ran very fast for a long time, as we've been doing." "A slow sort of country!" said the Queen. "Now, here, you see, it takes all the running you can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!"
Lewis Carroll, *"Through the Looking Glass"*

In spite of the territory covered in this book in what seems to be a fast and long run, we have barely scratched the surface of ECL^iPS^e CPS.

The ECL^iPS^e platform is offering to knowledgeable users much, much more than could be described in this elementary introduction, which concentrated on basic ideas only. The doubting Reader is kindly asked to have a close look at all the standard predicates listed in the *Alphabetical Predicate Index*, see Figure 5. A number of advanced topics has been presented rather cursorily (e.g. controlling search), a number of important features has not been presented at all like graphics, and interfacing with procedural languages. Finally the important subject of finding *sub-optimum solutions* by means of heuristics like *local search* including *Hill Climbing*, *Tabu Search* and *Simulated Annealing*, has simply been omitted.

The interested Reader may find more about it on the continuously updated ECL^iPS^e website and on the ECL^iPS^e discussion forum.

The books aim was educating ECL^iPS^e (and CLP) novices. The Author always believed that education in anything is not like filling a vessel, but rather like igniting a fire. It's up to the Reader to judge to what extend this book meets those claims. However, it's up to the Author to state that writing this book was a source of personal satisfaction and enjoyment.

Glossary

Absurdoland - a totally fictitious country, being a place where unusual, unbelievable and extraordinary things are happening, as recounted in many problem stories of this book.

Accumulator - an initially empty list (or zero variable) to which head (or values) are added on each recursion of the predicate containing the accumulator.

AI - *Artificial Intelligence*.

Algorithm - a solution method guaranteeing success.

Anonymous variable - a variable which does not need to be grounded.

AoA - Activity on Arcs network: uses directed arcs to represent activities.

Appearance of variable - the presence of some *predicate variable* in many places of the body of a rule, or in the same predicate in other rules.

Argument - a *variable* associated with a predicate.

Argument - an *atom* associated with a structure.

Argument - free - an argument with no assigned value from its domain. item[Argument - ground] - an argument with assigned value from its domain.

Arity - the number of arguments to a term. The notation *Name/Arity* is used to specify a term by giving its name and arity.

Arity - of predicate - number of variables in a predicate.

Arity - of relation - number of variables in Cartesian product.

Array - a generalization of variable, capable of storing multiple values as vectors or matrices.

Artificial Intelligence - a branch of computer science trying to emulate human performance usually deemed to be intelligent.

Assert - to save a grounded predicate in a database.

Assigning - pairing elements of some set with elements of another sets so as to fulfill *belongness constraints*.

Atom - a Prolog/CLP non-numerical (i.e. logical or symbolic) *constant* with zero *arity*, presented by a sequence of characters starting with a lower case letter or by any sequence of characters put between double quotes or single quotes.

Backtracking - the process of *degrounding* the recently *grounded variable* followed by making the *contracted state* equal to one corresponding to the nearest *choice point*.

Backtracking - Forward Checking - *backtracking* in CLP, initiated when, as the result of the last variable grounding, an empty *domain* appears.

Backtracking - Looking Ahead - *backtracking* in CLP, initiated when, as the result of the last variable grounding, for the next search step the appearance of an empty *domain* is predicted.

Backtracking - Standard - *backtracking* in Prolog, initiated when a grounded predicate fails.

Belongness constraint - a constraint stating that some items belong together as parts of some entity.

Body - see *rule*.

Boolean - see *Variable - Boolean*.

Built-in - see *Predicate - built-in*.

Branch and bound - a form of backtracking search with the additional constraint to find a state that minimizes some *objective function*.

Cartesian product - of n sets - the set of all possible *n-tuples*, each element of which belongs to a different set.

CCOP - continuous constraint *optimization* problem.

CCSP - continuous constraint satisfaction problem.

Choice point - a predicate having at least one variable with value not yet grounded to some value of its *domain*, serving as point of return when - during search - a recently *grounded* variable results in *failure*.

Clause - a basic building block of Prolog and CLP programs, being either a *fact* or a *rule*.

Closed World Assumption - - the assumption that the *head* of a not satisfied *rule* is considered to be false.

Combinatorial variable - see *Variable - discrete*.

Combinatorial explosion - - the effect of rapid *state space* growth caused by increasing number of *decision variables*.

Command mode - an execution mode for CLP programs, run in DOS-like command window.

Compound interest - addition of interest to the principal before next interest is calculated.

Configuring - selecting, from some sets, subsets fulfilling constraints of belongness and compatibility constraints.

Conjunction - an operation on logical operands that produces a true value if and only if all of its operands are true.

Consistency techniques - algorithms making a set of integer variables, defined by names and *domains*, to fulfill a set of constraints by properly adjusting the initial variable *domains*. Used for *constraint propagation* in CLP. Consistency techniques are not complete *inference method*.

Constant - an atom (starting with a small letter), an integer number, a real number, a list or array of atoms, of integer numbers, of real numbers.

Constraint - a *relation* over a set of *domain* variables, which constricts the combination of *domain* values to which the variables may be *grounded*. A constraint represents conditions which these variables must satisfy.

Constraint - active - a constraint that could initiate search in case it is either not consistent or not all of its variables have been grounded.

Constraint - consistent - a constraint with variables grounded in a way the constraint is satisfied.

Constraint - passive - a constraint that is used as a test in case all its variables are grounded.

Constraint propagation - a process initiated by making a constraint consistent by grounding its variables.

Constraint propagation in CLP - a process by which the value of a grounded constraint variable is modifying *domains* of relevant variables so that their constraints are satisfied. The modification consists of removing those values from all *domains* that violate the constraint. Constraint propagation in CLP may be performed without search, but it is s not a complete *inference method*.

Constraint propagation in Prolog - a process by which the grounding done by *unification* for a constraint variable in some rule is *spread* i.e. repeated for all instances of this variable in the body of this rule and for all other instances of the predicates in other rules. Constraint propagation in Prolog cannot be performed without search.

Continuous variable - see *Variable - continuous*.

COP constraint optimization problem.

Critical path - the shortest sequence of projects activities starting from the initial activity and ending with the final activity.

CSP - constraint satisfaction problem.

Decision variable - see *Variable - decision*.

Declarative programming - a programming paradigm based on describing problems to be solved, rather than describing how to go about solving them.

Decomment - remove `%` comment lines.

Degrounding a variable - making a *grounded variable* free.

Delayed goals - goals that could not have been instantiated because of insufficient information.

Direct enumeration - see *Search - exhaustive*.

Discrete variable - see *Variable - discrete*.

Disequation - `?Term1 \== ?Term2` - succeeds if `Term1` and `Term2` are not identical terms, `?ExprX \= ?ExprY` - succeeds if `ExprX` is not equal to `ExprY` where `Expr` - an integer arithmetic expression.

Disjunction - an operation on logical operands that produces a value of true if at least one of its operands is true.

Domain, continuous - a range of *values* a continuous *variable* may take.

Domain. discrete - a set of *values* a discrete *variable* may take.

Domain, finite - see *Domain. discrete*.

eplex - a solver for *LP*, *IP* and *MP* problems, integrated into ECL^iPS^e.

Exhaustive search - see *Search - exhaustive*.

Fact - a *predicate* with no *arguments* or with all arguments grounded, considered to be satisfied. Facts are used to express constraints.

fail - a predicate that always fails, used in Prolog to force backtracking in order to find alternate solutions.

Failure - a grounded predicate is not satisfied, i.e. results in a false clause.

Feasible solution - any solution satisfying all constraints of the problem.

Function - a special case of relation for n sets of variables. A function assigns a unique element (or none) of one set (the "output" set) to each *n-1 tuple* of the remaining *n-1* sets (the "input" sets).

Forward checking - initiate backtracking for failures to be unavoidable in the next search step.

Free - argument - see *Argument - free*.

Free predicate - see *Predicate - free*.

Free variable - see *Variable - free*.

Function - a special case of relation for n sets of variables. A function assigns a unique element (or none) of one set (the "output" set) to each *n-1 tuple* of the remaining *n-1* sets (the "input" sets).

FS - feasible state, see *state - feasible*.

FST - feasible state trajectory, see *state trajectory, feasible*.

Functor - a synonym for *predicate*, not used in this book.

Gantt chart - a graphical representation of resource allocation over time for concurrently performed tasks.

General Problem Solver - the ancestor of AI computer programs which separate its knowledge of problems (rules represented as input data) from its strategy of how to solve problems (a generic solver engine).

Goal - a query initiating the logical flow of a Prolog/CLP program. Goals have a boolean result of yes or no, succeed or fail.

Grounded predicate - see *Predicate - grounded*.

Grounded variable - see *Variable - grounded*.

Grounding of variable - assigning to the *variable* a *value* from its *domain*. See also *labeling*.

Head - see *rule*.

Heuristic - a problem-solving approach with no guarantee of success.

Identity of variables - see *variables - identity*.

Imperative programming - a programming paradigm based on declaring algorithms needed to solve problems.

Implication in logic - a logical operation with two variables called *Conclusion* and *Condition*. It returns false, if and only if the Conclusion is true, and the *Condition* is false.

Implication in rules - a logical operation with two variables called *Conclusion* and *Condition*. It returns false, if and only if the *Conclusion* and *Condition* have opposite logical values.

Inconsistency - the appearance of an empty *domain* for some *variable* in the process of *constraint propagation*.

Inference methods - methods used to discover information implied by data.

Inference methods - complete - methods guaranteeing that if a solution for a CSP exists, it will be determined.

Inference methods - incomplete - methods that sometimes may not manage to find a solution for a CSP, although such solution exists.

Inference system - part of the Prolog or CLP compiler used to infer conclusions from knowledge bases.

Infix notation - predicate names are written in between arguments.

Input of predicate - a variable determined outside the predicate in which it appears.

Input of program - a variable determined by the user of the Prolog or CLP program in which it appears.

Instantiated - a variable to which a predicate or list has been assigned.

Integer Programming - a set of numerical technique for the optimization of integer-valued linear objective functions subject to integer-valued linear equality and/or inequality constraints.

IP - see *Integer Programming*.

Iteration - applying a predicate repeatedly for consecutive data in a loop.

Job - a series of *tasks* to be performed in some order.

Job-shop - a specific environment of scheduling problems with a number of jobs consisting of tasks performed concurrently on the same set of machines.

Knowledge - an understanding of a subject needed to make rational decisions.

Knowledge base - a text file containing (in proper syntactic form) the entire knowledge needed by the inference system to solve the decision problem under consideration.

Labeling - consecutively grounding a set of variables to their domain values.

Linear Programming - a set of numerical technique for the optimization of real-valued linear objective functions subject to real-valued linear equality and/or real-valued linear inequality constraints.

List -a *tuple* starting with left-hand square bracket (`[`) and ending wit right-hand square bracket (`]`).

Logic - a science about what follows from what.

Logical values - constants `true` or `false`.

Logical variable - a *variable* that can be *grounded* to a logical value.

Looking ahead - initiate backtracking for failures to be unavoidable in the second next search step.

LP - see *Linear Programming*.

Makespan - the difference between start time and finish time for a sequence of jobs or tasks.

Mathematical programming problems - *linear programming* problems, *integer programming* problems or *mixed programming* problems.

Mixed Programming - a set of numerical technique for the optimization of real-valued linear objective functions subject to real-valued and integer-valued linear equality and/or linear inequality constraints.

MM - an abbreviation that represents one million (M stands for "a thousand", MM being "thousand thousands".).

Mode of variable - the role played by the variable as argument of built-in predicate (input, output,input instantiated, input grounded)

Modelling - translating verbal problem statements into Prolog or CLP programs.

MP - see *Mixed Programming*.

MU - Monetary Unit, a fictitious currency unit used throughout this book.

Name of variable - any series of letters starting with a capital letter or underscore.

Name of predicate - any series of letters and symbols, starting with a non-capital letter.

Neighbourhood constraints - constraints determining the position of each element of some set with respect to the remaining elements.

Nested predicate - see *predicate - nested*.

Non-numerical - logical or symbolic.

Number - an integer constant (like 9 or 123) or floating-point constant (like 3.14, 2.79) with decimal points only.

Objective function - a function of *decision variables* to be optimized while solving *COP*'s or *CCOP*'s.

Operation Research - an interdisciplinary system science technology that uses mathematical modeling, statistical analysis, and mathematical optimization to arrive at optimal or near-optimal solutions to complex decision-making problems.

Optimization - the best way to utilize limited resources (money, production capacity,time). Finding the best solution from all *feasible solutions*.

OS - optimum state, see s*state - optimum*.

OST - optimum state trajectory, see s*state - optimum trajectory*.

Output of predicate - a variable determined by the predicate in which it appears.

Output of program - a variable determined by the Prolog or CLP program in which it appears.

Permutation - any arrangement of a tuple of different values into a particular order.

Precedence constraint - a constraint stating the relative order of some items in space or in time.

Predicate - a *relation* between ordered *variables* referred to as arguments, declared by naming it, naming their arguments, arranging their order and defining them either by other predicates or by declaring their domains. Predicates are used to express constraints.

Predicate - built-in - a predicate designed by Prolog/CLP language designers and made available for ECL^iPS^e users.

Predicate - elementary - built-in predicates of elementary functionality provided by libraries `ic` and `branch_and_bound`, usually having no more than a single input *list*.

Predicate - free - a predicate with some free variables.

Predicate - global - built-in predicates of advanced functionality provided by libraries `ic_global`, `ic_cumulative`, `ic_edge_finder`, `ic_edge_finder3`, usually having many input *lists*.

Predicate - grounded - a predicate with all variables grounded.

Predicate - nested - a predicate that serves as argument of another predicate.

Predicate - private - a predicate defined by user, with a name different from names of ECL^iPS^e built-ins.

Predicate - satisfied - a grounded predicate that is a true clause.

Predicate - unsatisfied - a grounded predicate that is a false clause.

Prefix notation - predicate names are written in front of its arguments.

Procedural programming - see *Imperative programming*.

Propagation - see *Constraint - propagation*.

Q.E.D. - an initialization of the Latin phrase *Quod errat demonstrandum* meaning *what has been proved*; an abbreviation used to conclude proofs or arguments.

Quadratic programming - a set of numerical technique for the optimization of quadratic objective functions subject to real-valued linear equality and/or real-valued linear inequality constraints.

Query - the head of some rule, invoked to be satisfied. Queries are used to activate Prolog/CLP programs.

Reification - associating a *constraint* with a *Boolean* variable *grounded* to `1` if the constraint is satisfied, and *grounded* to `0` otherwise.

Recursion - defining a predicate by applying it as part of its definition.

Recursion- tail - the last thing a tail-recursive predicate does is to call itself.

Regrounding a variable - assigning to a *degrounded variable* a new (untested) *value* from its *domain*.

Relation - a subset of the *Cartesian product* of some sets.

Resource - anything necessary for performing some action. item[Resource constraint] - a constraint limiting the overall amount of resource available.

Retract - to remove a grounded predicate from a database.

Rule - a conditional statement with the meaning: *If conditions are true, then conclusion is true*, the *conclusion* being an ungrounded predicate referred to as the *head* of the rule, the *conditions* being a conjunction of grounded or ungrounded predicates referred to as the *body* of the rule. Rules are written in the form *conclusion:- conditions*, the symbol (*:-*) being a convenient way of writing the rule implication arrow (←).

Satisfied - having the logical value *true*.

Scheduling - ordering elements of some set so as to fulfill *precedence constraints* and *resource constraints*.

Search - the following sequence of steps: 1)grounding a selected *decision variable*, and 2)testing the satisfaction of relevant constraints: if some constraint fails, *backtracking* is initiated. Otherwise another decision variable is selected and grounded. Search is a complete *inference method*.

Search and propagation - a process of *search*and *propagation* performed alternately.

Search - exhaustive - generating consecutively all states of the state space and testing whether they satisfy all constraints of the problem.

Search space - see *state space*.

Solver - software for solving optimization problems.

Sequencing - ordering elements of some set so as to fulfill precedence constraints.

Semantics - meaning of symbols and clauses of a language.

Spreading a variable value - the grounding done for a predicate variable in some rule is repeated by *unification* for all instances of this variable in the body of this rule and for all other instances of the predicates in other rules.

State - complete - any grounding of domain values to all *decision variables*.

State - contracted - any grounding of domain values to some *decision variables*.

State - feasible - such assignment of domain values to decision variables that satisfies all constraints.

State - optimal - a feasible state for which some *objective function* achieves its optimum.

State - optimal trajectory - a feasible state trajectory, for which some *objective function* achieves its optimum.

State space - all groundings of domain values to all *decision variables*.

State space - contracted - all groundings of domain values to some *decision variables*.

State trajectory, feasible - a sequence of feasible states leading from some initial feasible state to some final feasible state of the state space.

State trajectory, optimal - a sequence of feasible states leading from some initial feasible state to some final feasible state of the state space, while optimizing some cost function.

State - unfeasible - an grounding of domain values to decision variables for which at lest one constraint is unsatisfied.

String - any sequence of characters enclosed in double quotes.

Structure - a tuple of a fixed number of atoms with a name.

Success - a grounded predicate is satisfied, i.e. corresponds to a true clause.

Syntax - feasible arrangements of symbols of a language.

Tail-recursion - recursive rules with the *head* calling itself at the end of the *body*.

Task - an elementary indivisible activity recognized in a job.

Tautology - a statement that is true just by the meaning of the words in it.

Term - a basic data type in Prolog and CLP: an *atom*, a *variable*, a *number*, a *predicate*, a *structure*, a *list*.

Timetabling - pairing elements of some set with elements of a set of time intervals.

top. - the main *query* used throughout this book.

Tuple - an ordered sequence of elements.

Unification - the process of matching elements in a way that makes two syntactically equivalent terms (most often predicates) equal

Unsatisfied - having the logical value *false*.

Value - a *constant*

Value choice heuristic - a heuristic that determines the order of domain values used for grounding variables while searching.

Value spreading - see *Spreading a variable value*.

Variable - an unknown that has a *name* staring with a small letter or underscore and a *domain*.

Variable - anonymous - a variable that do not need to be grounded.

Variable - Boolean - a variable with domain [0,1].

Variable - combinatorial - see *Variable - discrete*.

Variable - continuous - a variable with continuous domain.

Variable - decision - a variable used to formulate CSP, COP, CCSP and CCOP.

Variable - degrounded - a *grounded* variable that has been made again *free*.

Variable - degrounding - making a grounded variable free while restoring its value to its domain.

Variable - discrete - a variable with finite domain.

Variable - free - a variable with no assigned value from its domain.

Variable - grounded - a variable with assigned value from its domain.

Variable - grounding - assigning to a free variable a value from its domain.

Variable - identity - the sameness of a predicate variable is assured by assigning to it the same name in a rule and by assigning it to the same argument position in the predicate in other rules.

Variable - meaning - to make Prolog/CLP programs understandable, the meaning of all variables used should be precisely defined.

Variable - regrounded - a variable to which is assigned at least a second value from its *domain* in turn.

Variable - regrounding - assigning to a degrounded variable at least a second value from its *domain*.

Variable choice heuristic - a heuristic that determines the order of variables to be grounded while searching.

Bibliography

[Aggoun-93] Aggoun, A. and N. Baldiceanu, *Extending CHIP in Order to Solve Complex Scheduling and Placement Problems*, Mathl. Comput. Modelling, Vol. 17, No. 7, pp. 57-73, 1993.

[Apt-03] Apt, K. R., *Principles of Constraint Programming*, Cambridge University Press, Cambridge 2003.

[Apt-07] Apt, K. R. and M. G. Wallace, *Constraint Logic Programming using* ECL^iPS^e, Cambridge University Press, Cambridge 2007.

[Ahriz-13] Ahriz, H., *Z models for constraint and optimisation problems* http://www.comp.rgu.ac.uk/staff/ha/ZCSP/, 2013.

[Baker-09] Baker, K. R. and D. Trietsvch, *Principles of Sequencing and Scheduling*, John Wiley and Sons, Hoboken, 2009.

[Baldiceanu-94] Baldiceanu, N. and Contejean E., *Introducing Global Constraints in CHIP* Math.Comput.Modelling, Vol. 20, No. 12, pp. 97-123, 1994.

[Baldiceanu-10] Baldiceanu, N., *Global Constraints Catalog* http://www.emn.fr/x-info/sdemasse/gccat/, 2010.

[Baptiste-95] Baptiste, Ph., C. Le Pape and W. Nuijten, *Constraint-Based Optimization and Approximation for Job-Shop Scheduling*, Proceedings of the AAAI-SIGMAN Workshop on Intelligent Manufacturing Systems, IJCAI-95, Montreal, Canada, 1995.

[Baptiste-01] Baptiste, Ph., C. Le Pape and W. Nuijten, *Constraint-Based Scheduling*, Kluwer Academic Publishers, Boston, 2001.

[Bartak-10] Barták, R., *On-Line Guide to Prolog Programming*, http://kti.mff.cuni.cz/ bartak/prolog/, 2010.

[Bartak-10a] Barták, R., *On-Line Guide to Constraint Programming*, http://kti.mff.cuni.cz/ bartak/constraints/, 2010.

[Bellman-61] Bellman, R., *Adaptive Control Processes*, Princeton University Press, Princeton 1961.

[Bizam-75] Bizám, G. and J. Herczeg, *Games and logic*, A Polish translation (*Gry i logika)* from the Hungarian original, Wydawnictwa Naukowo-Techniczne, Warszawa, 1975.

[Brachman-04] Brachman, R. J. and H. J. Levesque, *Knowledge Representation and Reasoning*, Elsevier - Morgan Kaufmann, Amsterdam, 2004.

[Bratko-01] Bratko, I., *Prolog programming for Artificial Intelligence*, Addison Wesley, Harlow, 3rd ed., 2001.

[Carlier-89] Carlier, J. and E. Pinson, *An algorithm for solving the job-shop problem*, Management Science, 35(2), 164-176, 1989.

[Clark-84] Clark K.L. and F.G. McCabe. *Micro-PROLOG: programming in logic*. Prentice Hall, 1984.

[CHIP-07] COSYTEC Complex Systems Technologies. *CHIP V5*. http://www.cosytec.com/.

[Dechter-03] Dechter, R., *Constraint Processing*, Morgan Kaufmann, San Francisco, 2003.

[Duncan-90] Duncan, T., *Scheduling Problems and Constraint Logic Programmingf: A Simple Example and its Solution*, AIAI-TR-120,AIAI University of Edinburgh, 1990.

[ECLiPSe-10] CISCO, *The ECLiPSe Constraint Programming System*, http://www.eclipse-clp.org/, 2010.

[Edmund-98] Edmund, D., *Bob's Shish Kebabs*, Dell Logic Puzzles, June, 1998, p. 24.

[Edmund-10] Edmund, D., *Dough Edmund's Puzzle Page*, http://www.eclipse-clp.org/eclipse/examples, 2010.

[French-82] French, S., *Sequencing and Scheduling: An Introduction to the Mathematics of Job-Shop*, Ellis Horwood, 1982.

[Friedman-Hill-03] Friedman-Hill, E., *Jess in Action. Rule-Based Systems in Java*, Manning Publ. Co., Greewich, 2003.

[From-94] From, A., *Programmation par Contraintes*, Addison-Wesley, Paris, 1994.

[Gołuchowski-07] Gołuchowski, J., *Technologie informatyczne w zarządzaniu wiedzą w organizacji*, (Information Technologies in Kowledge Management), Wydawnictwo Akademii Ekonomicznej, Katowice, wyd. 2, 2007.

[Graham-78] Graham, R. L., *Combinatorial Scheduling Theory*, In L. A. Steen (Ed.), *Mathematics Today: Twelve Informal Essays*, Springer-Verlag, New York, 1978, pp. 183-211.

[Hansen-03] Hansen, J., *Constraint Programming versus Mathematical Programming*, Orbit, Vol. 3, Feb. 2003.

[Hooker-00] Hooker, J.N., *Logic-Based Methods for Optimization: Combining Optimization and Constraint Satisfaction*, John Wiley and Sons, New York, 2000.

[Hooker-07] Hooker, J.N., *Integrated Methods for Optimization*, Springer, Pittsburgh, 2007.

[Hunt-13] Hunt, W., *The Stable Marriage Problem*, http://www.csee.wvu.edu/ ksmani/courses/fa01/random/lecnotes/lecture5.pdf, 2013.

[Jaffar-94] Jaffar, J. and M. J. Maher, *Constraint Logic Programming: A Survey*, Journal of Logic Programming, 1994, 19-20, str. 503-581.

[Kjellerstrand-13] Kjellerstrand, H., *Constraint Programming Blog*, http://www.hakank.org/ECLiPSe/, 2013.

[Legierski-06] Legierski, W., *Automated Timetabling via Constraint Programming*, J. Skalmierski CS, Gliwice, 2006.

[Lines-92] Lines, M.E., *Think of a number*, IOP Publishing Ltd, London, 1992.

[Luger-98] Luger, G. F. and Stubblefield, W.A., *Artificial Intelligence. Structures and Strategies for Complex Problem Solving*, Addison Wesley Longman, Inc, Harlow, 3rd ed., 1998.

[Mackworth-92] Mackworth, A.K., *Constraint Satisfaction*, W: Shapiro S.C. (Ed.) *Encyclopedia of Artificial Intelligence* John Wiley and Sons, New York, 1992.

[Marriott-98] Marriott, K. and P. J. Stuckey, *Programming with Constraints: An Introduction*, The MIT Press, Cambridge Mass., 1998.

[Michalewicz-07] Michalewicz, Z. and M. Michalewicz, *Puzzle Based Learning*, Proceedings of the 2007 AaeE Conference, Melbourne, 2007.

[Michalewicz-08] Michalewicz, Z. and M. Michalewicz, *Puzzle-Based Learning: Introduction to Critical Thinking, Mathematics, and Problem Solving*, Hybrid Publishers, Melbourne, 2008.

[Milano-04] Milano, M.,(ed.) *Constrained and Integere Programming. Towards a Unified Methodology*, Kluwer Academic Publishers, Boston, 2004.

[Morgan-08] Morgan, T., *Business Rules and Information Systems.Aligning IT with Business Goals*, Addison-Wesley, Boston, 4th Printing, 2008.

[Mozart/Oz-10] *Mozart/Oz multi-paradigm language*, http://www.mozart-oz.org/home/doc/fdt/node37.html, 2010.

[Muth-63] Muth, J.F. and G. L. Thompson, *Industrial Scheduling*, Prentice-Hall, Inc., Englewood Cliffs, 1963.

[Niederliński-99] Niederliński, A., *Constraint Logic Programming - from Prolog to CHIP*, Proceedings of the Workshop on Constraint Programming for Decision and Control,Editor J. Figwer, Institute of Automation, Silesian University of Technology, Gliwice, 1999, str. 27-34.

[Niederliński-06] Niederliński, A., *rmes - Rule- and Model-Based Expert Systems*, PKJS.com.pl, Gliwice, 2008.

[Poole-98] Poole, D., A. Mackworth and R. Goebel, *Computational Intelligence. A Logical Approach*, Oxford University Press, New York, 1998.

[Puget-08] Puget, J-F., *Constraint Programming: A Great AI Success*, In Prade, H. (Ed.), Proceedings of the 13th European Conference on Atrtificial Intelligence ECAI98, pp. 698-705. John Wiley and Sons, 1998.

[Ross-03] Ross, R. G., *Principles of the Business Rule Approach*, Addison-Wesley, Boston, 2003.

[Rossi-06] Rossi, F., P. van Beek and T. Walsh, *Handbook of Constraint Programming*, Elsevier, Amsterdam, 2006.

[Russel-03] Russel, S. and O. Norvig, *Artificial Intelligence. A Modern Approach*, Prentice Hall, II-nd edition, Upper Saddle Roiver, 2003.

[Schimpf-10] Schimpf, J., *Sudoku*, http://www.eclipse-clp.org/eclipse/examples, 2010.

[Schimpf-10a] Schimpf, J., *Liars*, http://www.eclipse-clp.org/eclipse/examples, 2010.

[Simonis-10] Simonis, H., *ECLiPSE ELearning Website*, http://www.eclipse-clp.org/eclipse/examples, 2010.

[Steen-83] Steen, L. A. (Ed.), *Contemporary mathematics. Twelve essays*, Polish translation, WNT, Warszawa, 1983.

[Szczygieł-03] Szczygieł, T., *Angle Packing Problems: Constraint Logic Programming versus Classical Approaches*, JSCS, Gliwice, 2003.

[Taha-08] Taha, H. A.,Operations Research: An Introduction, *Operations Research: An Introduction*, Prentice Hall, Upper Saddle River, 8th. Edition, 2008.

[Toth-02] Toth, P. and D. Vigo, *The Vehicle Routing Problem*, SIAM, Philadelphia, 2002.

[Tsang-95] Tsang, E., *Foundation of Constraint Satisfaction*, Academic Press, London 1995.

[van Hentenryck-89] van Hentenryck, P., *Constraint Satisfaction in Logic Pogramming*, The MIT Press, Cambridge Mass., 1989.

[van Hentenryck-99] van Hentenryck, P., *The OPL Optimization Programming Language*, The MIT Press, Cambridge Mass., 1999.

[Visual Prolog-98] *Visual Prolog 5.1.* Prolog Development Center, Copenhagen, 1998, http//www.pdc.dk/vip.

[von Halle-02] von Halle, B., *Business Rules Applied*, John Wiley and Sons, New York, 2002.

[Wagner-75] Wagner, H. M., *Principles of Operations Resaerch*, Prentice Hall, Englewood Cliffs, 1975.

[Wikipedia-13] *Stable marriage problem*, http://en.wikipedia.org/wiki/`Stable_marriage_problem`, 2013

[Williams-99] Williams, H. P., *Model Building in Mathematical Programming*, John Wiley and Sons, Chichester, 4th ed., 1999.

[Winston-94] Winston, W.L., *Operations Research*, Duxbury Press, Belmont, 3rd ed., 1994.

[Wirth-75] Wirth, N., *"Algorithms + Data Structures = Programs"*, Prentice Hall, Englewood Cliffs, 1975.

[Wojcik-05] Wójcik, B., *Szeregowanie zadań metodami CLP,(Scheduling with CLP)*, MSc thesis, Institute of Automation, Silesian University of Technology, Gliwice, 2005.

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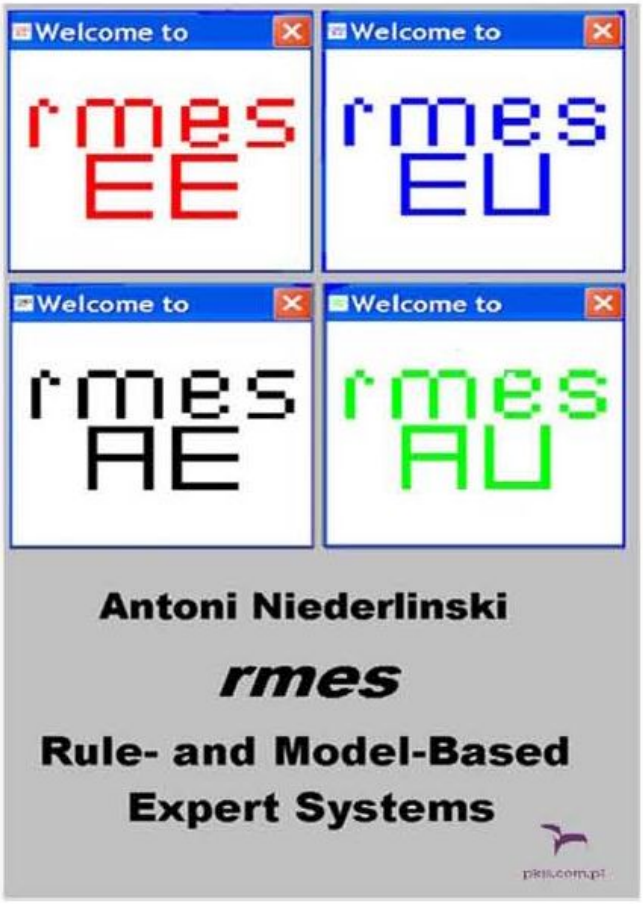

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